Soc 500: Applied Social Statistics
Week 1: Introduction and Probability

Brandon Stewart[1]

Princeton

September 3, 2024

[1] Many of these slides are adapted from slides by Matt Blackwell and Adam Glynn with contributions from Justin Grimmer, Erin Hartman, and Matt Salganik. Illustrations by Shay O'Brien.

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- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causal inference

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   - What is Probability?
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  - Sofia Avila
  - Christina Pao

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- Syllabus is a useful resource including philosophy of the class.

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Why R?

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- It will give you super powers (but not at first)

Artwork by @allison_horst

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- We all come from very different backgrounds. Please have patience with yourself and with others.

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  ask us even more questions

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Problem sets are the most important part of the class!

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Your Job: work hard and get help when you need it!

Staying in Touch

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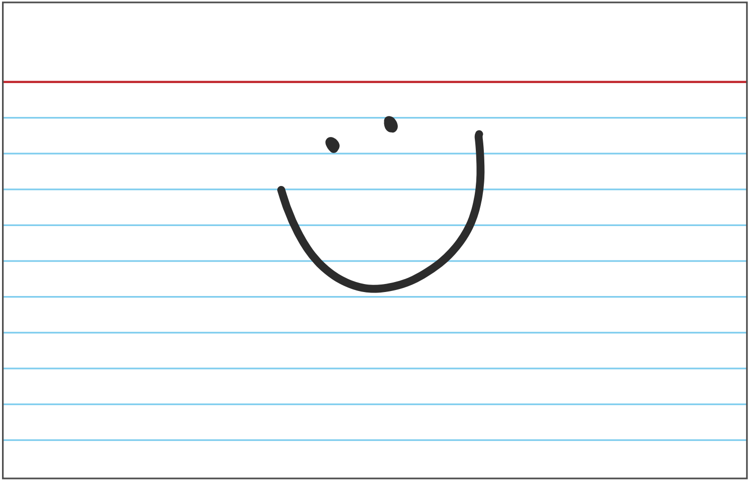
Can we go over Bayes' Rule again?

Staying in Touch



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- A somewhat obvious tip: don't skip the math!

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- Go over your psets and the pset solutions the moment they are graded as a habit and figure out what you don't know.

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Welcome To Class!

Be sure to read the syllabus for more details.

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- In practice, an essential part of research, policy making, political campaigns, selling people things. . .

Why study probability?

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It enables inference.

In Picture Form

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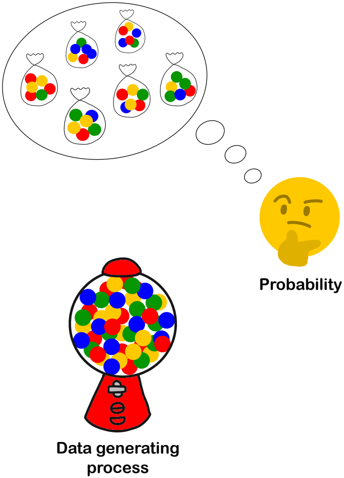

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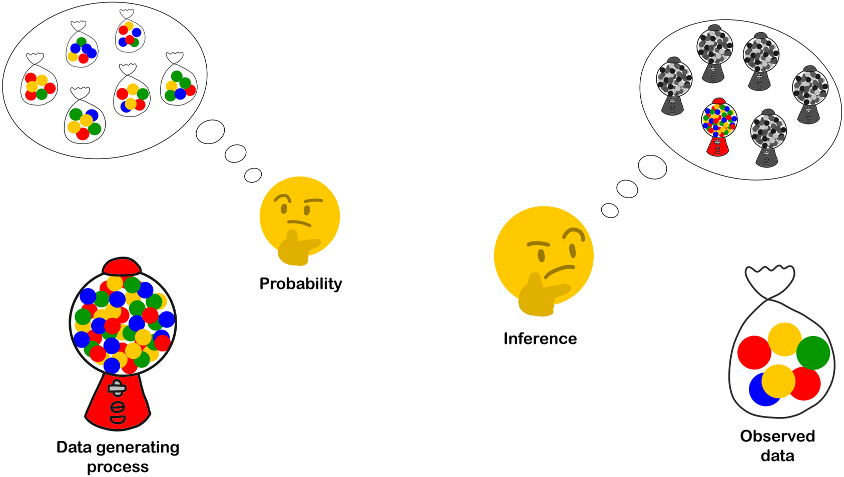

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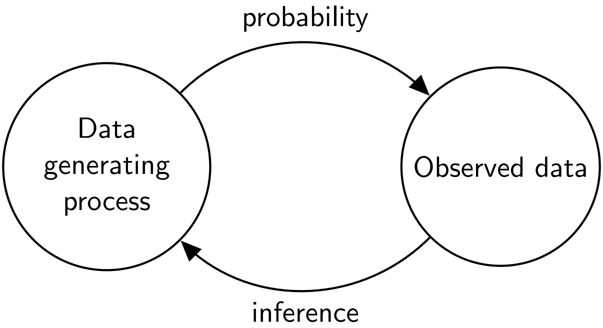

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Tea-Tasting Distribution

Success count	Permutations of selection	Number of permutations
0	oooo	$1 \times 1 = 1$
1	ooox, ooxo, oxoo, xooo	$4 \times 4 = 16$
2	ooxx, oxox, oxxo, xoxo, xxoo, xoox	$6 \times 6 = 36$
3	oxxx, xoxx, xxox, xxxo	$4 \times 4 = 16$
4	xxxx	$1 \times 1 = 1$
Total		70

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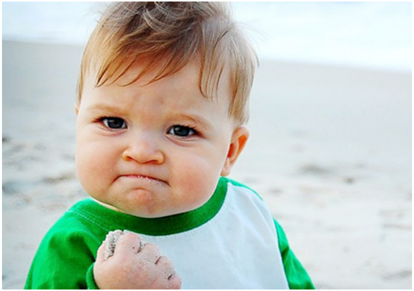

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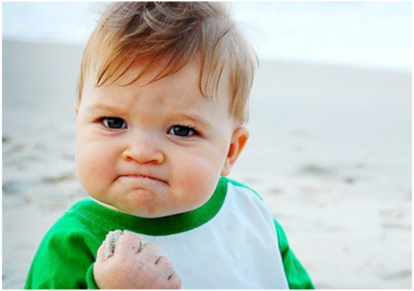

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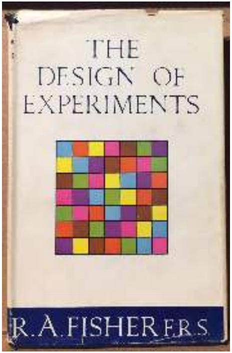

This became the Fisher Exact Test.

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- Statistics has been used intermittently as a force for progress and a force against progress.

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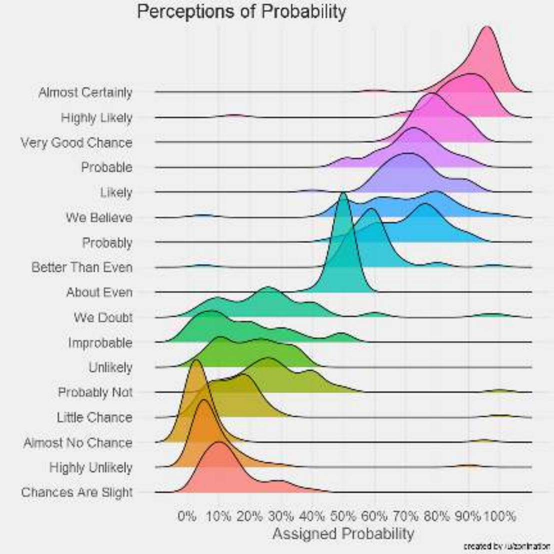
From 'Probably' to Probability



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- Thus: we need to know how probability gives rise to data

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All the rules of probability can be derived from these axioms.
To state them formally, we first need some definitions.

