

# SPIA 586a: Machine Learning for Policy Analysis

## Week 5: Prediction in a Counterfactual Population

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<sup>1</sup>Some causal inference material drawn from: Matt Blackwell, Justin Grimmer, Jens Hainmueller, Erin Hartman, and Kosuke Imai.

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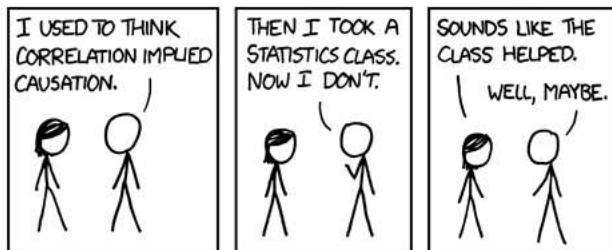
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- Long Run
  - ▶ target tasks  $\rightarrow$  data sources  $\rightarrow$  guiding values

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# Causation



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- Would sending police to this location lower crime?
- Does getting an A in this class help me get a job?

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# Key Pieces

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- Confounders (the common causes of treatment and outcome)

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- When it works though it can be a powerful view into the things that we care the most about.
- By convention we often care the counterfactual levels we care about **treated** and **control** and we often consider only binary **treatment variables** because continuous variables are often even more complicated!

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- When is this okay?
- The key trick of causal inference: finding sets of data where distribution of **observed** is equal to distribution of **counterfactual**.

# Secret Sauce: Separating Identification and Estimation

- **Identification**: How much can you learn about the estimand if you have an infinite amount of data? (i.e., do we have data from the right distribution to learn with?)
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- **Law of Decreasing Credibility (Manski)**: The credibility of inference decreases with the strength of the assumptions maintained

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$T_i$ : Unit assigned to:

$$T_i = \begin{cases} 1 & \text{Receive Aspirin} \\ 0 & \text{Receive Placebo} \end{cases}$$

$(T_i = 1)$

1-2

$(T_i = 0)$

PLACEBO

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$$Y_i(T_i) = \begin{cases} Y_i(1) & \text{Headache (or not) for unit } i \text{ with Aspirin} \\ Y_i(0) & \text{Headache (or not) for unit } i \text{ with placebo} \end{cases}$$

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$\tau_i$ : The treatment effect

$$\tau_i = Y_i(1) - Y_i(0)$$

Illustrated potential outcomes here and later courtesy of Erin Hartman

# Causal Inference is a Missing Data Problem

Example: Aspirin's Impact on Headaches

| Patient<br>$i$ | Pill<br>$T_i$                                                                     | Headache Status<br>$Y_i(\text{🔴})$ $Y_i(\text{🟡})$ $Y_i$                                                                                                            | Age<br>$X_{1i}$ | Academic<br>$X_{2i}$ |
|----------------|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|----------------------|
| 1              |  |   | 25              | Y                    |
| 2              |  |   | 55              | N                    |
| 3              |  |   | 62              | Y                    |
| 4              |  |   | 80              | N                    |
| 5              |  |   | 32              | Y                    |
| 6              |  |   | 45              | B                    |
| $\vdots$       | $\vdots$                                                                          | $\vdots$ $\vdots$ $\vdots$                                                                                                                                          | $\vdots$        | $\vdots$             |
| n              |  |   | 71              | N                    |

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Example: Aspirin's Impact on Headaches

| Patient<br>$i$ | Pill<br>$T_i$                                                                     | Headache Status<br>$Y_i(\text{red pill})$ $Y_i(\text{brown pill})$ $Y_i$                                                                                            | Age<br>$X_{1i}$ | Academic<br>$X_{2i}$ |
|----------------|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|----------------------|
| 1              |  |   | 25              | Y                    |
| 2              |  |   | 55              | N                    |
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| $\vdots$       | $\vdots$                                                                          | $\vdots$                                                                                                                                                            | $\vdots$        | $\vdots$             |
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Step 1: (Randomly) sample units

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|----------------|---|---------------|----------------------------------------------------------|-----------------|----------------------|
| 1              | 👤 | 🟡 I-2         | 😞 😞                                                      | 25              | Y                    |
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| ⋮              | ⋮ | ⋮             | ⋮ ⋮ ⋮                                                    | ⋮               | ⋮                    |
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Step 2: Randomly assign treatment

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| ⋮              | ⋮                                                                                 | ⋮                                                                                 | ⋮                                                                                                                                                                                                                                                     | ⋮               | ⋮                    |
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**Step 3:** Measure revealed potential outcome

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- The difficulty in all these cases is that we can't **observe** the **causal** part.

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- In other words: if we just **include** the **right** variables and **don't include** the **wrong** variables we can do prediction and be fine!
- Mechanically this ends up looking easy: predict the outcome with  $T = 1$  and predict the outcome with  $T = 0$  then take the difference! (more later!)

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- Don't include **post-treatment** variables unless you really know what you are doing.

# What are the “Right” Variables?

- Generally speaking we want to control for/condition on/use as predictors **pre-treatment confounders** and not **colliders**.
- How do we know which variables are which? (Use the Frameworks!)
  - ▶ **Potential Outcomes**: Those such that  $Y(0), Y(1) \perp\!\!\!\perp T|X$   
     $\rightsquigarrow$  intuitively, the pre-treatment things such that it is as though you have random assignments within levels of  $X$
  - ▶ **DAG Framework**: Those that block all backdoor paths  
     $\rightsquigarrow$  draw your causal model and the algorithm will tell you which sets of variables.
- In both cases the rough intuition: **common sources of variation** between treatment and outcome (i.e. it is okay to omit things that just cause treatment and just cause the outcome)
- Don't include **post-treatment** variables unless you really know what you are doing.
- NB: analyzing within a subset of data is the same as conditioning on the variable defining the subset!

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# An Empirical Analysis of Racial Differences in Police Use of Force

Roland G. Fryer Jr.

*Harvard University and National Bureau of Economic Research*

This paper explores racial differences in police use of force. On non-lethal uses of force, blacks and Hispanics are more than 50 percent more likely to experience some form of force in interactions with police. Adding controls that account for important context and civilian behavior reduces, but cannot fully explain, these disparities. On the most extreme use of force—officer-involved shootings—we find no racial differences either in the raw data or when contextual factors are taken into account. We argue that the patterns in the data are consistent with a model in which police officers are utility maximizers, a fraction of whom have a preference for discrimination, who incur relatively high expected costs of officer-involved shootings.

We can never be satisfied as long as the Negro is the victim of the unspeakable horrors of police brutality. (Martin Luther King Jr., August 28, 1963)

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**TheUpshot**

DATA DIVE

**Surprising New Evidence Shows Bias  
in Police Use of Force but Not in  
Shootings**

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**Evidence:**

**Claim:**

**Why wrong:**

Fryer 2019. Fuller critique by Knox et al. 2020 and Durlauf and Heckman 2020.

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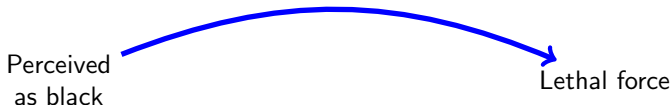
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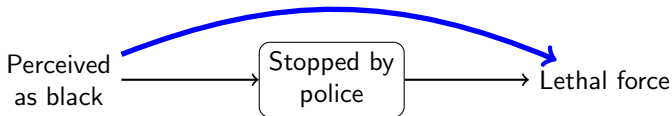


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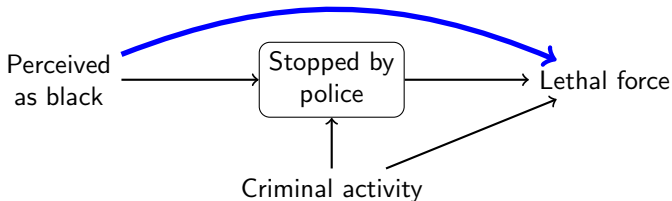


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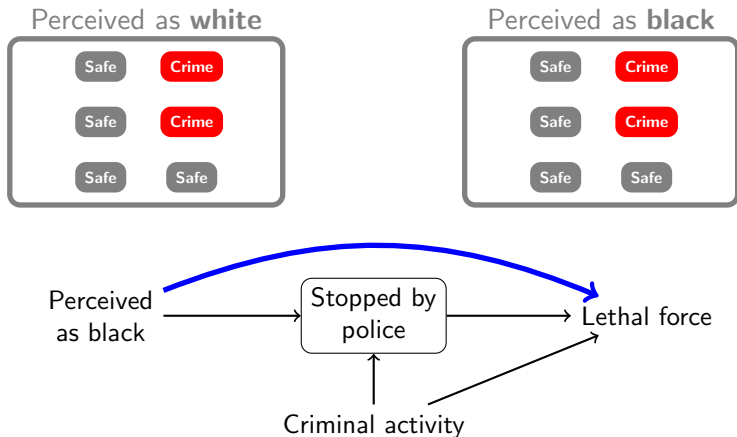
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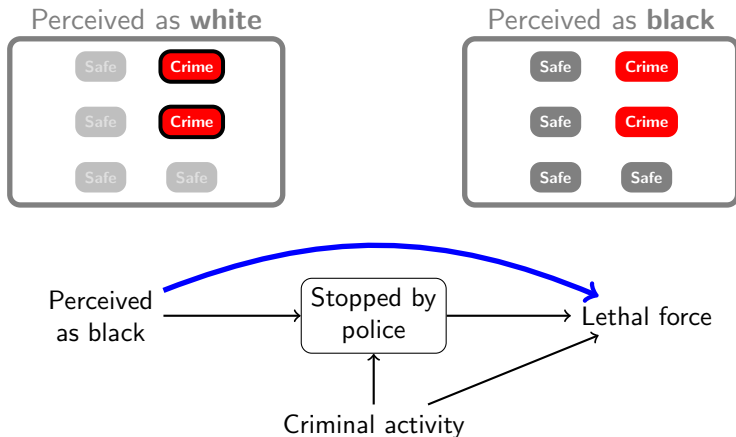
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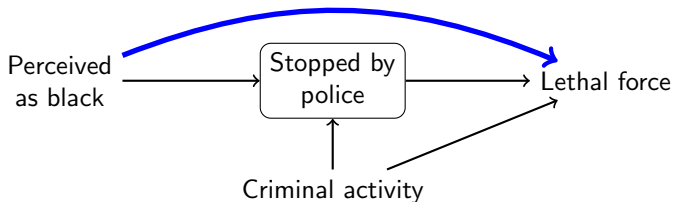
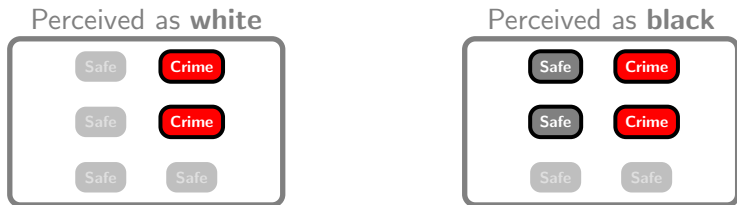
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“We use the term ‘racial differences’ 114 times in lieu of the more prescriptive wording—‘racial discrimination.’ We use the phrase ‘conditional on an interaction’ 20 times...I am not sure how many more ways we would have needed to caveat our results to satisfy [the critics].”

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**Caveats** are the problem. Estimand is only defined in terms of the regression model being fit.

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**What is your estimand?**

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The **purpose** of the  
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A common answer:



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**What is your estimand?**

A common answer:

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— We estimated  $\beta_1$

$$Y = \beta_0 + X_1\beta_1 + X_2\beta_2 + \epsilon$$



The **purpose** of the statistical analysis

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**Epistemological  
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A **unit-specific**  
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$$Y_i(t)$$

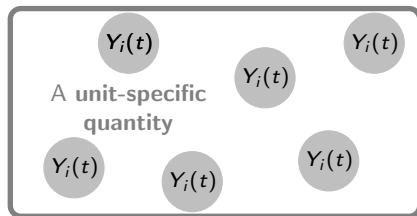
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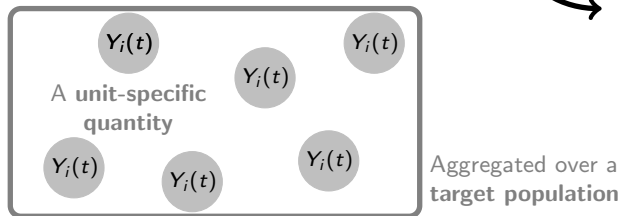
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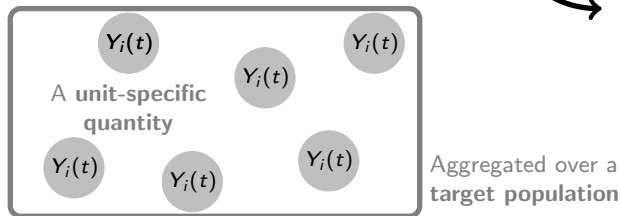


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What was Fryer's estimand?

