Week 10: Causality with Measured Confounding

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Princeton

November 12–14, 2024

[1]These slides are heavily influenced by Matt Blackwell, Jens Hainmueller, Erin Hartman, Kosuke Imai, Gary King, Ian Lundberg, Richard Nielsen, and Matt Salganik.

Where We've Been and Where We're Going...

- Last Week
  - diagnostics
- This Week
  - review to this point
  - parametric g methods
  - matching as nonparametric preprocessing
  - fixed effects
  - sensitivity analysis
- Next Week
  - approaches with unmeasured confounding
- Long Run
  - causal frameworks $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causal inference

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  - Dec 3–5: Diff-in-Diff + Wrap-Up (Week 12), Pset 11 due 12/5, Precept 12/6
  - For Dec 3: Read Lundberg, Ian, Rebecca Johnson, and Brandon M. Stewart. "What is your estimand? Defining the target quantity connects statistical evidence to theory." *American Sociological Review* 86.3 (2021): 532-565.

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  - Dec 6: Review Session during "Normal" Precept Time
  - Dec 14?–19: Final Exam Period (Discussion)

1. Regression with Measured Confounding
   - Regression with Constant Effects
   - Additive Regression with Heterogeneous Effects
   - Imputation Estimators
2. Matching
   - Fundamentals of Matching
   - Two Approaches to Matching
3. Fixed Effects
   - Non-parametric Identification and Fixed Effects

From Stratification to Regression

- Nonparametric identification yielded this estimand:

$$E[Y_i(1) - Y_i(0)] = E\Big[E[Y_i \mid T_i = 1, \mathbf{X}_i] - E[Y_i \mid T_i = 0, \mathbf{X}_i]\Big]$$

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- Ignorability tell us that the counterfactual quantities can be written in terms of these quantities based on observed data, but we still need to estimate those conditional expectations!

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2. Change all treatment values to 1 and 0
3. Predict for everyone
4. Take the sample mean of the difference.

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With additive regression, the parametric g-formula estimator simplifies to $\hat{\beta}$.

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This no longer collapses to a coefficient!

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Positivity	$P(T = t \mid \mathbf{X} = \mathbf{x}) > 0$

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Estimation	$E(Y \mid T, \mathbf{X})$ follows the assumed model

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Let's consider the implications of estimation with regression under different assumptions about the model.

Identification under Selection on Observables: Regression

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   - τ will provide well-defined linear approximation to the average causal response function $E[Y|T=1,X] - E[Y|T=0,X]$. Approximation may be very poor if $E[Y|T,X]$ is misspecified and then τ may be biased for the ATE.

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3. Heterogeneous treatment effects (τ differs for different values of X)
   - even If outcomes are linear in X, τ converges to the conditional-variance-weighted average of the underlying causal effects rather than the ATE.

Ideal Case: Constant Effects Model With Linearly Separable Confounding

Assume a data generating process with constant effects and linearly separable confounding:

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$$\begin{aligned} Y_i &= \beta_0 + \tau T_i + \eta_i \\ &= \beta_0 + \tau T_i + X_i'\beta + u_i \end{aligned}$$

- Thus, a regression where T_i and X_i are entered linearly can recover the ATE. (The regression model matches the data generating process)

Heterogeneous effects, binary treatment

- Completely randomized experiment:

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- Error term now includes two components:
  1. "Baseline" variation in the outcome: $(Y_i(0) - \mu_0)$
  2. Variation in the treatment effect, $(\tau_i - \tau)$
- We can verify that under experiment, $E[\varepsilon_i | T_i] = 0$

Heterogeneous effects, binary treatment

- Completely randomized experiment:

$$\begin{aligned} Y_i &= T_i Y_i(1) + (1 - T_i) Y_i(0) \\ &= Y_i(0) + (Y_i(1) - Y_i(0)) T_i \\ &= \mu_0 + \tau_i T_i + (Y_i(0) - \mu_0) \\ &= \mu_0 + \tau T_i + (Y_i(0) - \mu_0) + (\tau_i - \tau) \cdot T_i \\ &= \mu_0 + \tau T_i + \varepsilon_i \end{aligned}$$

- Error term now includes two components:
  1. "Baseline" variation in the outcome: $(Y_i(0) - \mu_0)$
  2. Variation in the treatment effect, $(\tau_i - \tau)$
- We can verify that under experiment, $E[\varepsilon_i | T_i] = 0$
- Thus, OLS estimates the ATE with no covariates.

Adding covariates

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- What about the regression estimand, τ_R? How does it relate to the ATE/ATT?

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- Let's investigate this under a saturated regression model:

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- Linear in X_i by construction!