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(Note we defined illogical guesses to be prob$= 0$)

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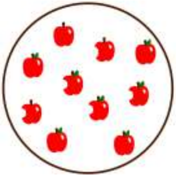

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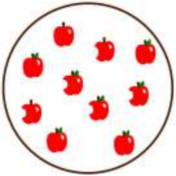

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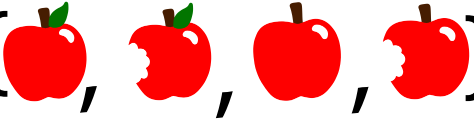
$\Omega = \{$  $\}$

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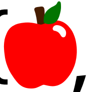
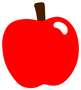
$\Omega = \{$ , , ,  $\}$

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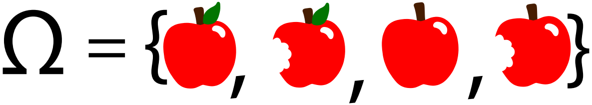

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Example:

If $A = \{\omega | \omega \text{ has a leaf}\}$:

 $\in A$,  $\in A$,  $\not\in A$,  $\not\in A$

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Important complement: $\Omega^c = \emptyset$, where $\emptyset$ is the empty set.

Unions and Intersections

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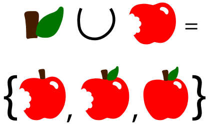

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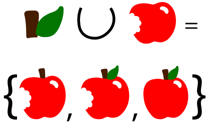

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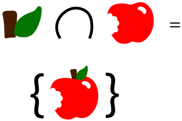

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Sample spaces can have infinite events where we will often write the different events using subscripts of the same letter: $A_1, A_2, \ldots A_\infty$ (e.g. imagine an event that was the count of some object)

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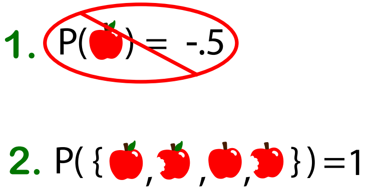

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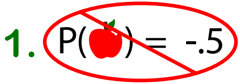

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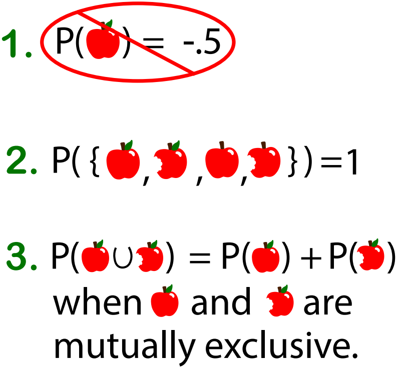

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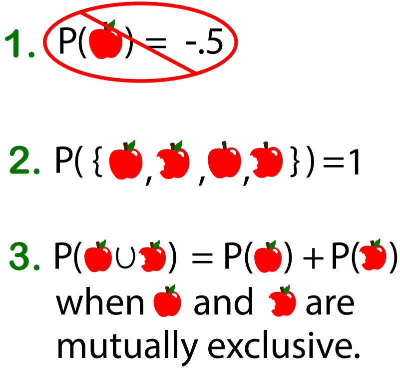

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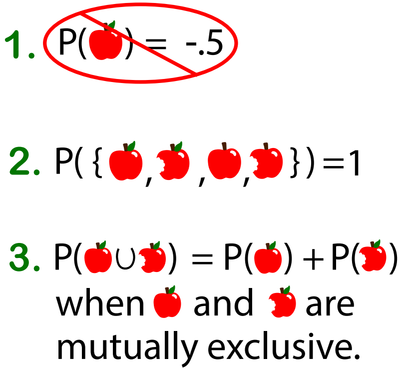

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Three Big Ideas

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Marginal, joint, and conditional probabilities

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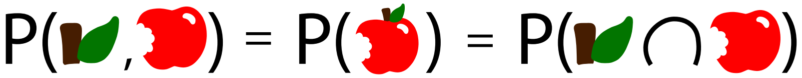

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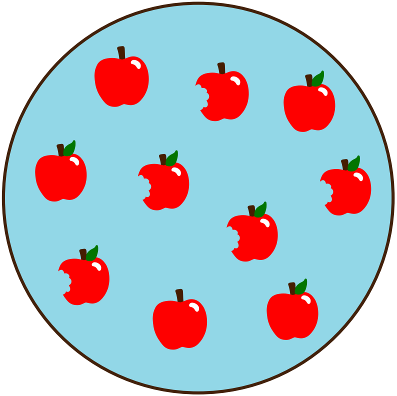

P() = 7/10

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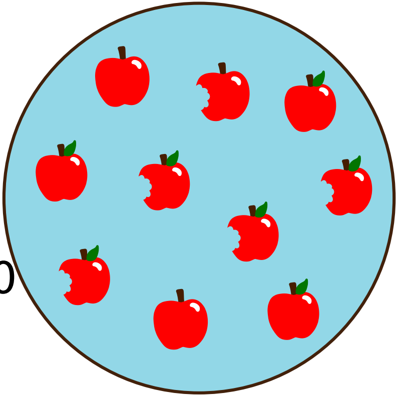

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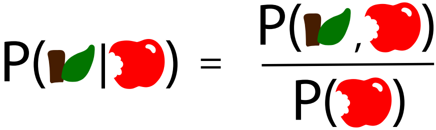
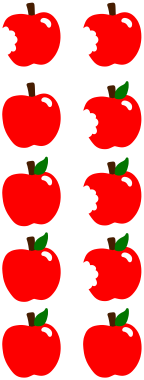
Hopefully this second formulation is intuitive!

Conditional Probability: A Visual Example





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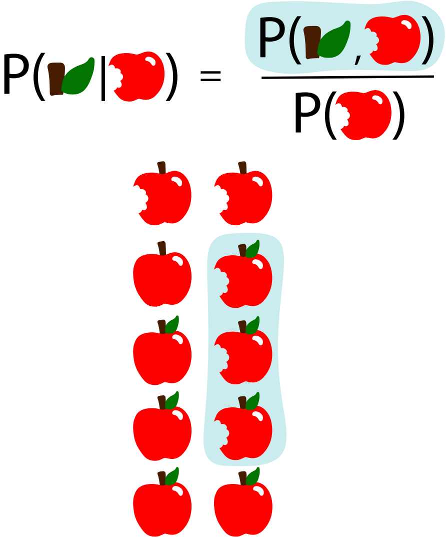

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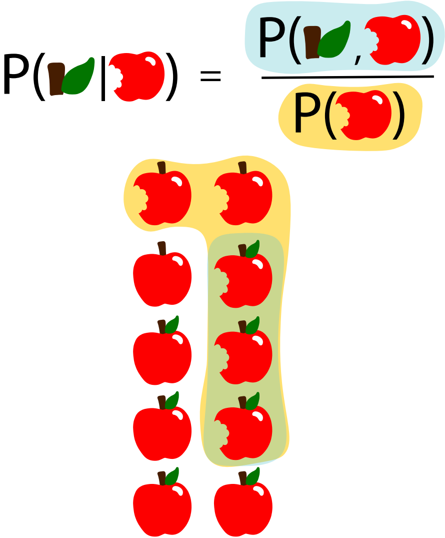

Law of Total Probability (LTP)

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With 2 Events:

$$\begin{aligned} P(B) &= P(B, A) + P(B, A^c) \\ &= P(B|A)P(A) + P(B|A^c)P(A^c) \end{aligned}$$

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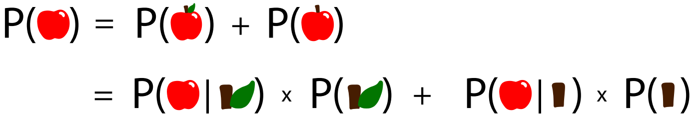

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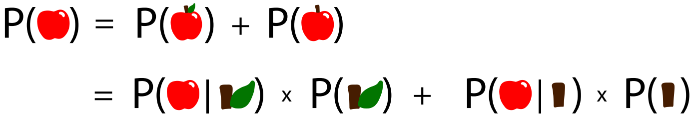