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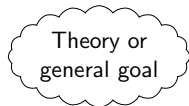


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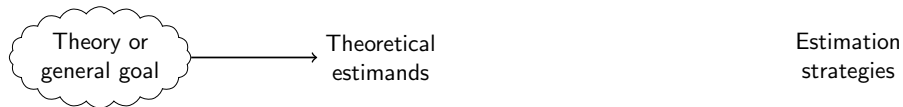
It could have been a lot of things (and that is the problem)

# Research framework



Estimation  
strategies

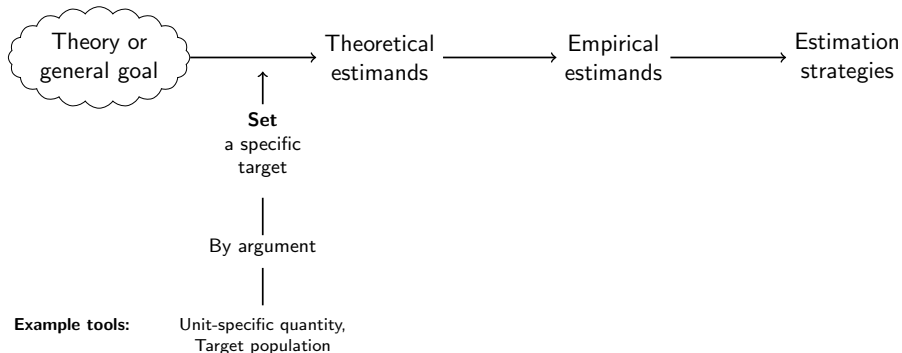
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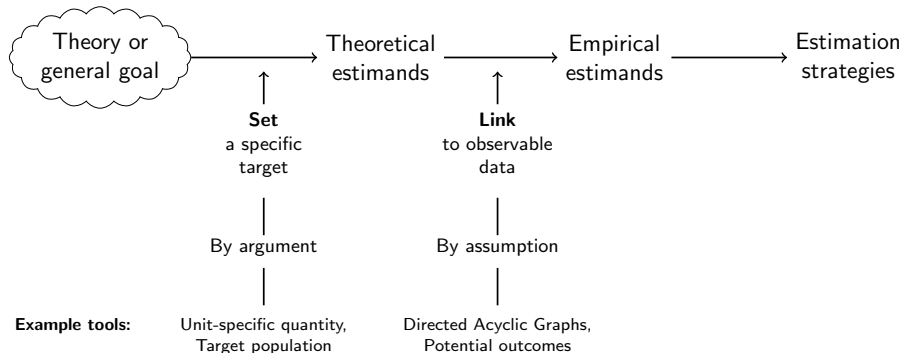
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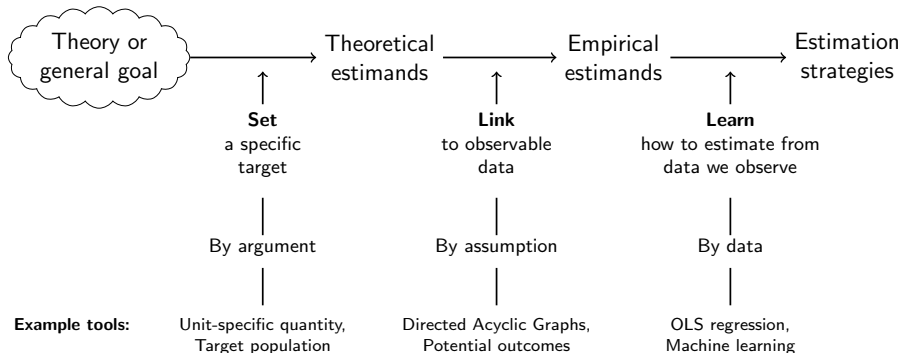
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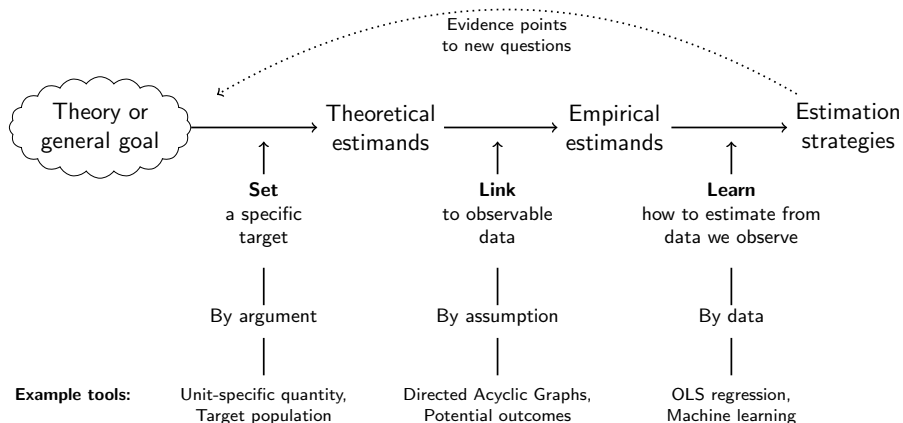
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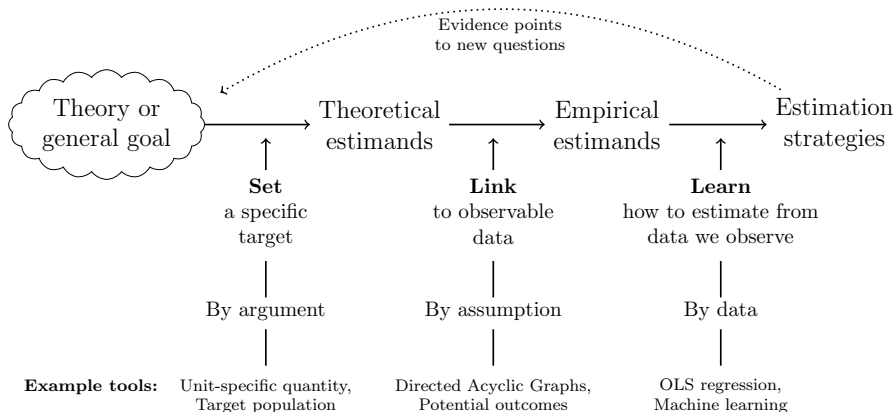


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# Theoretical Estimands (By Argument)

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## (1) Unit-Specific Quantity

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- (1) **Unit**-Specific Quantity
- (2) Aggregated Over a Target **Population**

# Theoretical Estimands (By Argument)

(1) **Unit**-Specific Quantity

(2) Aggregated Over a Target **Population**

$$\frac{1}{n} \sum_{i=1}^n Y_i$$

Mean over every  $i$   
among U.S. adults  
(target population)

Whether each  $i$   
is employed  
(unit-specific quantity)

# Theoretical Estimands (By Argument)

(1) **Unit**-Specific Quantity

(2) Aggregated Over a Target **Population**

$$\frac{1}{n} \sum_{i=1}^n \left( \underset{\substack{\uparrow \\ \text{Employment if} \\ \text{enrolled in} \\ \text{job training}}}{Y_i(1)} - \underset{\substack{\uparrow \\ \text{Employment if} \\ \text{not enrolled in} \\ \text{job training}}}{Y_i(0)} \right)$$

Mean over every  $i$   
among U.S. adults  
(target population)

(unit-specific quantity)

# Many Options

| Estimand name                        | Mathematical statement                                                                                              | DAG                                    | Reference                 | Colloquial terms                            |
|--------------------------------------|---------------------------------------------------------------------------------------------------------------------|----------------------------------------|---------------------------|---------------------------------------------|
| Average treatment effect             | $\frac{1}{n} \sum_i \left( Y_i(d') - Y_i(d) \right)$                                                                | $D \rightarrow Y$                      | Morgan and Winship (2015) | Effect                                      |
| Conditional average treatment effect | $\frac{1}{n_x} \sum_{i: X_i=x} \left( Y_i(d') - Y_i(d) \right)$                                                     | $X \rightarrow D \rightarrow Y$        | Athey and Imbens (2016)   | Effect heterogeneity or moderation          |
| Causal interaction                   | $\frac{1}{n} \sum_i \left( \left( Y_i(a', d') - Y_i(a', d) \right) - \left( Y_i(a, d') - Y_i(a, d) \right) \right)$ | $A \rightarrow Y$<br>$D \rightarrow Y$ | Vanderweele (2015)        | Joint treatment effect                      |
| Controlled direct effect             | $\frac{1}{n} \sum_i \left( Y_i(d', m) - Y_i(d, m) \right)$                                                          | $M \rightarrow Y$<br>$D \rightarrow Y$ | Acharya et al. (2016)     | Mediation (Illustrations: Example 2)        |
| Natural direct effect                | $\frac{1}{n} \sum_i \left( Y_i(d', M_i(d)) - Y_i(d, M_i(d)) \right)$                                                | $M \rightarrow Y$<br>$D \rightarrow Y$ | Imai et al. (2011)        | Mediation (Part B of the Online Supplement) |
| Effect of time-varying treatment     | $\frac{1}{n} \sum_i \left( Y_i(d'_1, d'_2) - Y_i(d_1, d_2) \right)$                                                 | $D_1 \rightarrow D_2 \rightarrow Y$    | Wodtke et al. (2011)      | Cumulative effect                           |

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- Pager (2003) explores “the ways in which the effects of race and criminal record interact to produce new forms of labor market inequalities”

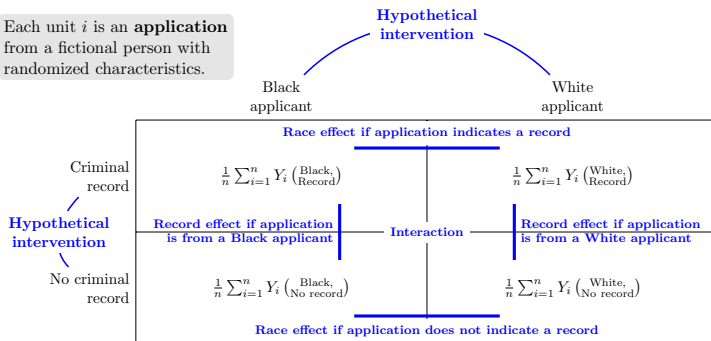
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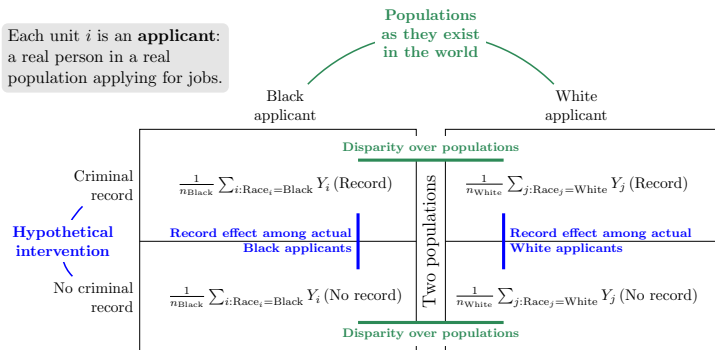
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Each unit  $i$  is an **application** from a fictional person with randomized characteristics.



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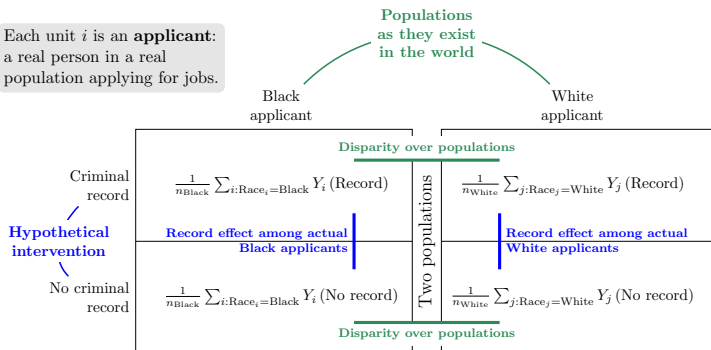
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Each unit  $i$  is an **applicant**:  
a real person in a real  
population applying for jobs.



Different **estimands** suggest different policy and theory.

# Target Population (Angrist and Evans 1998)

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Effect of motherhood  
on employment

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First two births  
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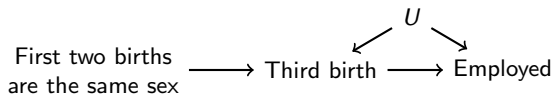
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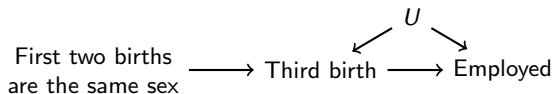
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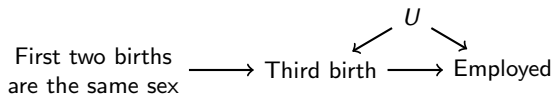


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## Precise estimand



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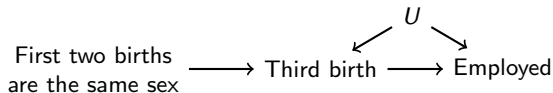
## Vague estimand

Effect of motherhood  
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## Precise estimand

Effect of having **3 vs. 2 children**

unit-specific  
quantity



# Target Population (Angrist and Evans 1998)

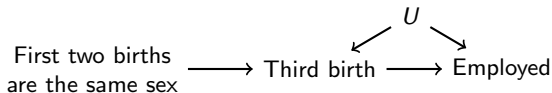
## Vague estimand

Effect of motherhood  
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target population

## Precise estimand

Effect of having 3 vs. 2 children  
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**You have to argue either:**

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## Empirical Estimands (Assumption)

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Mean  
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Potential  
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Potential  
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Link the Theoretical Estimand to **Observable** Data

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Mean among applications assigned to the condition (White, Record)      Mean among applications assigned to the condition (Black, No record)

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This includes causal identification, missing data, etc.