Investigating the regression coefficient

- How can we investigate τ_R? Well, we can rely on the regression anatomy:

$$\tau_R = \frac{\text{Cov}(Y_i, T_i - E[T_i|X_i])}{\text{Var}(T_i - E[T_i|X_i])}$$

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- With a little work we can show:

$$\tau_R = \frac{E\left[\tau(X_i)(T_i - E[T_i|X_i])^2\right]}{E[(T_i - E[T_i|X_i])^2]} = \frac{E[\tau(X_i)\sigma_t^2(X_i)]}{E[\sigma_t^2(X_i)]}$$

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- $\sigma_t^2(x) = \text{Var}[T_i|X_i = x]$ is the conditional variance of treatment assignment.

ATE versus OLS

$$\tau_R = E[\tau(X_i)W_i] = \sum_x \tau(x) \frac{\sigma_t^2(x)}{E[\sigma_t^2(X_i)]} P[X_i = x]$$

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- Maximum variance with $P[T_i = 1|X_i = x] = 1/2$.

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  - Implies that the OLS weights are 1.
- Incorrect linearity assumption in X_i will lead to more bias.

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- This is a feature, not a bug, of regression: we can't learn anything from those cases anyway (i.e. it is automatically handling issues of common support).
- The downside is that we have to be aware of what happened!

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Jensen (2003), "Democratic Governance and Multinational Corporations: Political Regimes and Inflows of Foreign Direct Investment."

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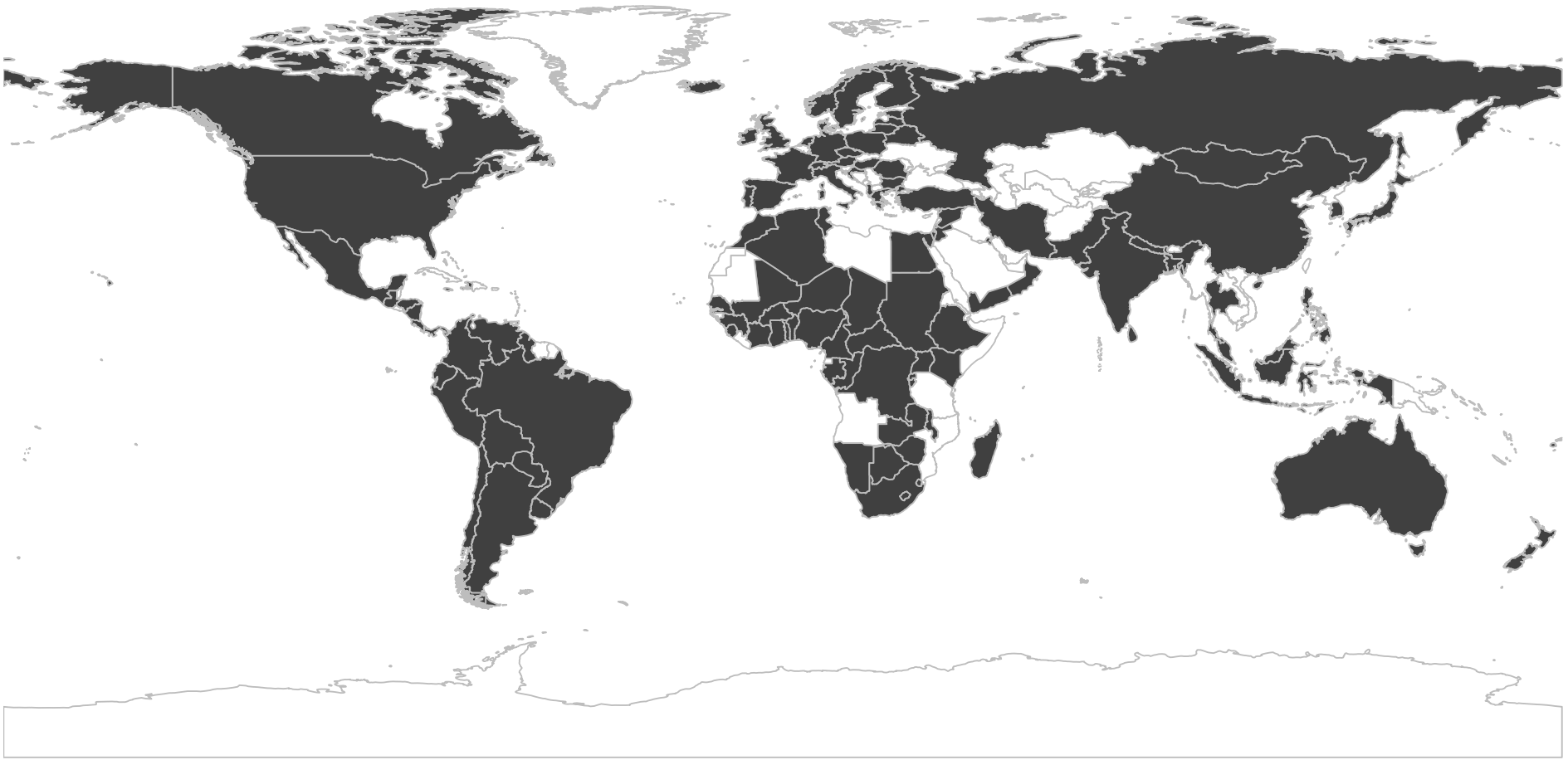
Jensen estimates that a 1 unit increase in polity score corresponds to a 0.020 increase in net FDI inflows as a percentage of GDP ($p < 0.001$).