In general, if $\{A_i : i = 1, 2, 3, \dots\}$ forms a partition of the sample space, then

$$
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Engaged and Disengaged Respondents

PNAS RESEARCH ARTICLE | POLITICAL SCIENCES OPEN ACCESS

Current research overstates American support for political violence

Sean J. Westwood, Justin Grimmer, Matthew Tyler, and Clayton Nall

Edited by Anthony Fowler, University of Chicago, Chicago, IL; received [illegible]; accepted [illegible] by Editorial Board Member Margaret Levi

Political scientists, pundits, and citizens worry that America is entering a new period of violent partisan conflict. Provocative survey data show that a large share of Americans (between 8% and 40%) support politically motivated violence. Yet, despite media attention, political violence is rare, amounting to a little more than 1% of violent hate crimes in the United States. We reconcile these seemingly conflicting facts with four large survey experiments ($n = 4,904$), demonstrating that self-reported attitudes on political violence are biased upward because of respondent disengagement and survey questions that allow multiple interpretations of political violence. Addressing question wording and respondent disengagement, we find that the median of existing estimates of support for partisan violence is nearly 6 times larger than the median of our estimates (18.5% versus 2.9%). Critically, we show the prior estimates overstate support for political violence because of random responding by disengaged respondents. Respondent disengagement also inflates the relationship between support for violence and previously identified correlates by a factor of 4. Partial identification bounds imply that, under generous assumptions, support for violence among engaged and disengaged respondents is, at most, 6.86%. Finally, nearly all respondents support criminally charging suspects who commit acts of political violence. These findings suggest that, although recent acts of political violence dominate the news, they do not portend a new era of violent conflict.

political violence | affective polarization | democratic norms

Significance

Recent political events show that members of extreme political groups support partisan violence, and survey evidence supposedly shows widespread public support. We show, however, that after accounting for survey-based measurement error, support for partisan violence is far more limited. Prior estimates overstate support for political violence because of random responding by disengaged respondents and because of a reliance on hypothetical questions about violence in general instead of questions on specific acts of political violence. These same issues also cause the magnitude of the relationship between previously identified correlates and partisan violence to be overstated. As policy makers consider interventions designed to dampen support for violence, our results provide critical information about the magnitude of the problem.

Provocative recent work (1–3)—cited in PNAS (4, 5), *The American Journal of Political Science* (6), 60 other articles and books, and 40 news articles that, together, have garnered over 2,281,159 Twitter engagements—asserts that large segments of the American population now support politically motivated violence. These studies report that up to 44% of Americans would endorse hypothetical violence in some undetermined future event (1–3, 7). This survey work fits within a media landscape that regularly raises the specter of political violence. Since 2016, we counted 7,863 mentions of political violence on news television, more than 630 news stories about political violence, and over 10 million tweets on the topic of the January 6 riot alone (see *SI Appendix*, section 1 for details for all counts in this paragraph). Political violence, however, remains exceedingly rare in the United States, amounting to 48 incidents (8) in 2019 (the most recent year for which data are available) compared to 4,526 incidents of nonpolitical violent hate crimes (9) and 1,203,808 total violent crimes (10) documented by the Department of Justice.

Three Big Ideas

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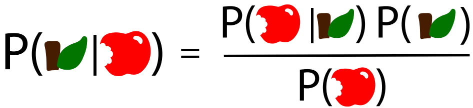
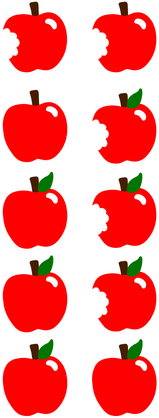
- Bayes' rule: if $P(B) > 0$, then:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes' Rule Mechanics





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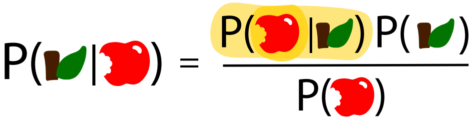

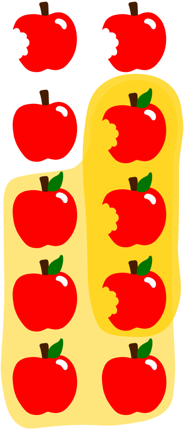

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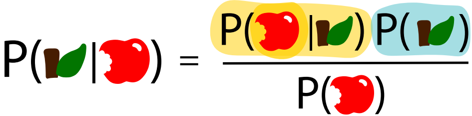

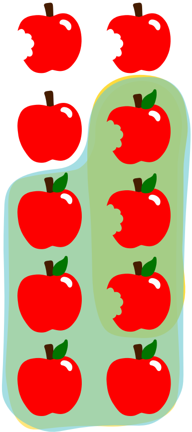

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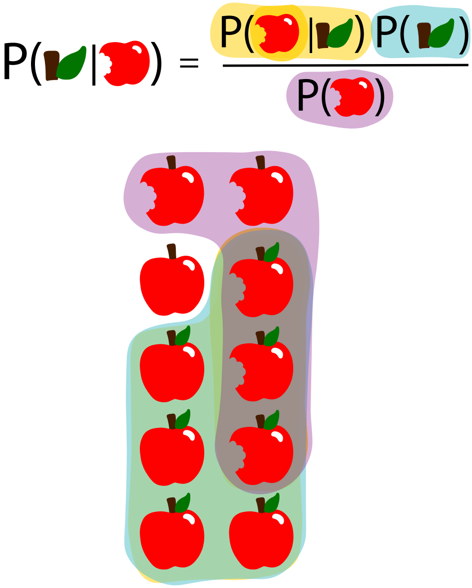
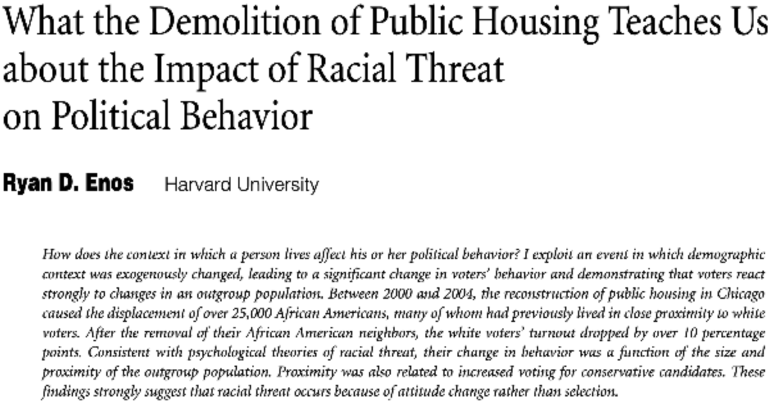


Example: Race and Names


What the Demolition of Public Housing Teaches Us about the Impact of Racial Threat on Political Behavior

Ryan D. Enos Harvard University

How does the context in which a person lives affect his or her political behavior? I exploit an event in which demographic context was exogenously changed, leading to a significant change in voters' behavior and demonstrating that voters react strongly to changes in an outgroup population. Between 2000 and 2004, the reconstruction of public housing in Chicago caused the displacement of over 25,000 African Americans, many of whom had previously lived in close proximity to white voters. After the removal of their African American neighbors, the white voters' turnout dropped by over 10 percentage points. Consistent with psychological theories of racial threat, their change in behavior was a function of the size and proximity of the outgroup population. Proximity was also related to increased voting for conservative candidates. These findings strongly suggest that racial threat occurs because of attitude change rather than selection.

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$\boxed{\text{Independence is a massively important concept in statistics.}}$

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Deploy with Caution!

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- Set us up to do inference (i.e. run that backwards!).
- A mathematical model that hopefully corresponds well to real behavior $\rightarrow$ sometimes this correspondence is direct and other times it is more like a loose analogy.
- It can feel frustrating to start with probability if you came to class wanting to analyze data, but it is the single most important piece of infrastructure.