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- Evidence of discrimination toward women in admissions?

# Bias?

- Within departments:

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- Marginal relationships (admissions and gender)  $\neq$  conditional relationship given third variable (department)

## Sex Bias in Graduate Admissions: Data from Berkeley

Measuring bias is harder than is usually assumed,  
and the evidence is sometimes contrary to expectation.

P. J. Bickel, E. A. Hammel, J. W. O'Connell

Determining whether discrimination because of sex or ethnic identity is being practiced against persons seeking passage from one social status or locus to another is an important problem in our society today. It is legally important and morally important. It is also often quite difficult. This article is an exploration of some of the issues of measurement and assessment involved in one example of the general problem, by means of which we hope to shed some light on the difficulties. We

deceision to admit or to deny admission. The question we wish to pursue is whether the decision to admit or to deny was influenced by the sex of the applicant. We cannot know with any certainty the influences on the evaluators in the Graduate Admissions Office, or on the faculty reviewing committees, or on any other administrative personnel participating in the chain of actions that led to a decision on an individual application. We can, however, say that if the admissions decision and the sex

by using a familiar statistic, chi-square. As already noted, we are aware of the pitfalls ahead in this naïve approach, but we intend to stumble into every one of them for didactic reasons.

We must first make clear two assumptions that underlie consideration of the data in this contingency table approach. Assumption 1 is that in any given discipline male and female applicants do not differ in respect of their intelligence, skill, qualifications, promise, or other attribute deemed legitimately pertinent to their acceptance as students. It is precisely this assumption that makes the study of "sex bias" meaningful, for if we did not hold it any differences in acceptance of applicants by sex could be attributed to differences in their qualifications, promise as scholars, and so on. Theoretically one could test the assumption, for example, by examining presumably unbiased estimators of academic qualification such as Graduate Record Examination scores, undergraduate grade point averages, and so on. There are, however, enormous practical difficulties in this. We therefore predicate our discussion on the validity of assumption 1.

Bickel, Peter J., Eugene A. Hammel, and J. William O'Connell. "Sex bias in graduate admissions: Data from Berkeley." *Science* 187, no. 4175 (1975): 398-404.

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- This general pattern repeats in **many** debates, often because of the limits of data collection.

| Study                | Empirical regularity                                                                               | Misleading conclusion                                                                 | Directed Acyclic Graph                                                                                                                                                         |
|----------------------|----------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Fryer (2019)         | Among those they stop, police shoot the same proportion of Black individuals as White individuals. | Police do not discriminate against Black individuals when using lethal force.         | <pre> graph LR     A[Perceived as Black] --&gt; B[Stopped by police]     C[Criminal activity] --&gt; B     B --&gt; D[Lethal force]     A --&gt; D </pre>                      |
| Bickel et al. (1975) | Among those who apply, Berkeley departments admit a higher proportion of women than of men.        | Admissions committees do not discriminate against women.                              | <pre> graph LR     A[Female] --&gt; B[Applied to Berkeley]     C[Strong candidate] --&gt; B     B --&gt; D[Accepted]     B --&gt; E[Perceived as female]     E --&gt; D </pre> |
| Chetty et al. (2020) | Among those with equal childhood incomes, Black and White women earn similar amounts as adults.    | Equalizing childhood incomes would eliminate the racial gap in women's adult incomes. | <pre> graph LR     A[Black] --&gt; B[Childhood income]     C[Other family advantages] --&gt; B     B --&gt; D[Adult income]     A --&gt; D </pre>                              |

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# Estimation (By Data)

**Learn** the Empirical Estimand Using Data

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|                                                                                   |                                                                                                                              |
|-----------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------|
| Mean over<br>entire sample                                                        | Expected outcome among cases<br>with the covariate values $\vec{x}_i$ of unit $i$<br>who are factually treated ( $D = 1$ )   |
| ↘                                                                                 | ↙                                                                                                                            |
| $\theta = \frac{1}{n} \sum_{i=1}^n \mathbf{E}(Y \mid \vec{X} = \vec{x}_i, D = 1)$ |                                                                                                                              |
| $- \frac{1}{n} \sum_{i=1}^n \mathbf{E}(Y \mid \vec{X} = \vec{x}_i, D = 0)$        |                                                                                                                              |
| ↗                                                                                 | ↖                                                                                                                            |
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Mean over  
entire sample



Regression prediction  
at observed covariate values  $\vec{x}_i$   
with treatment set to  $D = 1$



$$\hat{\theta} = \frac{1}{n} \sum_{i=1}^n \hat{\mathbf{E}}(Y \mid \vec{X} = \vec{x}_i, D = 1) \\ - \frac{1}{n} \sum_{i=1}^n \hat{\mathbf{E}}(Y \mid \vec{X} = \vec{x}_i, D = 0)$$



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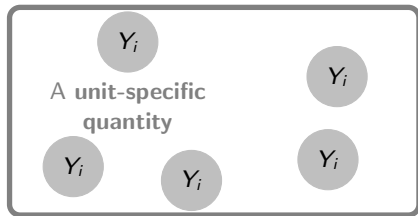
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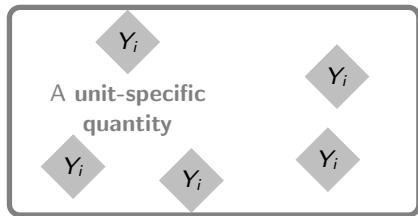
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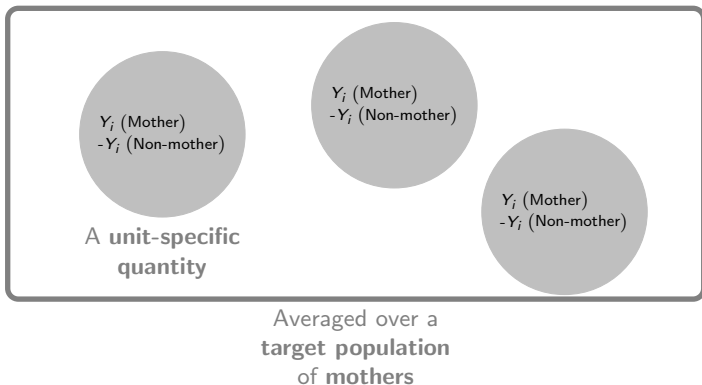
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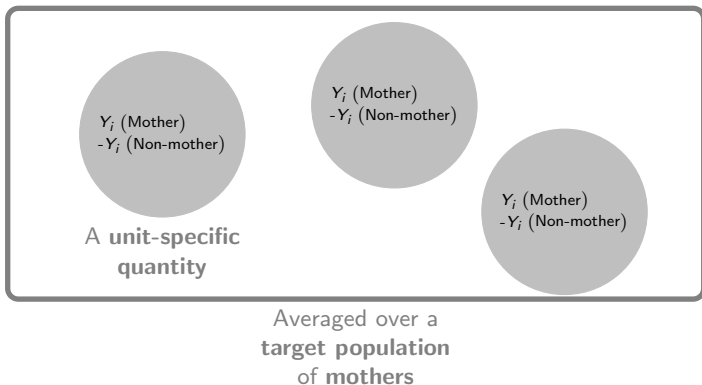
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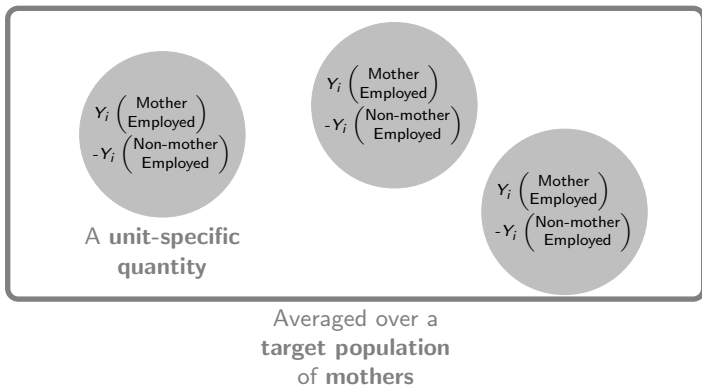
Added complexity: Wages are undefined for the non-employed.



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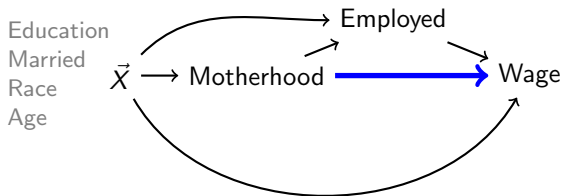
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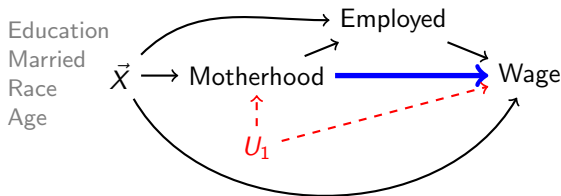


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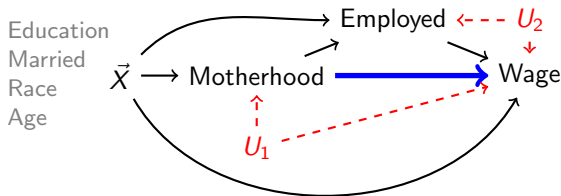
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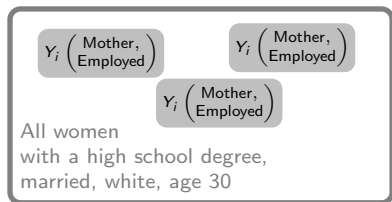


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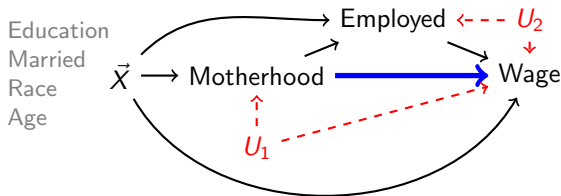


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Potential outcomes

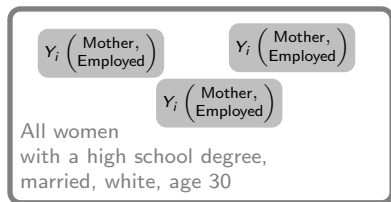


Focus on  
one  $\vec{X} = \vec{x}_i$

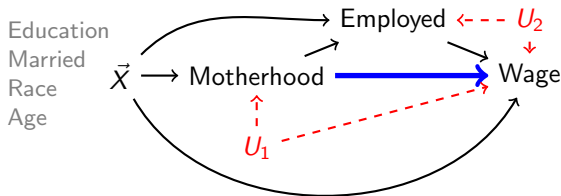


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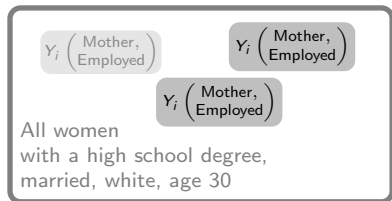


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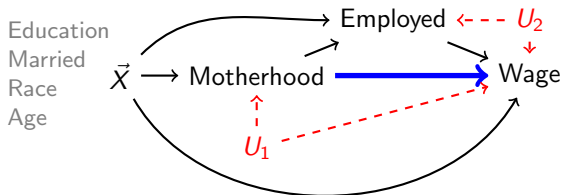


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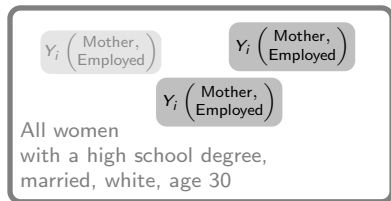


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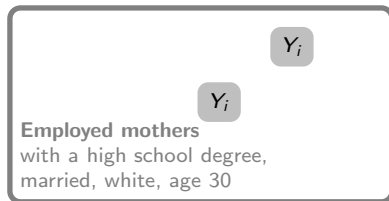


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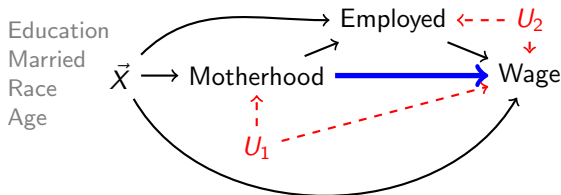
### Potential outcomes



### Realized outcomes



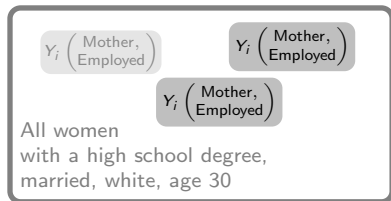
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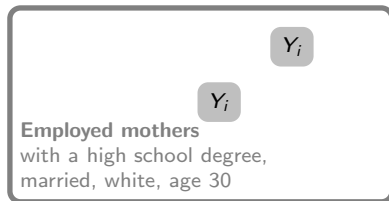
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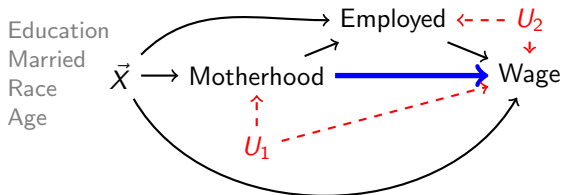
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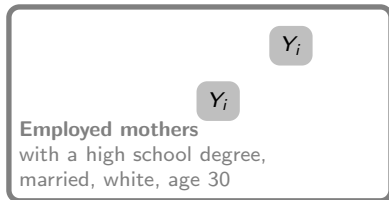


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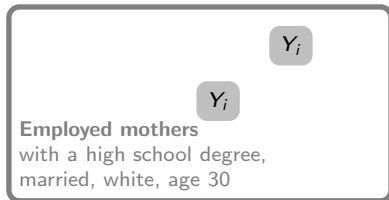
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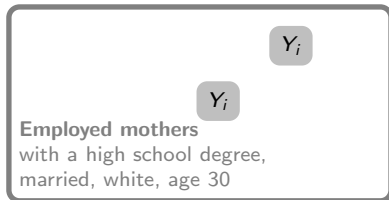


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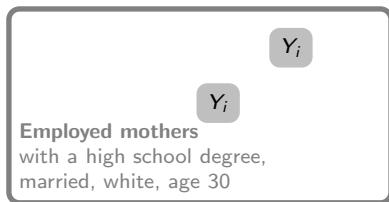
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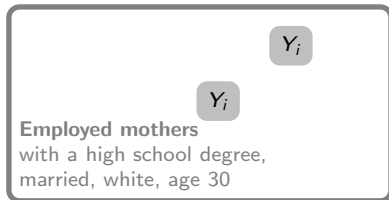
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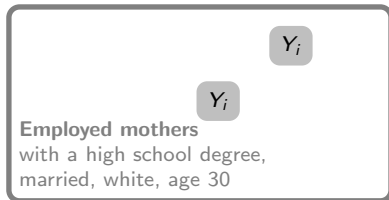
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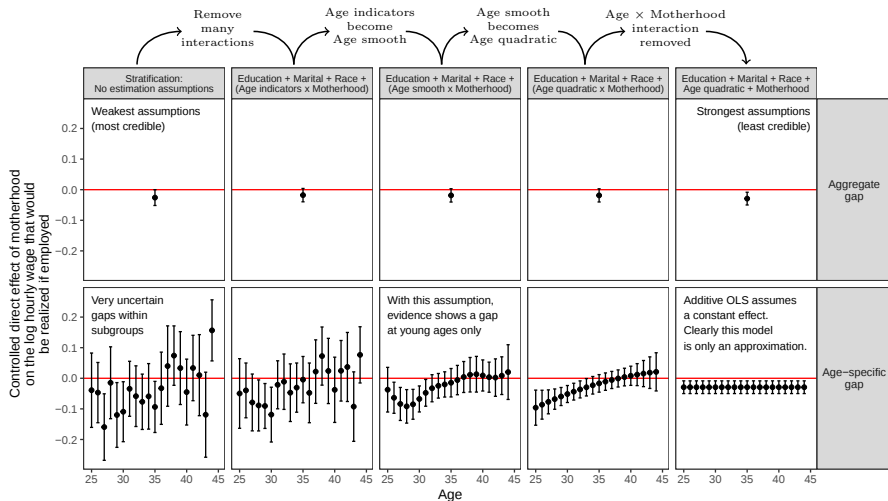
This is called an **imputation estimator**

Hahn, 1998

Abadie & Imbens 2006

Also called the parametric  $g$ -formula in biostatistics, Hernán & Robins 2020

Increasingly strong estimation assumptions: Less credibility



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  - Potential Outcomes
- 2 Prediction Policy Problems
- 3 What Is Your Estimand?
  - A Motivating Example
  - Framework
  - Theoretical Estimand
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- An interesting and complex case. It is “easier” because one counterfactual is known—if the defendant is detained they cannot fail to appear.
- Three requirements: being clear about the link between predictions and decisions; specifying the scope of payoff functions; and constructing unbiased decision counterfactuals.

*It is telling that in our application, much of the work happens after the prediction function was estimated.*

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- Don't train on any demographic other than age. Does this help? hurt?

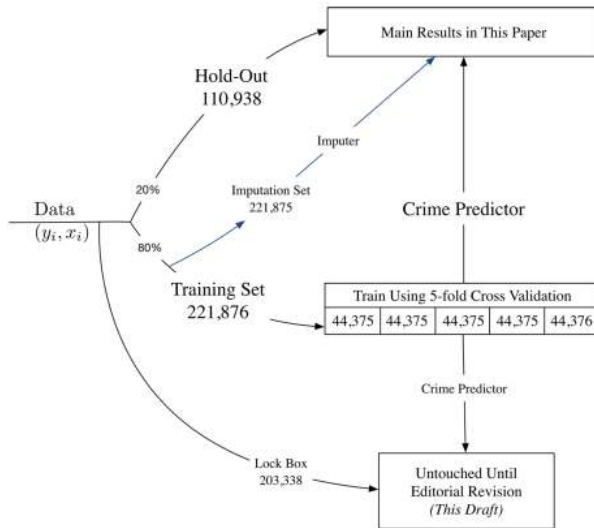


FIGURE I

Partition of New York City Data (2008–13) into Data Sets Used for Prediction and Evaluation

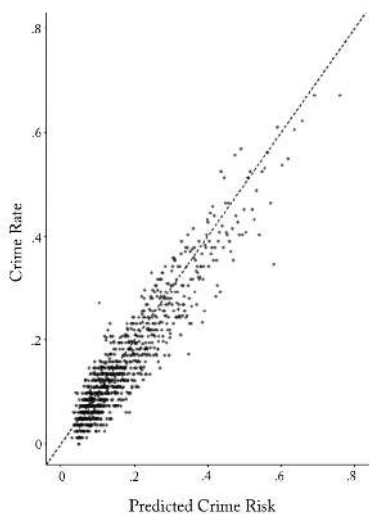
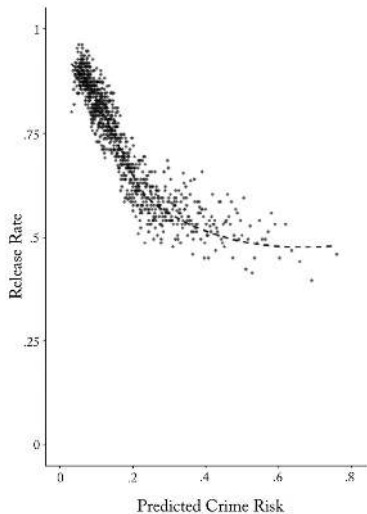


FIGURE II

How Machine Predictions of Crime Risk Relate to Judge Release Decisions and Actual Crime Rates

**TABLE II**  
**COMPARING LOGISTIC REGRESSION TO MACHINE-LEARNING PREDICTIONS**  
**OF CRIME RISK**

| Predicted risk percentile | ML/ logit overlap | Average observed crime rate for cases identified as high risk by: |                    |                    |                    |                    |
|---------------------------|-------------------|-------------------------------------------------------------------|--------------------|--------------------|--------------------|--------------------|
|                           |                   | Both ML and logit                                                 | ML only            | Logit only         | All ML cases       | All logit cases    |
| 1%                        | 30.6%             | 0.6080<br>(0.0309)                                                | 0.5440<br>(0.0209) | 0.3996<br>(0.0206) | 0.5636<br>(0.0173) | 0.4633<br>(0.0174) |
| 5%                        | 59.9%             | 0.4826<br>(0.0101)                                                | 0.4090<br>(0.0121) | 0.3040<br>(0.0114) | 0.4531<br>(0.0078) | 0.4111<br>(0.0077) |
| 10%                       | 65.9%             | 0.4134<br>(0.0067)                                                | 0.3466<br>(0.0090) | 0.2532<br>(0.0082) | 0.3907<br>(0.0054) | 0.3589<br>(0.0053) |
| 25%                       | 72.9%             | 0.3271<br>(0.0038)                                                | 0.2445<br>(0.0058) | 0.1608<br>(0.0049) | 0.3048<br>(0.0032) | 0.2821<br>(0.0031) |

*Notes.* The table shows the results of fitting a machine-learning (ML) algorithm or a logistic regression to our training data set, to identify the highest-risk observations in our test set. Both machine-learning and logistic regression models are trained on the outcome of failure to appear (FTA). Each row presents statistics for the top part of the predicted risk distribution indicated in the first column: top 25% ( $N = 20,423$ ); 10% (8,173); 5% (4,087); and 1% (818). The second column shows the share of cases in the top X% of the predicted risk distribution that overlap between the set identified by ML and the set identified by logistic regression. The subsequent columns report the average crime rate observed among the released defendants within the top X% of the predicted risk distribution as identified by both ML and logit, ML only, and logit only, and all top X% identified by ML (whether or not they are also identified by logistic regression) and top X% identified by logit.

5. In practice algorithms would be decision aids, not decision makers. Our calculations simply highlight the scope of the potential gains. Understanding the determinants of compliance with prediction tools is beyond the scope of this article, though recent work has begun to focus on it ([Dietvorst, Simmons, and Masey 2015](#); [Yeomans et al. 2016](#); [Logg 2017](#)).

28. This logistic regression uses the same set of covariates as are given to the machine-learning algorithm, using a standard linear additive functional form. Interestingly, estimating a model that adds all two-way interactions between the predictors causes the logistic regression model to overfit the training data set and predict poorly out of sample, even in a pared-down model with just 5 or 10 covariates. Many of these interactions seem to be picking up statistical noise in the training set, although the machine-learning algorithm is able to find the interactivity that reflects real signal.

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- Big data alone is **not sufficient** we have to understand the task and the process!

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- It is often said that **causal inference** is a **missing data** problem, but **missing data** is also a **causal inference** problem!
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- In this class we will be relatively easy going about it, but be very cautious of **high levels of missingness** or where **selection into missingness** feels highly correlated with the outcome or treatment.
- Simple approaches based on imputation aren't guaranteed to work without further assumptions the same way prediction isn't guaranteed to work for causal inference!

# This Week in Review

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- Prediction in a counterfactual population

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Next Week:

- Salganik Chapter 4 of *Bit by Bit*: “Running Experiments”

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# The Core Idea: Linear/Logistic Regression with Adaptive Non-Linear Basis Functions

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$$\hat{y} = \hat{\beta}_0 + \sum_{m=1}^M \hat{\beta}_m \hat{\phi}_m(X)$$

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- appear in earnest in the mid 1980's
- universal approximator
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- reborn in  $\approx 2010$  as deep learning

# High Level Summary

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- impressive performance on complex data that can defy intuition

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# Single Layer Feedforward Neural Network

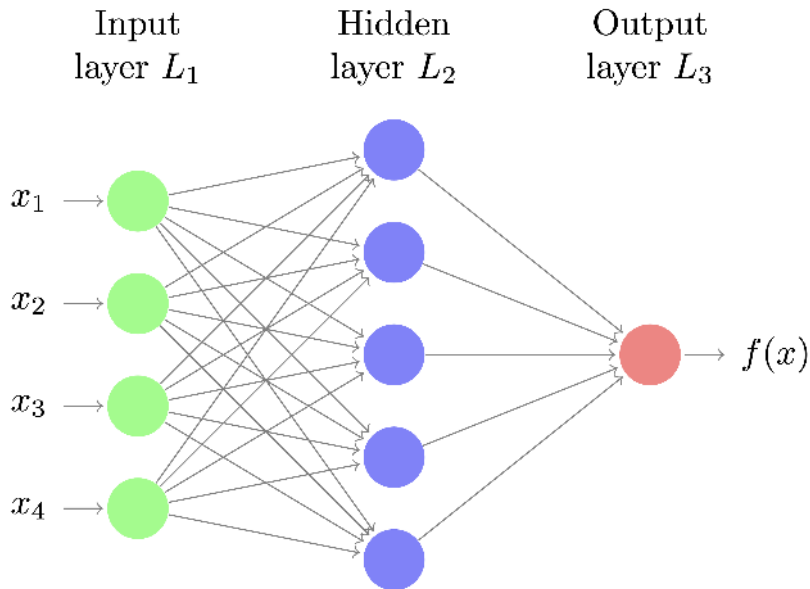


Image Credit: Efron and Hastie *Computer Age Statistical Inference: Algorithms, Evidence, and Data Science*

# Single Layer Feedforward Neural Network

$$z_{i,\ell} = g \left( w_0^{(1)} + \sum_{j=1}^4 w_{\ell,j}^{(1)} x_j \right)$$

# Single Layer Feedforward Neural Network

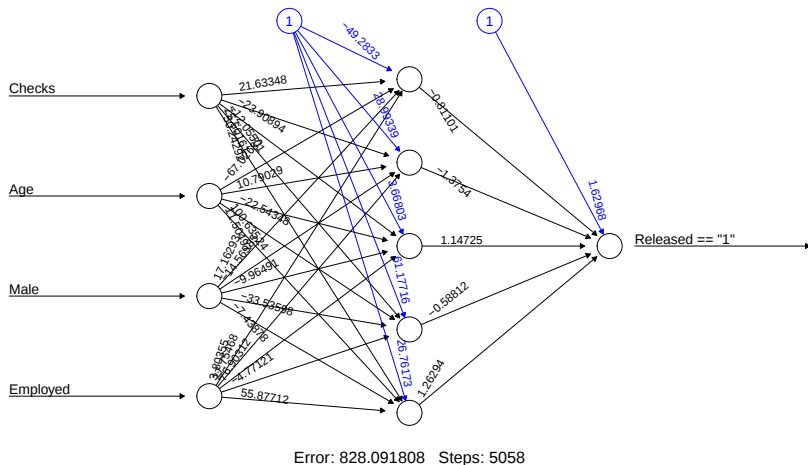
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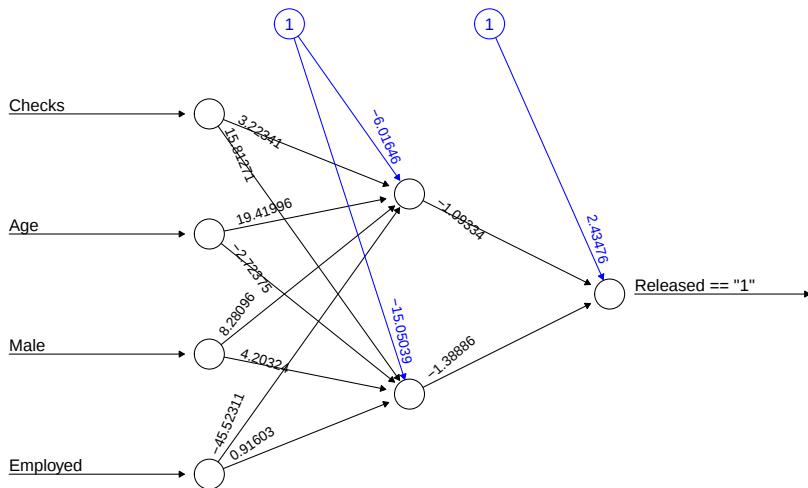
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$g()$  **must** be non-linear (called the **activation function**),  
 $h()$  will generally be the identity, logistic, or softmax.