This Week in Review

- Course logistics
- Core ideas in statistics
- Foundations of probability
- Three big probability concepts

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Next week: random variables!

i.e. how we link more complex real-world sample spaces to numerical recipes that we can reuse.

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   - What is Probability?
   - Sample Spaces and Events
   - Probability Functions
4. Three Big Ideas in Probability
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   - Bayes' Rule
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6. Definition of Random Variables
7. Discrete Distributions
8. Continuous Distribution

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F(μn!)
WITH

The New York Times

Comey and McCabe, Who Infuriated Trump, Both Faced Intensive I.R.S. Audits

The former F.B.I. director and his deputy, both of whom former President Donald J. Trump wanted prosecuted, were selected for a rare audit program that the tax agency says is random.

"Maybe it's a coincidence or maybe somebody misused the I.R.S. to get at a political enemy," said James B. Comey, the former F.B.I. director. "Given the role Trump wants to continue to play in our country, we should know the answer to that question." Jared Soares for The New York Times

TheUpshot

How Unlikely Is It That the Audits of Comey and McCabe Were a Coincidence? A Statistical Exploration.

The chances are minuscule. But minuscule is not zero.

Give this article 1K

James B. Comey was fired as F.B.I. director by President Donald J. Trump in 2017. Mr. Comey was also selected for a rare type of tax audit for his 2017 return. Doug Mills/The New York Times

"It's a little like the irresistible force and the immovable object," said Andrew Gelman, a professor of statistics and political science at Columbia University, when told in the abstract about this exercise. "On the one hand, you're saying it's completely random. On the other hand, you suspect it's not."

Mr. Gelman, like every other statistician who spoke with The Times about this problem, said the biggest hurdle was not any of the details but defining the question itself.

When we try to calculate the probability of a given event *because* we suspect it may not be random, we end up in the complicated position of trying to imagine how we would have predicted the likelihood of the event *before* it happened, said David Spiegelhalter. He heads the Winton Centre for Risk and Evidence Communication at the University of Cambridge, an organization dedicated to improving the way quantitative evidence is used in society.

The math is easy, he said, but formulating the question is tricky, bordering on "meaningless," in large part because of how hard it is to pin down the group we care about.

"'What's the chance of this happening?' is an easy statement to make," he said. "It's a familiar statement to make. But, actually, it's a very difficult question to answer."

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We will do this by introducing a random variable X to be Barack Obama's position on the 2008 New Hampshire primary ballot.

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We will generally suppress the function notation and just refer to X.

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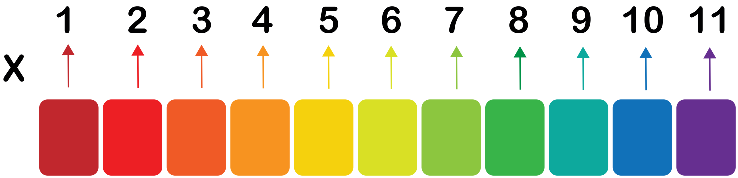

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- Other times the sample space is already numeric so its more obvious (e.g. how many minutes until the train arrives).

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  random variables are about bridging the abstract math and the concrete world. that can be hard, but it is super important and better than the alternative!

NH Ballot Order Example

Candidates:

- Joe Biden
- Hillary Clinton
- Chris Dodd
- John Edwards
- Mike Gravel
- Dennis Kucinich
- Barack Obama
- Bill Richardson

$$X = \left\{ \vphantom{\begin{matrix} \\ \\ \\ \\ \\ \\ \\ \end{matrix}} \right.$$

A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P,Q,R,S,T,U,V,W,X,Y,Z

X is a random variable indicating Obama's position on the ballot. Highlighted letters are those leading to a given ballot position. Highlighted individual is first.

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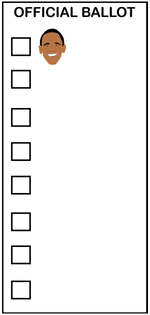

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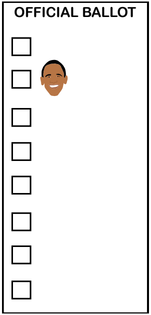
- Barack Obama
- Bill Richardson

$$X = \begin{cases} 1 \\ 2 \end{cases}$$



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X is a random variable indicating Obama's position on the ballot. Highlighted letters are those leading to a given ballot position. Highlighted individual is first.

NH Ballot Order Example

Candidates:

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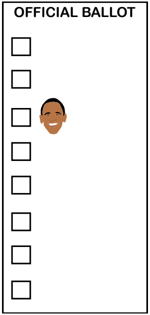
$$X = \begin{cases} 1 \\ 2 \\ 3 \\ \\ \\ \\ \\ \end{cases}$$



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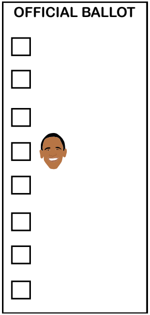
$$X = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \end{cases}$$



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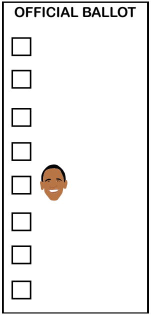
$$X = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{cases}$$



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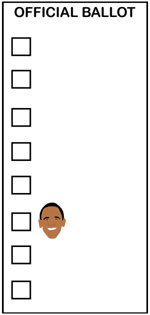
$$X = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \\ \\ \end{cases}$$



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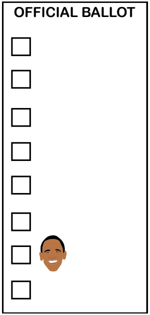
$$X = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{cases}$$



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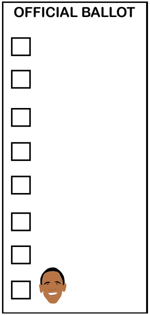
$$X = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ {\color{red}8} \end{cases}$$



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- A probability mass function (PMF) and a cumulative distribution function (CDF) are two common ways to define the probability distribution for a discrete random variable.
- Probability mass functions provide a compact way to represent information about how likely various outcomes are.

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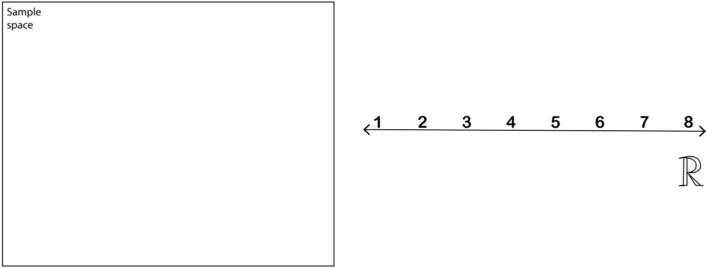

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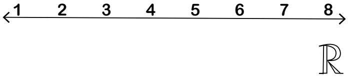

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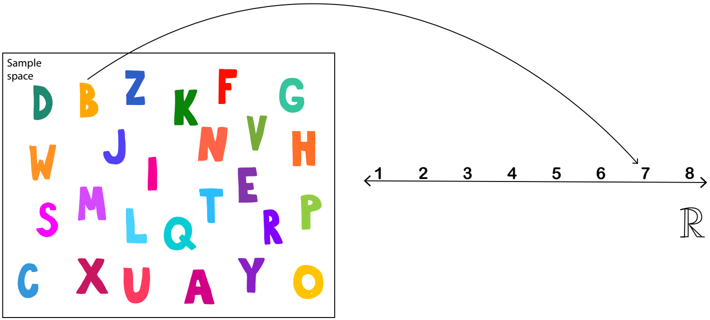