# Single Layer Feedforward Neural Network

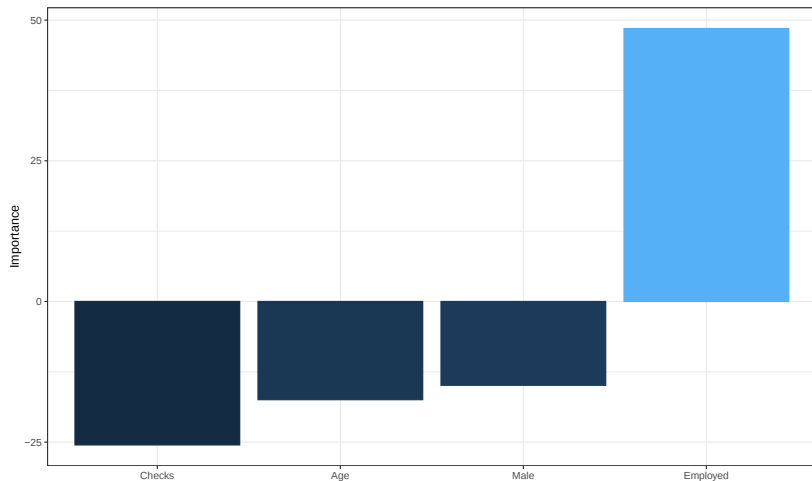


# Single Layer Feedforward Neural Network



Error: 809.07176 Steps: 1259

# Single Layer Feedforward Neural Network



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- All handled by modern software packages.

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# Mini-batch Stochastic Gradient Descent

- Core Idea: update each step with a small piece of the data rather than the full thing.
- An **epoch** is a sequence of steps which account for one full pass through the data.
- So if we have 100K data points and batch sizes of 100, there are 1000 steps per epoch.
- Sometimes convenient to specify train time in epochs rather than steps.

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It reduces to a linear model!

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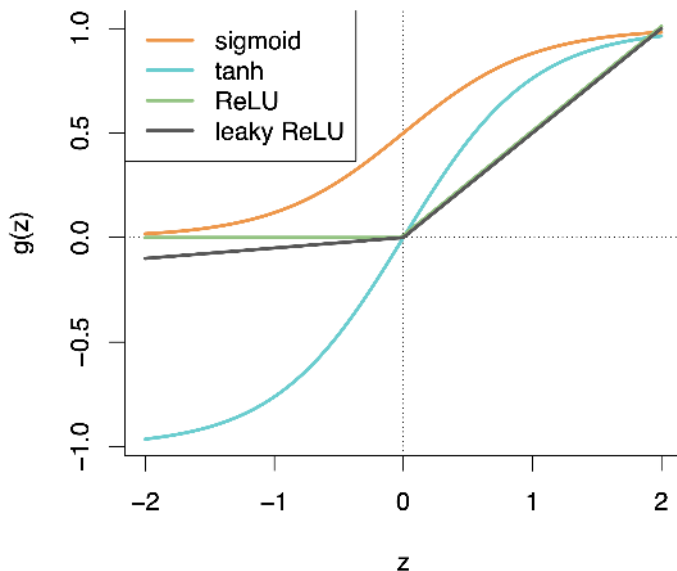


Image Credit: Efron and Hastie *Computer Age Statistical Inference: Algorithms, Evidence, and Data Science*

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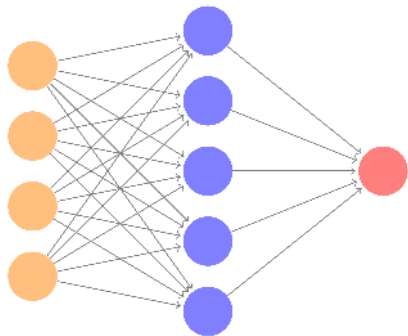
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Some are approximately equivalent to ridge penalty.

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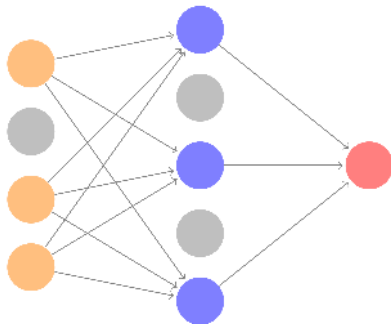
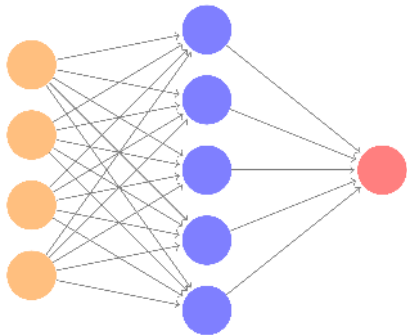


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- `torch` and `torchvision`: an R version of PyTorch

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Two important notes:

- returns pointers vs. regular R objects
- we use fast settings to decrease computation time

# 1) Standardize the Inputs

```
x <- scale(model.matrix(Salary ~ . - 1, data = Gitters))  
y <- Gitters$Salary
```

## 2) Define Model Structure

```
modnn <- nn_module(  
  initialize = function(input_size) {  
    self$hidden <- nn_linear(input_size, 50)  
    self$activation <- nn_relu()  
    self$dropout <- nn_dropout(0.4)  
    self$output <- nn_linear(50, 1)  
  },  
  forward = function(x) {  
    x %>%  
      self$hidden() %>%  
      self$activation() %>%  
      self$dropout() %>%  
      self$output()  
  }  
)
```

## 3-4) Define Other Model Components

```
modnn <- modnn %>%  
  setup(  
    loss = nn_mse_loss(),  
    optimizer = optim_rmsprop,  
    metrics = list(luz_metric_mae())  
  ) %>%  
  set_hparams(input_size = ncol(x))
```

## 5) Fit the Model

```
fitted <- modnn %>%  
  fit(  
    data = list(x[-testid, ],  
               matrix(y[-testid], ncol = 1)),  
    valid_data = list(x[testid, ],  
                     matrix(y[testid], ncol = 1)),  
    epochs = 20  
  )
```

# What Can We Do With This?

- Call `predict(fitted, newdata = )` to get predictions.

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- **New!** Use `as_array()` to convert the pointer to a regular R object!
- Use `plot(fitted)` to show error metrics during optimization.

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complex data and **high signal-to-noise** ratio.

# Multiple Layer Feedforward Neural Network

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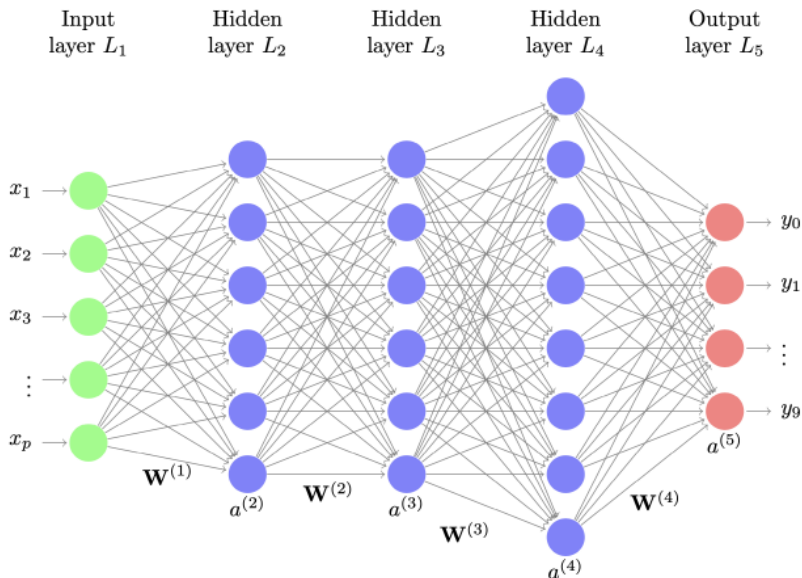
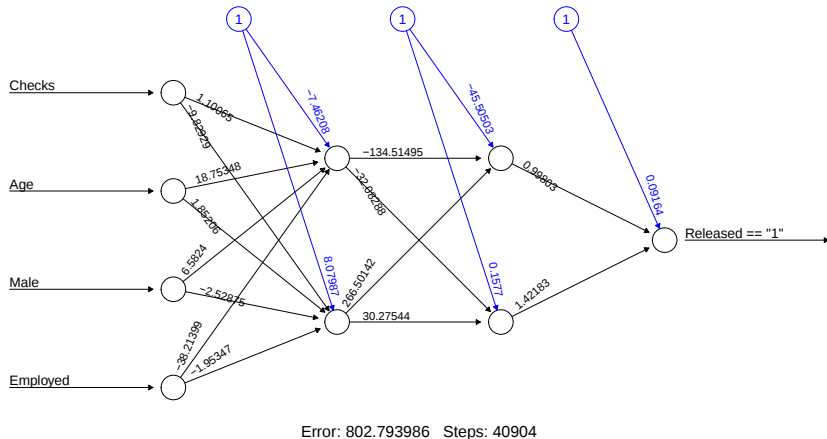


Image Credit: Efron and Hastie *Computer Age Statistical Inference: Algorithms, Evidence, and Data Science*

# Multiple Layer Feedforward Neural Network

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## Example: MNIST Digits

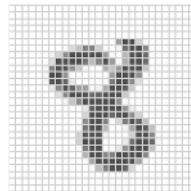
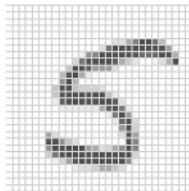
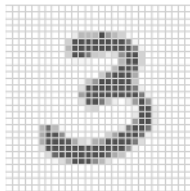
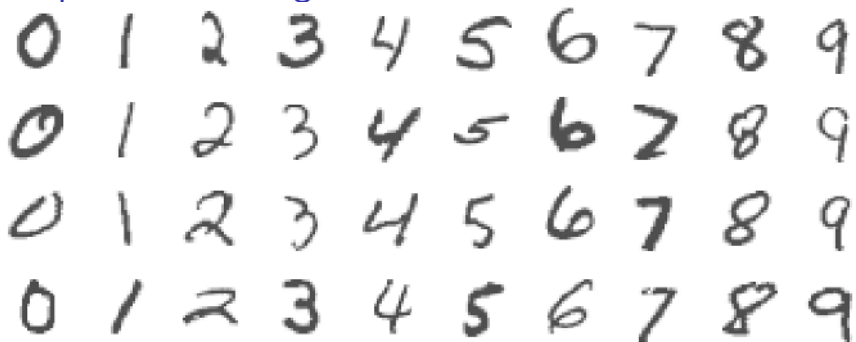


Image Credit: G. James, D. Witten, T. Hastie and R. Tibshirani. *An Introduction to Statistical Learning, with applications in R: Second Edition*

# Implementation using torch

```
M_NN_struct <- nn_module(  
  initialize = function() {  
    self$linear1 <- nn_linear(in_features = 28*28, out_features = 256)  
    self$linear2 <- nn_linear(in_features = 256, out_features = 128)  
    self$linear3 <- nn_linear(in_features = 128, out_features = 10)  
  
    self$drop1 <- nn_dropout(p = 0.4)  
    self$drop2 <- nn_dropout(p = 0.3)  
  
    self$activation <- nn_relu()  
  },  
  forward = function(x) {  
    x %>%  
    self$linear1() %>%  
    self$activation() %>%  
    self$drop1() %>%  
    self$linear2() %>%  
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    self$drop2() %>%  
    self$linear3()  
  }  
)
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# Implementation using torch