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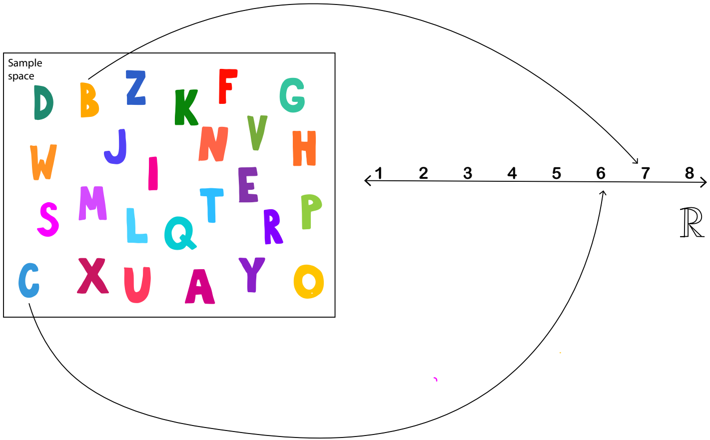

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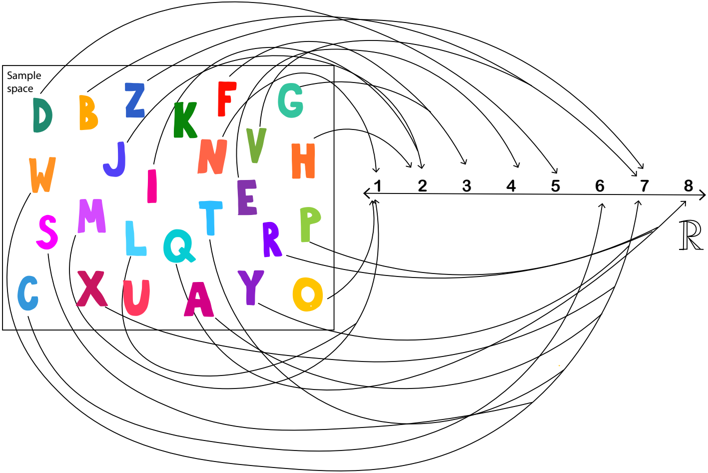

Example: New Hampshire

Candidates:

- Joe Biden
- Hillary Clinton
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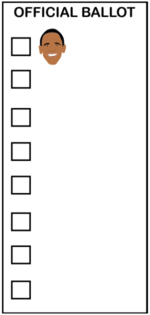
$$p_X(x) = \begin{cases} 4/26 & x = 1 \\ & \end{cases}$$



A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P,Q,R,S,T,U,V,W,X,Y,Z

Probability of the random variable equaling a number is just the probability of the underlying event (subset of the sample space).

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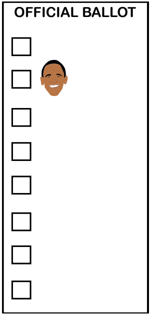

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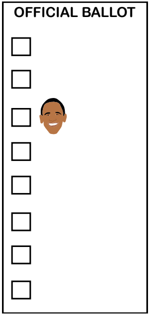

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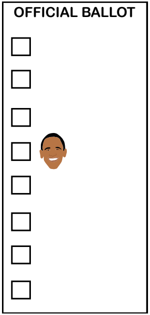

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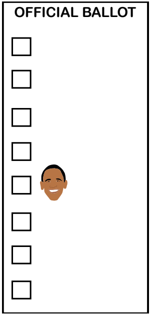

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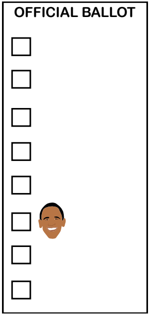

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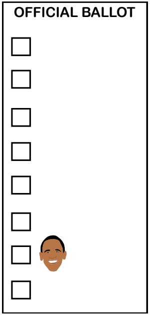

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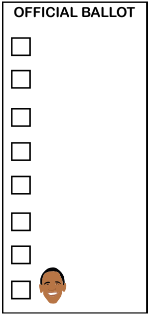

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Discrete Probability Mass Functions

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Definition (Probability Mass Function)

The probability mass function (PMF) of a discrete random variable X is the function p_X given by,

$$p_X(x) = P(X = x)$$

Understanding the Notation:

- $X = x$ is defining an event.

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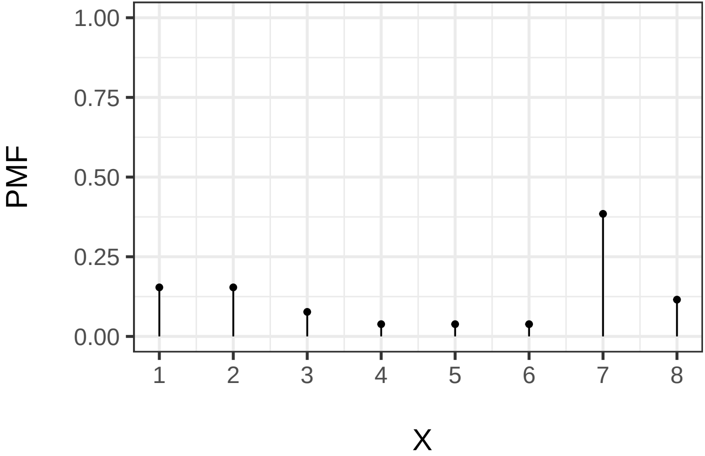
- $\sum_x p_X(x) = 1$.

NH Obama Ballot Position PMF Plot



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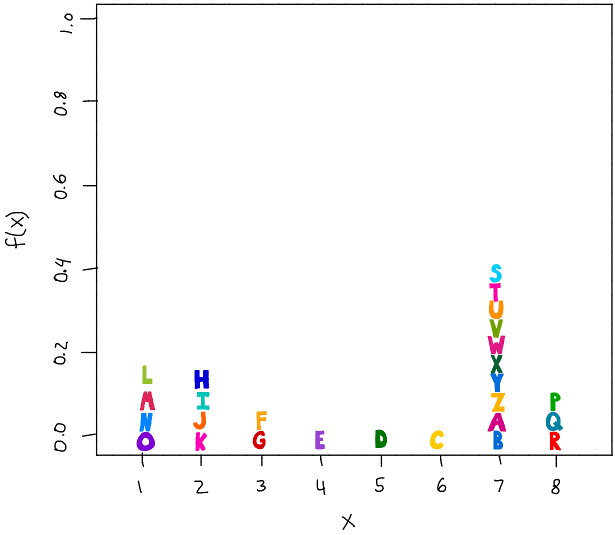

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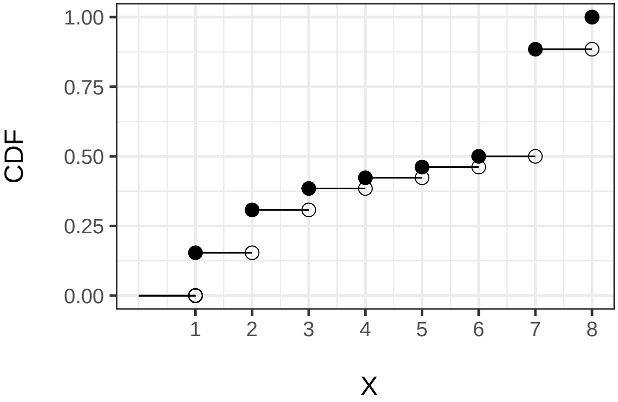
- right-continuous
- converges to 0 and 1 in the limits

NH Obama Ballot Position CDF Plot



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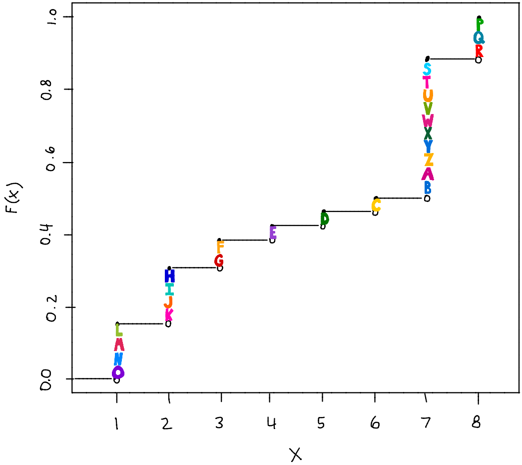

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- We can summarize these distributions with one number

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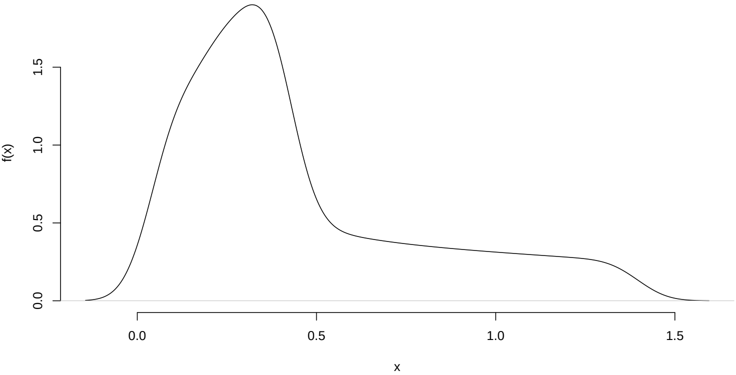
- They are similar to the discrete case with a few subtle differences.