```
modelnn <- modelnn %>%  
  setup(  
    loss = nn_cross_entropy_loss(),  
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    metrics = list(luz_metric_accuracy())  
  )
```

# Alternative Architectures

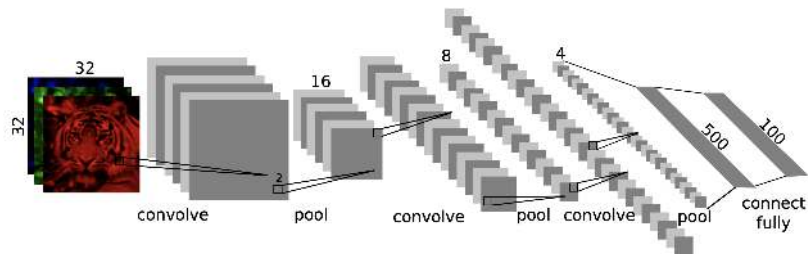


Image Credit: Efron and Hastie *Computer Age Statistical Inference: Algorithms, Evidence, and Data Science*

# Convolutional Neural Networks (CNNs)

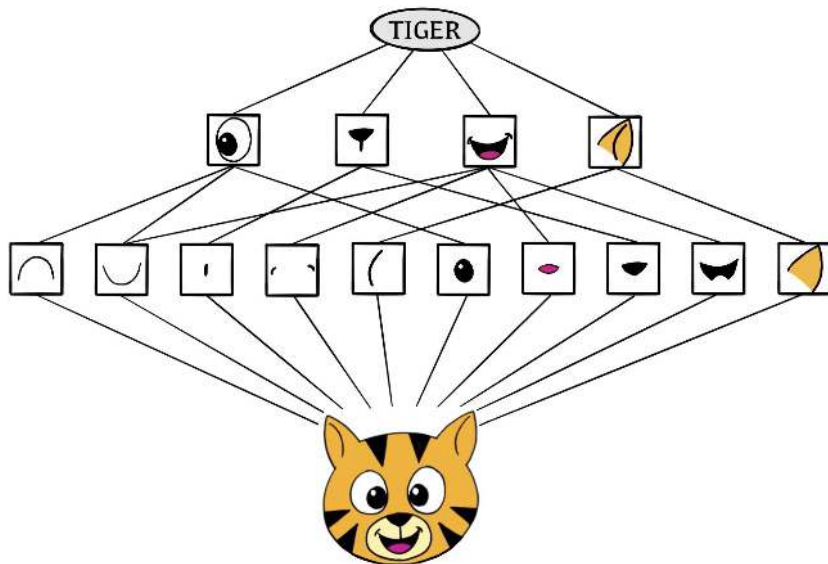
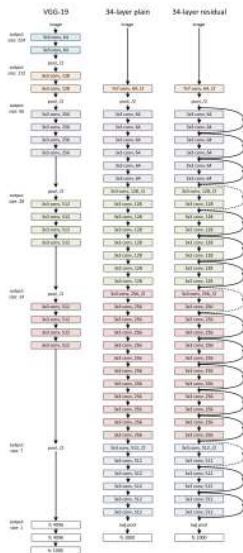


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# Resnet Architecture



He, K., Zhang, X., Ren, S. and Sun, J., 2016. Deep residual learning for image recognition. In Proceedings of the IEEE conference on computer vision and pattern recognition (pp. 770-778).

# A Story About Architecture

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## *Turing Award Won by 3 Pioneers in Artificial Intelligence*



From left, Yann LeCun, Geoffrey Hinton and Yoshua Bengio. The researchers worked on key developments for neural networks, which are reshaping how computer systems are built. From left, Facebook, via Associated Press; Aaron Vincent Blikstein for The New York Times; Chad Bechman/Getty Images.

# But Also a Story About Data

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## Fei-Fei Li

From Wikipedia, the free encyclopedia

*The native form of this [personal name](#) is Li Feifei. This article uses [Western name order](#) when mentioning individuals.*

**Fei-Fei Li** (simplified Chinese: 李飞飞; traditional Chinese: 李飛飛; born 1976) is a Chinese-born American computer scientist, non-profit executive, and writer. She is the Sequoia Capital Professor of Computer Science at [Stanford University](#).<sup>[2]</sup> Li is a Co-Director of the Stanford Institute for Human-Centered Artificial Intelligence, and a Co-Director of the Stanford Vision and Learning Lab.<sup>[3][4]</sup> She served as the director of the [Stanford Artificial Intelligence Laboratory](#) (SAIL)<sup>[5]</sup> from 2013 to 2018. In 2017, she co-founded AI4ALL, a nonprofit organization working to increase diversity and inclusion in the field of artificial intelligence.<sup>[6][7]</sup> Her research expertise includes [artificial intelligence](#) (AI), [machine learning](#), [deep learning](#), [computer vision](#) and [cognitive neuroscience](#).<sup>[8]</sup> She was the leading scientist and principal investigator of [ImageNet](#).<sup>[9]</sup>

**Fei-Fei Li**



Li at AI for Good in 2017

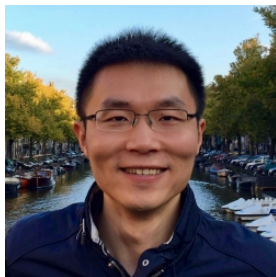
|                    |                                                            |
|--------------------|------------------------------------------------------------|
| <b>Born</b>        | 1976 (age 44–45) <sup>[1]</sup><br><a href="#">Beijing</a> |
| <b>Nationality</b> | American                                                   |
| <b>Alma mater</b>  | <a href="#">Princeton University</a> (B.A. in Physics)     |

# Princeton Connections!

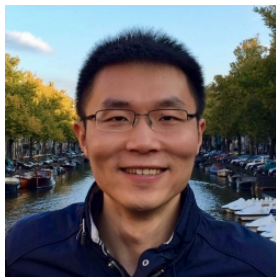
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# Tuning Parameters

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- number of nodes per layer
- regularization

# Tuning Parameters

- number of layers
- number of nodes per layer
- regularization
- optimization technique

# Tuning Parameters

- number of layers
- number of nodes per layer
- regularization
- optimization technique
- architecture

Powerful but difficult to use

## Powerful but difficult to use

*If you have ever tried to train a NN on your own, then you may know how painful and uncertain it can be. I am not talking about following the existing tutorials and demos, when all hyperparameters are already tuned for you, but about taking some data and creating something from scratch. Even with modern DL [deep learning] high-level toolkits, where all best practices such as proper weights initialization and optimizers' betas, gammas, and other options are set to sane defaults, and tons of other stuff hidden under the hood, there are still lots of decisions that you can make, hence lots of things can go wrong. As a result, your network almost never works from the first run and this is something that you should get used to.*

(Lapan 2018, 66)

# Summary

# Summary

- neural networks

# Summary

- neural networks
- deep learning as an extension of basic neural networks

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- neural networks
- deep learning as an extension of basic neural networks

Going Deeper:

G. James, D. Witten, T. Hastie and R. Tibshirani. *An Introduction to Statistical Learning, with applications in R: Second Edition* Chapter 10

# Review of Methods

|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    |           |              |        |        |         |
| GAMs          |           |              |        |        |         |
| Ridge/Lasso   |           |              |        |        |         |
| Dec. Trees    |           |              |        |        |         |
| Rand. Forests |           |              |        |        |         |
| Boosting      |           |              |        |        |         |
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# Review of Methods

|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         |              |        |        |         |
| GAMs          | Y         |              |        |        |         |
| Ridge/Lasso   |           |              |        |        |         |
| Dec. Trees    |           |              |        |        |         |
| Rand. Forests |           |              |        |        |         |
| Boosting      |           |              |        |        |         |
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| Rand. Forests | Y         |              |        |        |         |
| Boosting      |           |              |        |        |         |
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| Rand. Forests | Y         |              |        |        |         |
| Boosting      | Y         |              |        |        |         |
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| Dec. Trees    | Y         |              |        |        |         |
| Rand. Forests | Y         |              |        |        |         |
| Boosting      | Y         |              |        |        |         |
| Neural Nets   | Y         |              |        |        |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            |        |        |         |
| GAMs          | Y         |              |        |        |         |
| Ridge/Lasso   | N         |              |        |        |         |
| Dec. Trees    | Y         |              |        |        |         |
| Rand. Forests | Y         |              |        |        |         |
| Boosting      | Y         |              |        |        |         |
| Neural Nets   | Y         |              |        |        |         |

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|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            |        |        |         |
| GAMs          | Y         | N            |        |        |         |
| Ridge/Lasso   | N         |              |        |        |         |
| Dec. Trees    | Y         |              |        |        |         |
| Rand. Forests | Y         |              |        |        |         |
| Boosting      | Y         |              |        |        |         |
| Neural Nets   | Y         |              |        |        |         |

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| GAMs          | Y         | N            |        |        |         |
| Ridge/Lasso   | N         | N            |        |        |         |
| Dec. Trees    | Y         |              |        |        |         |
| Rand. Forests | Y         |              |        |        |         |
| Boosting      | Y         |              |        |        |         |
| Neural Nets   | Y         |              |        |        |         |

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|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
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| GAMs          | Y         | N            |        |        |         |
| Ridge/Lasso   | N         | N            |        |        |         |
| Dec. Trees    | Y         | Y            |        |        |         |
| Rand. Forests | Y         |              |        |        |         |
| Boosting      | Y         |              |        |        |         |
| Neural Nets   | Y         |              |        |        |         |

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|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            |        |        |         |
| GAMs          | Y         | N            |        |        |         |
| Ridge/Lasso   | N         | N            |        |        |         |
| Dec. Trees    | Y         | Y            |        |        |         |
| Rand. Forests | Y         | Y            |        |        |         |
| Boosting      | Y         |              |        |        |         |
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|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            |        |        |         |
| GAMs          | Y         | N            |        |        |         |
| Ridge/Lasso   | N         | N            |        |        |         |
| Dec. Trees    | Y         | Y            |        |        |         |
| Rand. Forests | Y         | Y            |        |        |         |
| Boosting      | Y         | Y            |        |        |         |
| Neural Nets   | Y         |              |        |        |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            |        |        |         |
| GAMs          | Y         | N            |        |        |         |
| Ridge/Lasso   | N         | N            |        |        |         |
| Dec. Trees    | Y         | Y            |        |        |         |
| Rand. Forests | Y         | Y            |        |        |         |
| Boosting      | Y         | Y            |        |        |         |
| Neural Nets   | Y         | Y            |        |        |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            | None   |        |         |
| GAMs          | Y         | N            |        |        |         |
| Ridge/Lasso   | N         | N            |        |        |         |
| Dec. Trees    | Y         | Y            |        |        |         |
| Rand. Forests | Y         | Y            |        |        |         |
| Boosting      | Y         | Y            |        |        |         |
| Neural Nets   | Y         | Y            |        |        |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            | None   |        |         |
| GAMs          | Y         | N            | Low    |        |         |
| Ridge/Lasso   | N         | N            |        |        |         |
| Dec. Trees    | Y         | Y            |        |        |         |
| Rand. Forests | Y         | Y            |        |        |         |
| Boosting      | Y         | Y            |        |        |         |
| Neural Nets   | Y         | Y            |        |        |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            | None   |        |         |
| GAMs          | Y         | N            | Low    |        |         |
| Ridge/Lasso   | N         | N            | Low    |        |         |
| Dec. Trees    | Y         | Y            |        |        |         |
| Rand. Forests | Y         | Y            |        |        |         |
| Boosting      | Y         | Y            |        |        |         |
| Neural Nets   | Y         | Y            |        |        |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            | None   |        |         |
| GAMs          | Y         | N            | Low    |        |         |
| Ridge/Lasso   | N         | N            | Low    |        |         |
| Dec. Trees    | Y         | Y            | Low    |        |         |
| Rand. Forests | Y         | Y            |        |        |         |
| Boosting      | Y         | Y            |        |        |         |
| Neural Nets   | Y         | Y            |        |        |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            | None   |        |         |
| GAMs          | Y         | N            | Low    |        |         |
| Ridge/Lasso   | N         | N            | Low    |        |         |
| Dec. Trees    | Y         | Y            | Low    |        |         |
| Rand. Forests | Y         | Y            | Medium |        |         |
| Boosting      | Y         | Y            |        |        |         |
| Neural Nets   | Y         | Y            |        |        |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning | Smooth | Var Sel |
|---------------|-----------|--------------|--------|--------|---------|
| Regression    | N         | N            | None   |        |         |
| GAMs          | Y         | N            | Low    |        |         |
| Ridge/Lasso   | N         | N            | Low    |        |         |
| Dec. Trees    | Y         | Y            | Low    |        |         |
| Rand. Forests | Y         | Y            | Medium |        |         |
| Boosting      | Y         | Y            | High   |        |         |
| Neural Nets   | Y         | Y            |        |        |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning    | Smooth | Var Sel |
|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      |        |         |
| GAMs          | Y         | N            | Low       |        |         |
| Ridge/Lasso   | N         | N            | Low       |        |         |
| Dec. Trees    | Y         | Y            | Low       |        |         |
| Rand. Forests | Y         | Y            | Medium    |        |         |
| Boosting      | Y         | Y            | High      |        |         |
| Neural Nets   | Y         | Y            | Very High |        |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning    | Smooth | Var Sel |
|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      |         |
| GAMs          | Y         | N            | Low       |        |         |
| Ridge/Lasso   | N         | N            | Low       |        |         |
| Dec. Trees    | Y         | Y            | Low       |        |         |
| Rand. Forests | Y         | Y            | Medium    |        |         |
| Boosting      | Y         | Y            | High      |        |         |
| Neural Nets   | Y         | Y            | Very High |        |         |

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|               | Nonlinear | Interactions | Tuning    | Smooth | Var Sel |
|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      |         |
| GAMs          | Y         | N            | Low       | Y      |         |
| Ridge/Lasso   | N         | N            | Low       |        |         |
| Dec. Trees    | Y         | Y            | Low       |        |         |
| Rand. Forests | Y         | Y            | Medium    |        |         |
| Boosting      | Y         | Y            | High      |        |         |
| Neural Nets   | Y         | Y            | Very High |        |         |

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|               | Nonlinear | Interactions | Tuning    | Smooth | Var Sel |
|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      |         |
| GAMs          | Y         | N            | Low       | Y      |         |
| Ridge/Lasso   | N         | N            | Low       | Y      |         |
| Dec. Trees    | Y         | Y            | Low       |        |         |
| Rand. Forests | Y         | Y            | Medium    |        |         |
| Boosting      | Y         | Y            | High      |        |         |
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|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      |         |
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| Dec. Trees    | Y         | Y            | Low       | N      |         |
| Rand. Forests | Y         | Y            | Medium    |        |         |
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| Regression    | N         | N            | None      | Y      |         |
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| Ridge/Lasso   | N         | N            | Low       | Y      |         |
| Dec. Trees    | Y         | Y            | Low       | N      |         |
| Rand. Forests | Y         | Y            | Medium    | N      |         |
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|               | Nonlinear | Interactions | Tuning    | Smooth | Var Sel |
|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      |         |
| GAMs          | Y         | N            | Low       | Y      |         |
| Ridge/Lasso   | N         | N            | Low       | Y      |         |
| Dec. Trees    | Y         | Y            | Low       | N      |         |
| Rand. Forests | Y         | Y            | Medium    | N      |         |
| Boosting      | Y         | Y            | High      | N      |         |
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|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      |         |
| GAMs          | Y         | N            | Low       | Y      |         |
| Ridge/Lasso   | N         | N            | Low       | Y      |         |
| Dec. Trees    | Y         | Y            | Low       | N      |         |
| Rand. Forests | Y         | Y            | Medium    | N      |         |
| Boosting      | Y         | Y            | High      | N      |         |
| Neural Nets   | Y         | Y            | Very High | Y      |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning    | Smooth | Var Sel |
|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      | N       |
| GAMs          | Y         | N            | Low       | Y      |         |
| Ridge/Lasso   | N         | N            | Low       | Y      |         |
| Dec. Trees    | Y         | Y            | Low       | N      |         |
| Rand. Forests | Y         | Y            | Medium    | N      |         |
| Boosting      | Y         | Y            | High      | N      |         |
| Neural Nets   | Y         | Y            | Very High | Y      |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning    | Smooth | Var Sel |
|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      | N       |
| GAMs          | Y         | N            | Low       | Y      | N       |
| Ridge/Lasso   | N         | N            | Low       | Y      |         |
| Dec. Trees    | Y         | Y            | Low       | N      |         |
| Rand. Forests | Y         | Y            | Medium    | N      |         |
| Boosting      | Y         | Y            | High      | N      |         |
| Neural Nets   | Y         | Y            | Very High | Y      |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning    | Smooth | Var Sel |
|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      | N       |
| GAMs          | Y         | N            | Low       | Y      | N       |
| Ridge/Lasso   | N         | N            | Low       | Y      | N/Y     |
| Dec. Trees    | Y         | Y            | Low       | N      |         |
| Rand. Forests | Y         | Y            | Medium    | N      |         |
| Boosting      | Y         | Y            | High      | N      |         |
| Neural Nets   | Y         | Y            | Very High | Y      |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning    | Smooth | Var Sel |
|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      | N       |
| GAMs          | Y         | N            | Low       | Y      | N       |
| Ridge/Lasso   | N         | N            | Low       | Y      | N/Y     |
| Dec. Trees    | Y         | Y            | Low       | N      | Y       |
| Rand. Forests | Y         | Y            | Medium    | N      |         |
| Boosting      | Y         | Y            | High      | N      |         |
| Neural Nets   | Y         | Y            | Very High | Y      |         |

# Review of Methods

|               | Nonlinear | Interactions | Tuning    | Smooth | Var Sel |
|---------------|-----------|--------------|-----------|--------|---------|
| Regression    | N         | N            | None      | Y      | N       |
| GAMs          | Y         | N            | Low       | Y      | N       |
| Ridge/Lasso   | N         | N            | Low       | Y      | N/Y     |
| Dec. Trees    | Y         | Y            | Low       | N      | Y       |
| Rand. Forests | Y         | Y            | Medium    | N      | ~N      |
| Boosting      | Y         | Y            | High      | N      |         |
| Neural Nets   | Y         | Y            | Very High | Y      |         |

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| GAMs          | Y         | N            | Low       | Y      | N       |