Calculus Review: Integration

Suppose we have some function $f(x)$



Calculus Review: Integration

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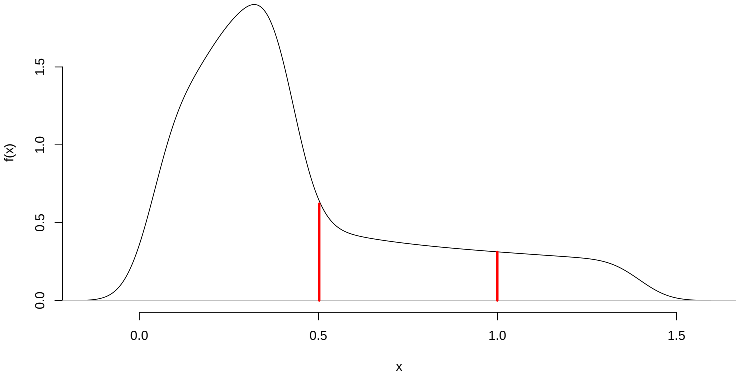

What is the area under $f(x)$ between $\frac{1}{2}$ and 1?

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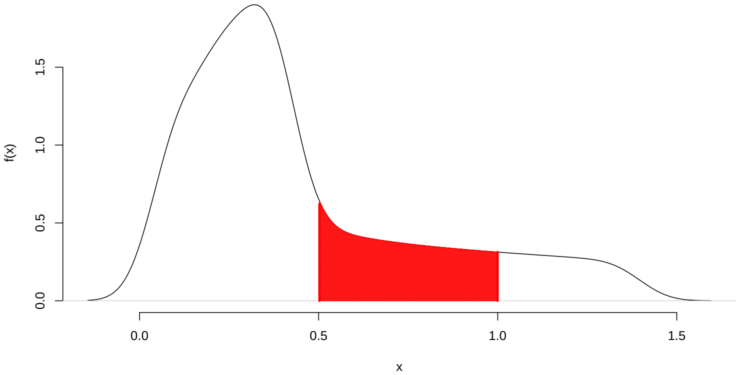

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Area under curve $= \int_{1/2}^{1} f(x)dx = F(1) - F(1/2)$

Continuous Random Variable

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Definition (Continuous Distribution)

A random variable has a continuous distribution if its CDF is differentiable. We also allow there to be endpoints (or finitely many points) where the CDF is continuous but not differentiable, as long as the CDF is differentiable everywhere else. (Blizstein and Hwang Definition 5.1.1)

Probability Density Function (PDF)

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Key Properties:

- non-negative
- integrates to 1. $\int_{-\infty}^{\infty} f_X(x)dx = 1$
- for any measurable set of real numbers B,

$$P(X \in B) = \int_B f_X(x)dx$$

Defining the CDF in terms of the PDF

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Definition (CDF of a Continuous Random Variable)

For a continuous random variable X define its cumulative distribution function $F_X(x)$ as,

$$F_X(t) = P(X \leq t) = \int_{-\infty}^{t} f_X(x)dx$$

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$$\begin{tikzcd} \text{pdf} \arrow[r, bend left, "\text{Integrate}"] & \text{cdf} \arrow[l, bend left, "\text{Differentiate}"] \end{tikzcd}$$

A Visual Example

Imagine you choose a number completely at random between 0 and 1 with all equally sized sets of values being equally likely.

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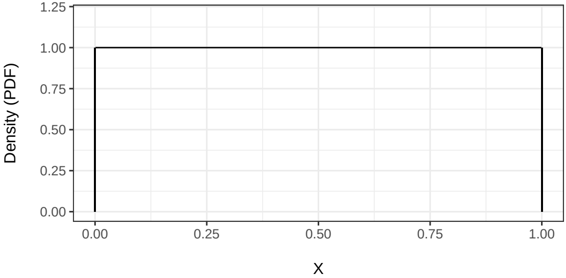
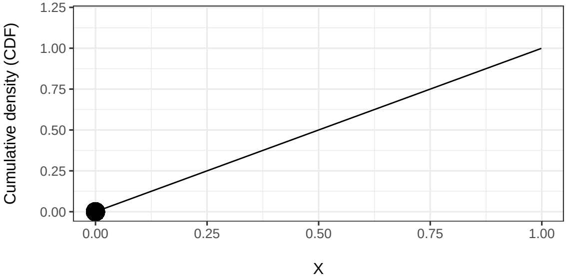
Imagine you choose a number completely at random between 0 and 1 with all equally sized sets of values being equally likely. This is a standard uniform distribution which has the CDF,

$$F_X(x) = x$$

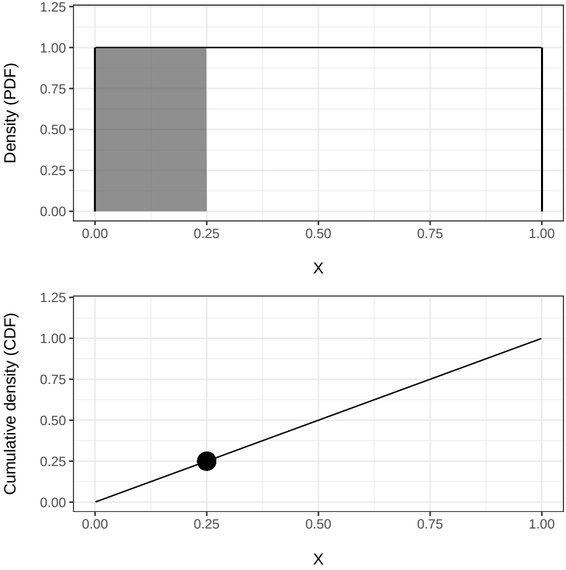
with support over $[0, 1]$.



Density (PDF)
1.25
1.00
0.75
0.50
0.25
0.00
0.00
0.25
0.50
0.75
1.00
X



Cumulative density (CDF)
1.25
1.00
0.75
0.50
0.25
0.00
0.00
0.25
0.50
0.75
1.00
X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0.25
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1.00
X
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0.75
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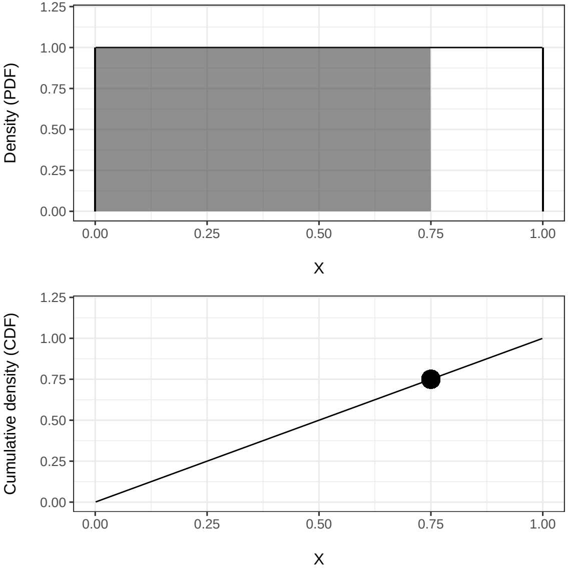
0.25
0.00
0.00
0.25
0.50
0.75
1.00
X



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0.25
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0.50
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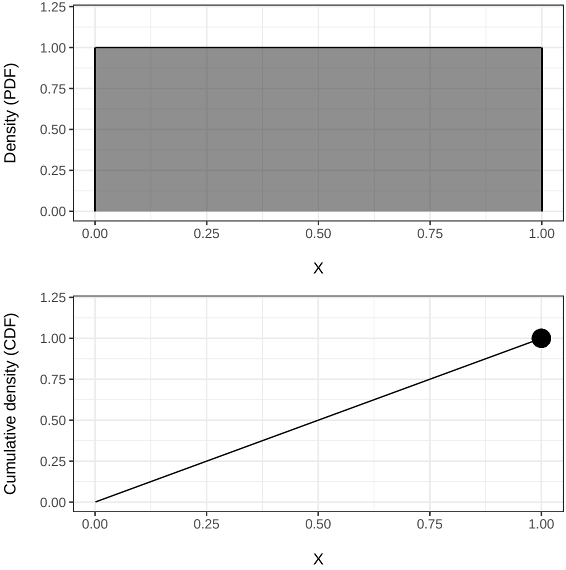
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0.75
1.00
X

What is the probability that the number is between 0.25 and 0.5?

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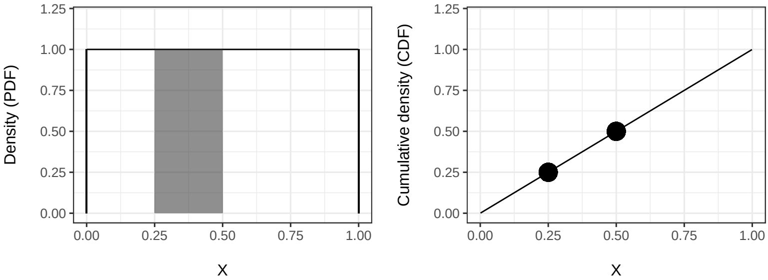

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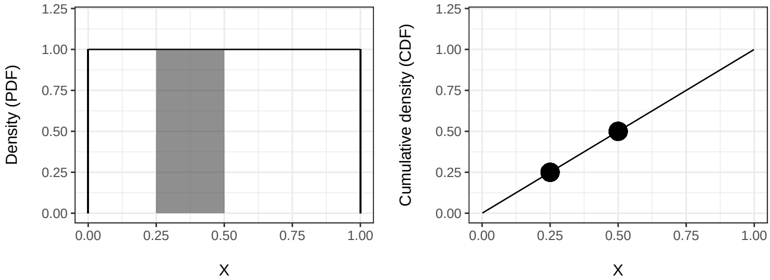

$$F_X(.5) - F_X(.25) = .25$$

The Core PMF/PDF Difference

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This means (perhaps counterintuitively) that a probability density function:

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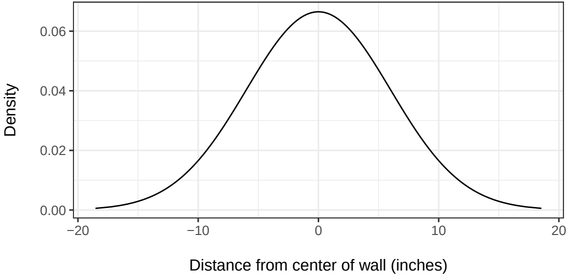
Let's explain!

A Numerical Example

Let's suppose we have someone throwing darts and we measure how far they are from the center of the wall in inches. In this case, perhaps the darts will be distributed with the following PDF.



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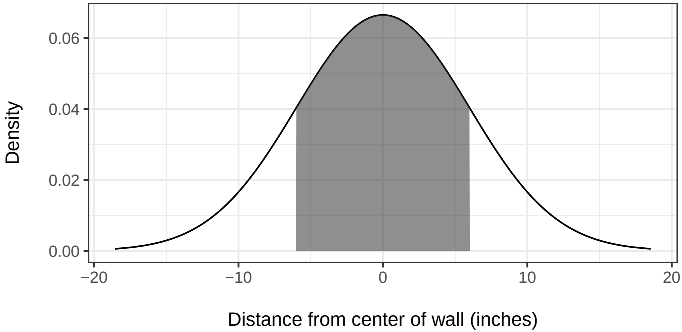

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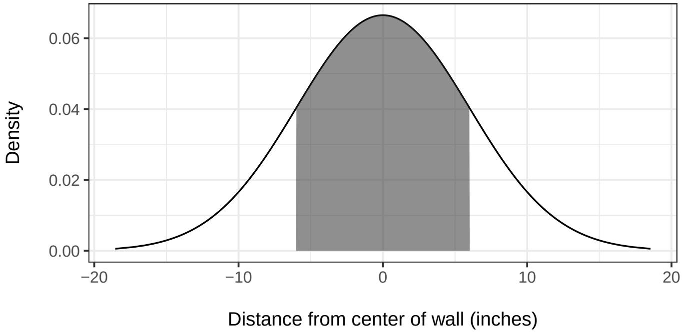

$$P(X \in (-6, 6)) \quad =$$

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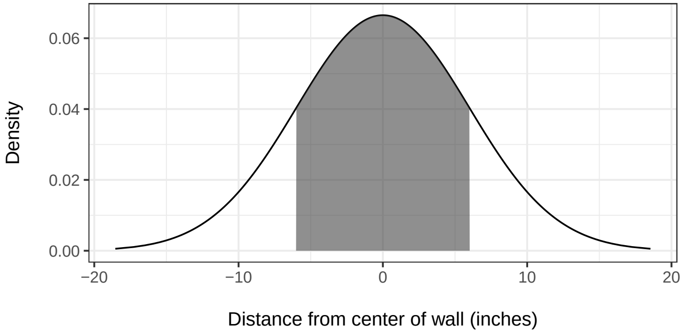

$$P(X \in (-6, 6)) \quad = \quad \int_{-6}^{6} f_X(x) dx$$

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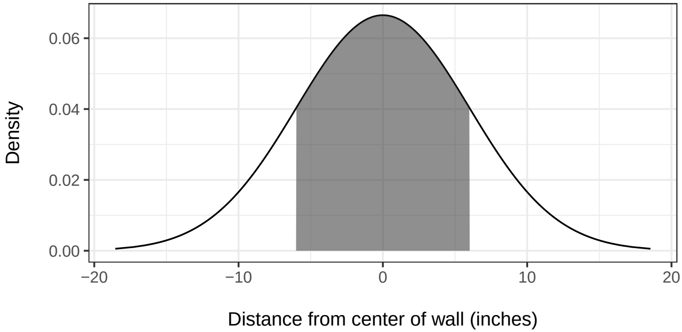

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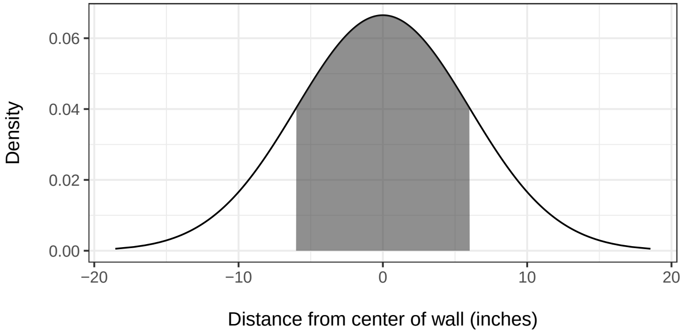