| Ridge/Lasso   | N         | N            | Low       | Y      | N/Y     |
| Dec. Trees    | Y         | Y            | Low       | N      | Y       |
| Rand. Forests | Y         | Y            | Medium    | N      | ~N      |
| Boosting      | Y         | Y            | High      | N      | ~N      |
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| GAMs          | Y         | N            | Low       | Y      | N       |
| Ridge/Lasso   | N         | N            | Low       | Y      | N/Y     |
| Dec. Trees    | Y         | Y            | Low       | N      | Y       |
| Rand. Forests | Y         | Y            | Medium    | N      | ~N      |
| Boosting      | Y         | Y            | High      | N      | ~N      |
| Neural Nets   | Y         | Y            | Very High | Y      | N       |

# Things to Consider When Picking A Method

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- How **many predictors** do you have? How **Informative** are they?
- How much **prior knowledge** do you have?
- How **smooth** is the output function?
- Who do I have to **explain** this too?

# The Big Assumption: Ignorability

**Identification by randomization:**

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- If treatment is randomized, then treatment is unrelated to any and all underlying characteristics, observed and unobserved (and even unknown)

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$$\{Y_i(0), Y_i(1)\} \perp\!\!\!\perp T_i$$

- This is sometimes called **unconfoundedness** or **ignorability**
- Another way of thinking of it: The distributions of the potential outcomes  $(Y_i(1), Y_i(0))$  are the same for the treatment and control group.
- Yet another way of thinking of it: The treatment and control group are exchangeable, or balanced (on observables and unobservables) on average

# How do we proceed?

## Identification by conditional independence:

- If treatment is not randomized, then treatment may be related underlying characteristics, observed and unobserved, which are related to the potential outcomes

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- 2 Prediction Policy Problems
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  - Framework
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  - Empirical Estimand
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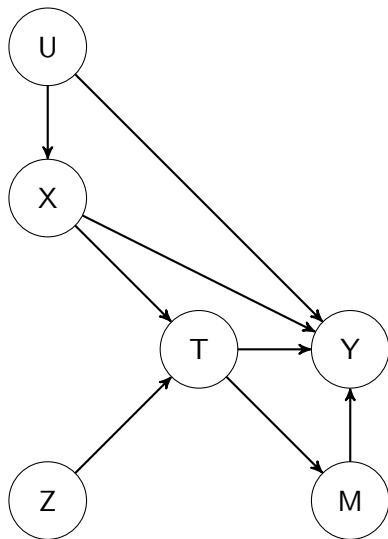
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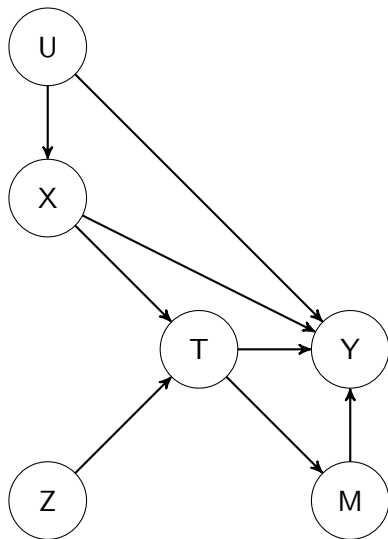
Code Idea: **causal reasoning before statistical reasoning**

# Components of a DAG

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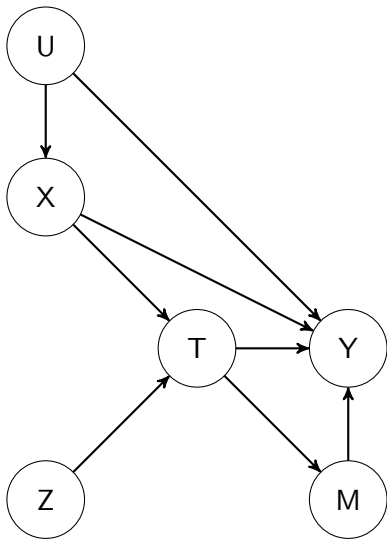


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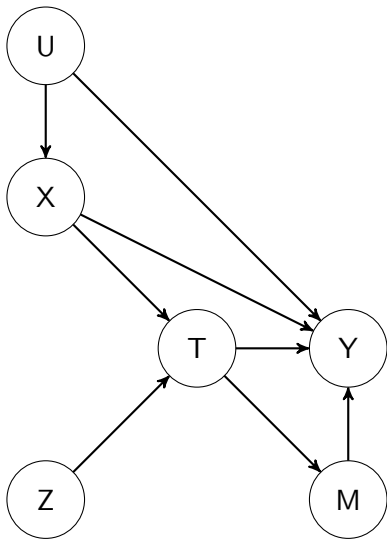
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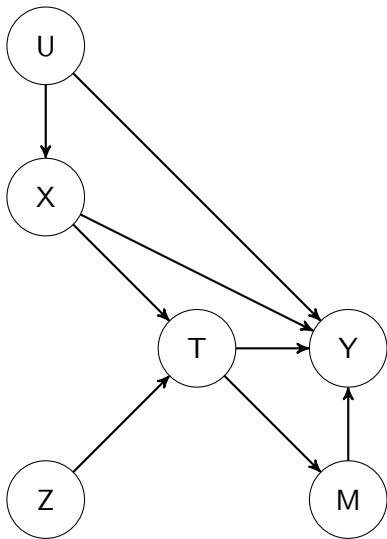
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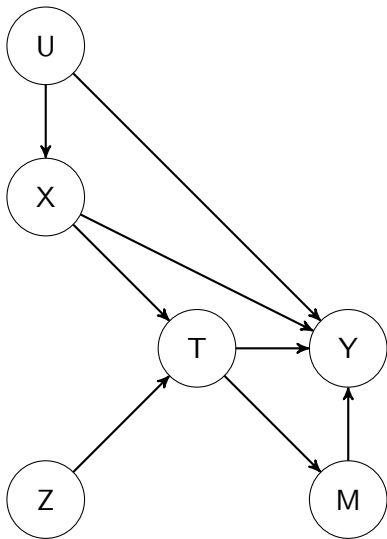
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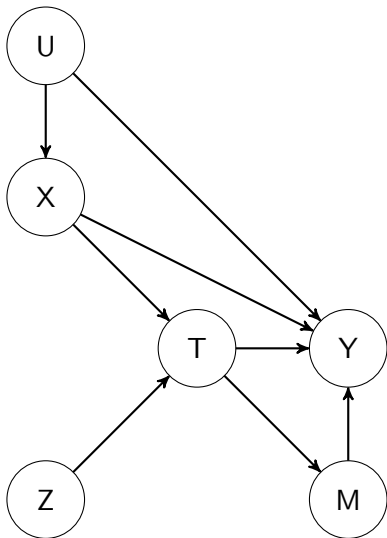
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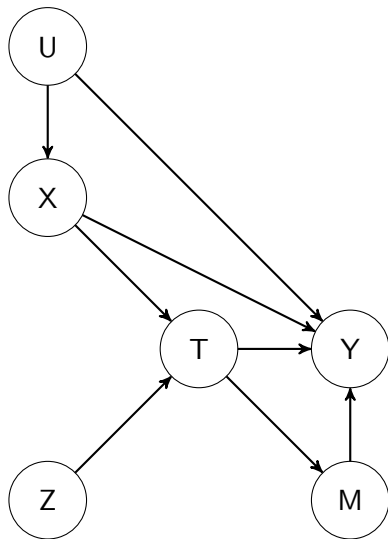
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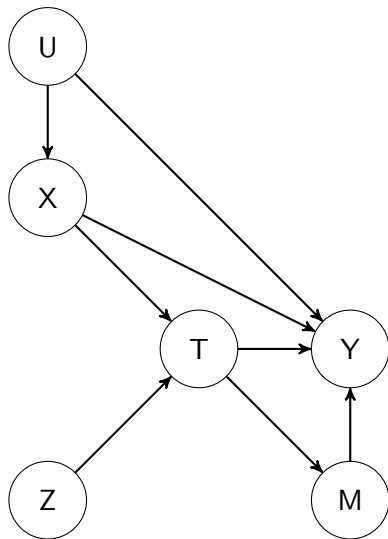
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- all relationships are **non-parametric**

# Relationships in a DAG



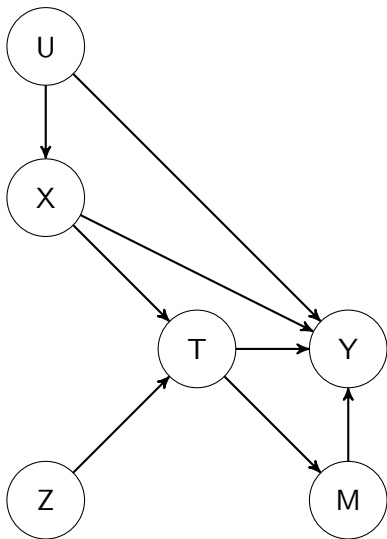
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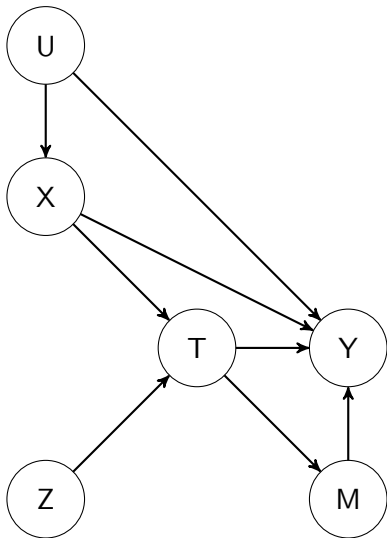
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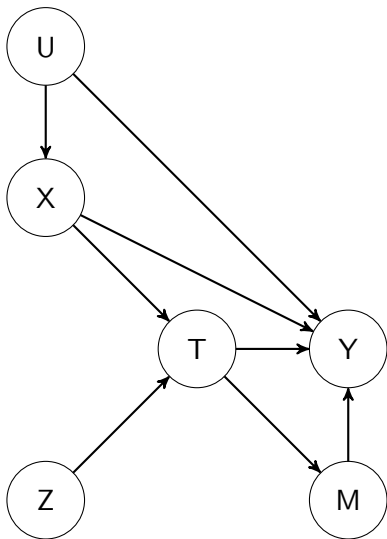
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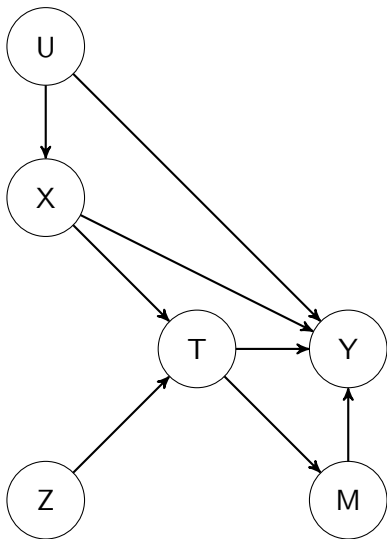
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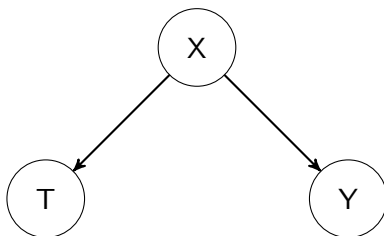
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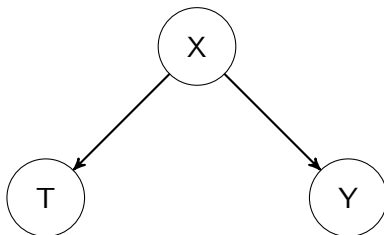
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- We will talk in depth about two types of relationships: **confounders** and **colliders**

# Confounders



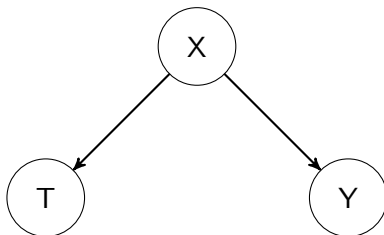
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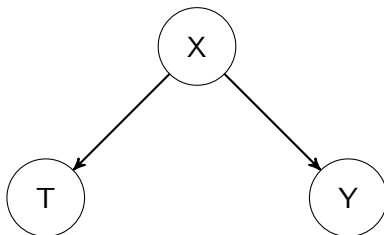
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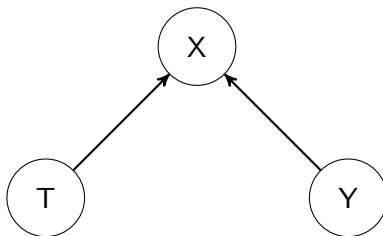
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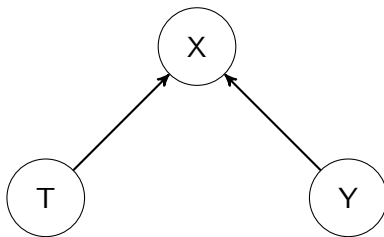
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- We can think of conditioning on a confounder as blocking the flow of association.

# Colliders



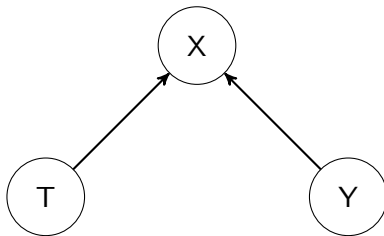
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- If we control for  $X$  they become associated and create a connection between  $T$  and  $Y$

# Colliders are scary because you can induce dependence



## Endogenous Selection Bias: The Problem of Conditioning on a Collider Variable

Felix Elwert<sup>1</sup> and Christopher Winship<sup>2</sup>

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email: elwert@wisc.edu

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email: cwinship@hs.harvard.edu

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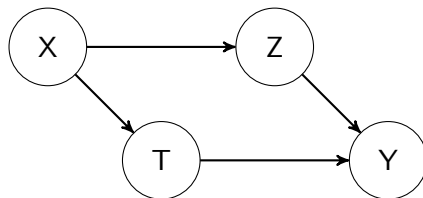
### Keywords

causality, directed acyclic graphs, identification, confounding,  
selection

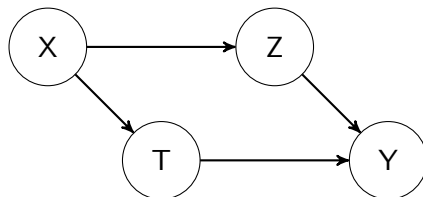
### Abstract

Endogenous selection bias is a central problem for causal inference. Recognizing the problem, however, can be difficult in practice. This article introduces a purely graphical way of characterizing endogenous selection bias and of understanding its consequences (Hernán et al. 2004). We use causal graphs (direct acyclic graphs, or DAGs) to highlight that endogenous selection bias stems from conditioning (e.g., controlling, stratifying, or selecting) on a so-called collider variable, i.e., a variable that is itself caused by two other variables, one that is (or is associated with) the treatment and another that is (or is associated with) the outcome. Endogenous selection bias can result from direct conditioning on the outcome variable, a post-outcome variable, a post-treatment variable, and even a pre-treatment variable. We highlight the difference between endogenous selection bias, common-cause confounding, and overcontrol bias and discuss numerous examples from social stratification, cultural sociology, social network analysis, political sociology, social demography, and the sociology of education.

## From Confounders to Back-Door Paths

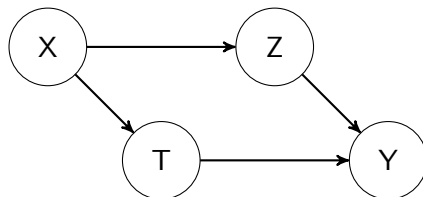


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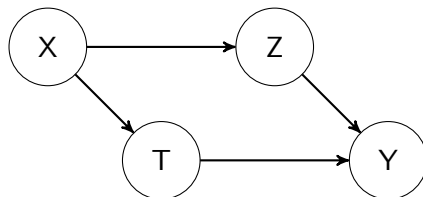
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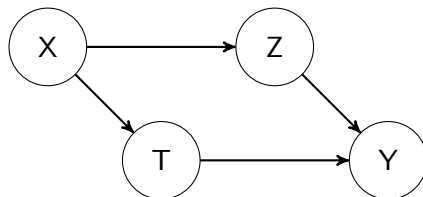
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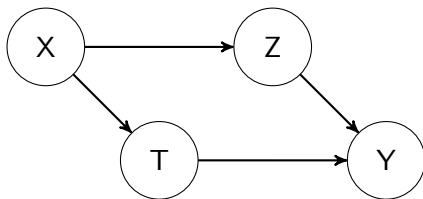
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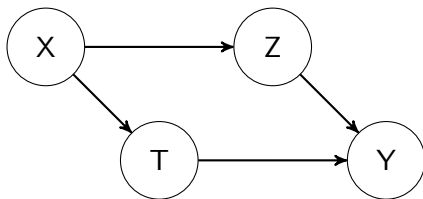
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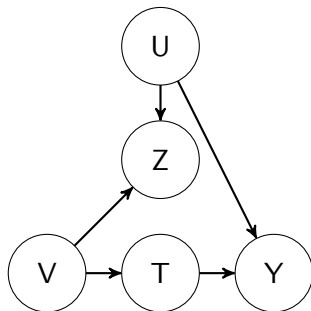
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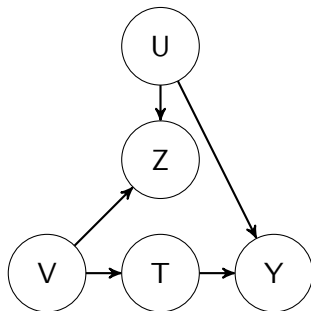
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- We want to **block** the back-door path to leave only the causal effect

# Colliders and Back-Door Paths

- $Z$  is a **collider** and it lies along a back-door path from  $T$  to  $Y$

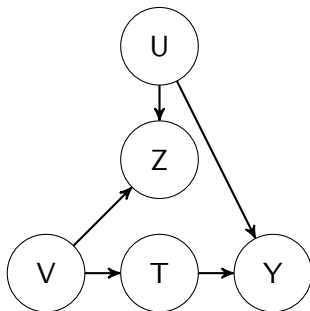


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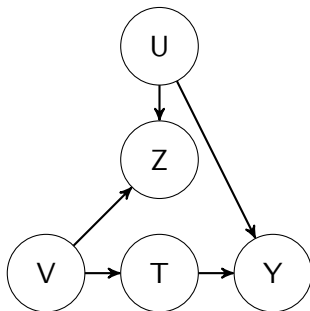
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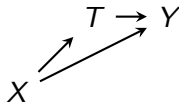
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- So how do we know which back-door paths to block?

## Blocking backdoor paths: College and earnings

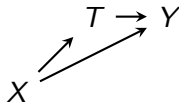
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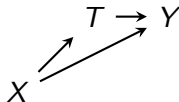
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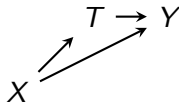
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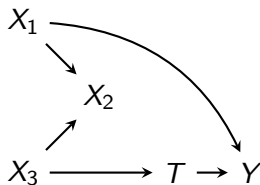
What do we need to include to block all backdoor paths between college and earnings?



Ability, parents' income, parents' education, extended family who pay for college and help you find a job, neighborhood characteristics that affect high school quality and also the availability of local jobs, ... **lots of things!**

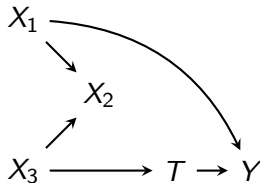
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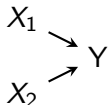
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**No need to condition!**  $X_2$  already blocks this path. it is a **collider**.

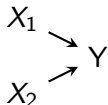
## Colliders: Be careful!



$Y$  is a **collider**.  $X_1$  and  $X_2$  are not associated, but they are when we hold  $Y$  constant.

What situations might produce this?

## Colliders: Be careful!

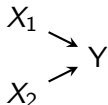


$Y$  is a **collider**.  $X_1$  and  $X_2$  are not associated, but they are when we hold  $Y$  constant.

What situations might produce this?

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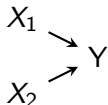


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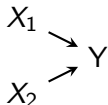


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**Should we worry about this design?** It depends on our **theory** about how these variables are related. We can argue about identification with a DAG.

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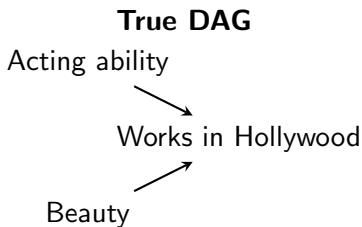
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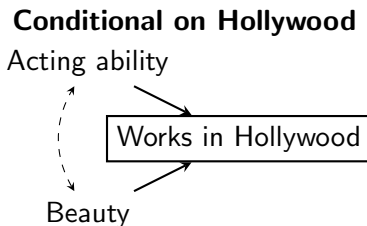
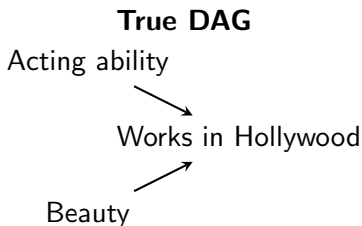


**Conditional on Hollywood**

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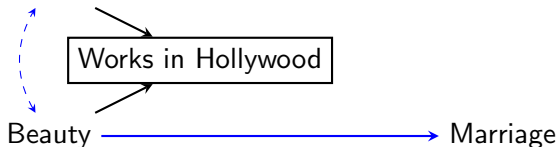


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Example extended from Elwert & Winship 2014

This is an example of conditioning on a collider! We induce a negative association between acting ability and beauty.

Acting ability



Under the assumptions above, our results are driven by collider conditioning!

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  - ▶ Tricky: identification results for identification only holds when variable is completely controlled for (which may be difficult!)

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Morgan and Winship (2014) *Counterfactuals and Causal Inference: Methods and Principles for Social Research*. Cambridge University Press.