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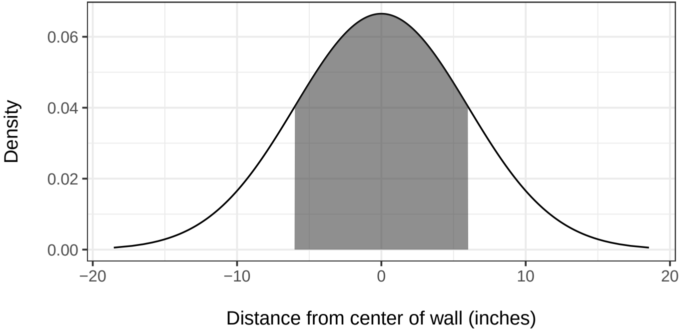
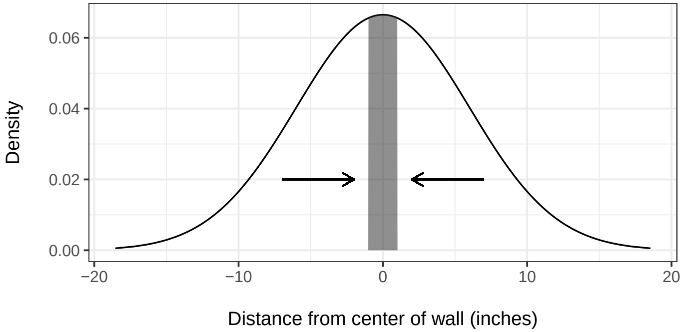


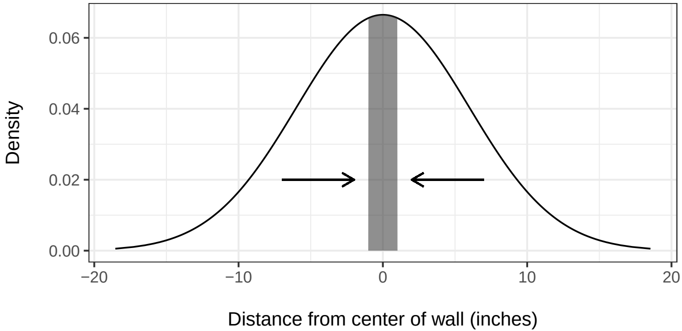
$$
\begin{aligned}
P(X \in (-6, 6)) &= \int_{-6}^{6} f_X(x)dx \\
&= F_X(6) - F_X(-6) \\
&= P(X < 6) - P(X < -6) \\
&= 0.683
\end{aligned}
$$

One inch?



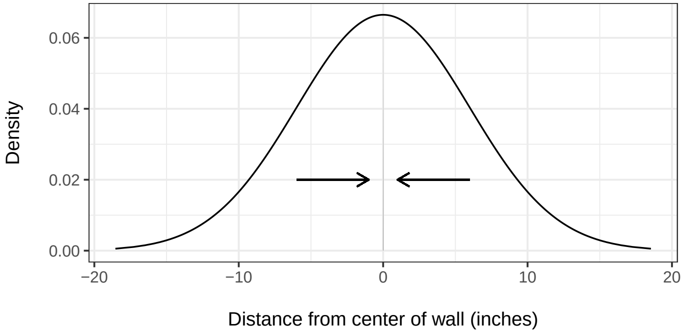
Density
0.06
0.04
0.02
0.00
−20
−10
0
10
20
Distance from center of wall (inches)

One inch?



$$P(X \in (-1, 1)) = 0.0664135$$

1/100th of an inch?



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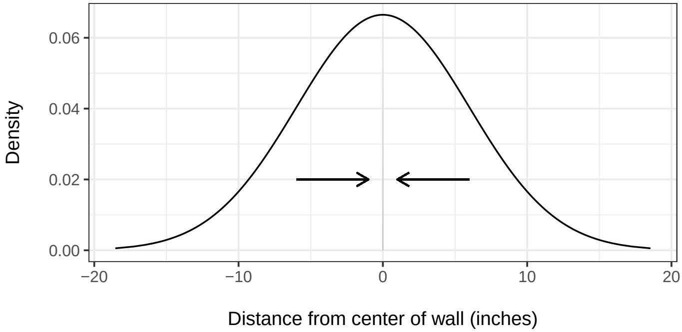
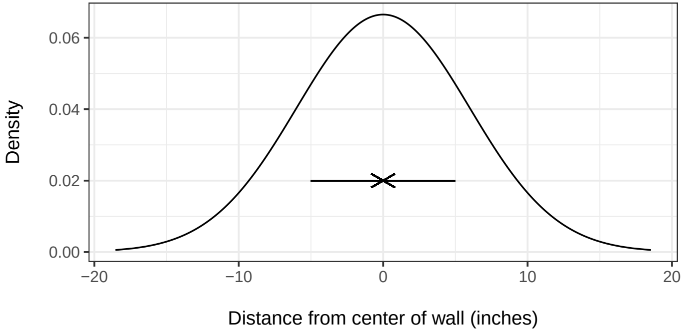


$$P(X \in (-.01, .01)) = 0.0006649037$$

A perfect bullseye?



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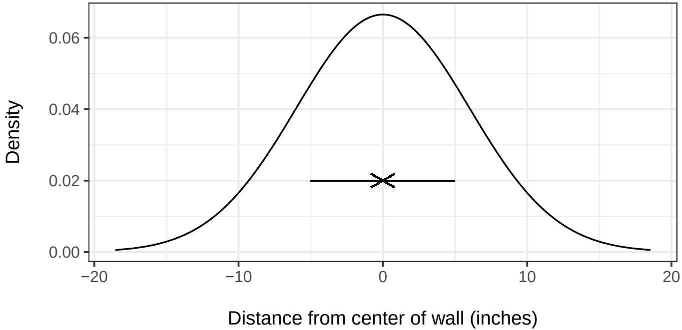

$$P(X = 0) = 0$$

The probability that a continuous variable takes on a discrete value is 0!

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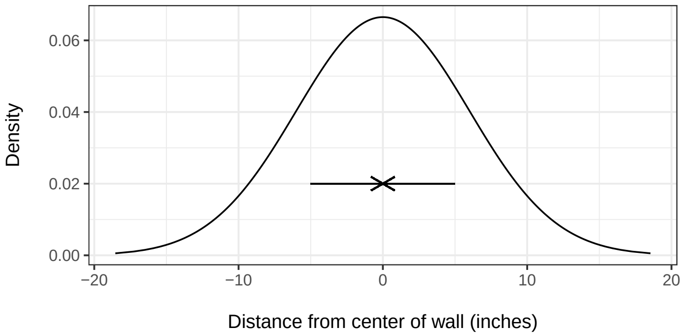

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Why?

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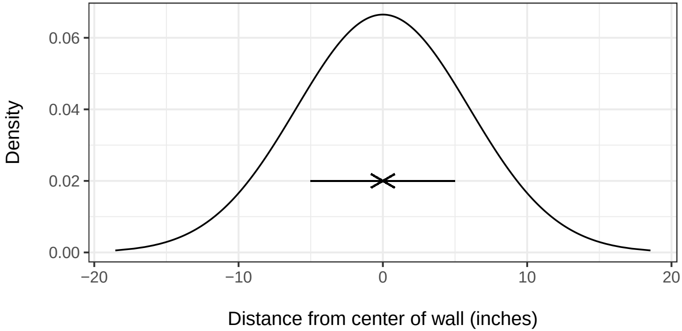

$$P(X = 0) = 0$$

The probability that a continuous variable takes on a discrete value is 0!

Why?

Because the width of the range we are calculating is zero, the area is zero.

The Practical Upshot

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- Reconciling the continuous/discrete divide is the purview of measure theory which is a layer deeper than we are going to go in this class.
- As with discrete random variables there are common families of distributions.

References

- Enos, Ryan D. "What the demolition of public housing teaches us about the impact of racial threat on political behavior." *American Journal of Political Science* (2015).
- Salsburg, David. *The Lady Tasting Tea: How Statistics Revolutionized Science in the Twentieth Century* (2002).