Nominal and Effective Samples



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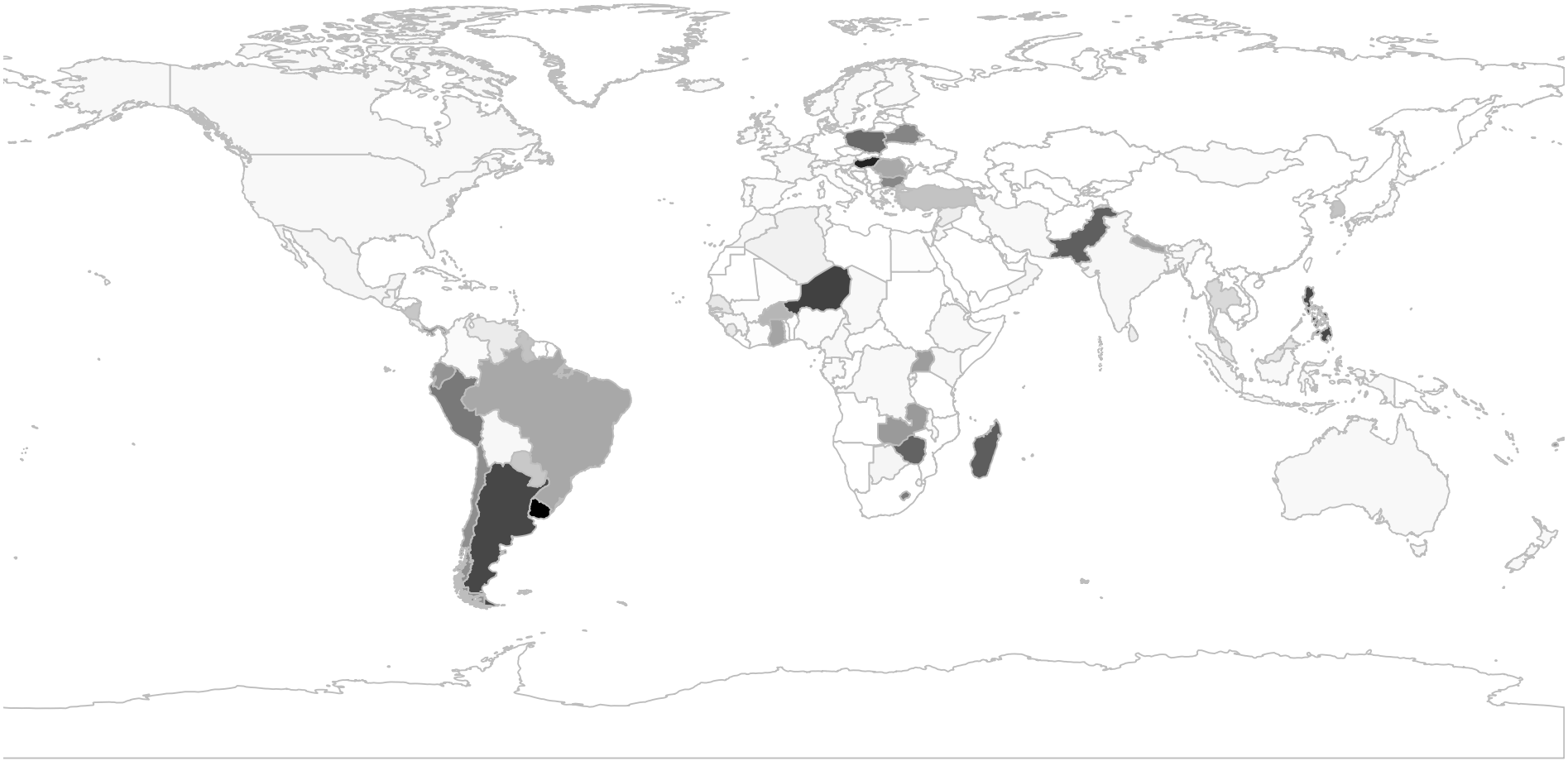

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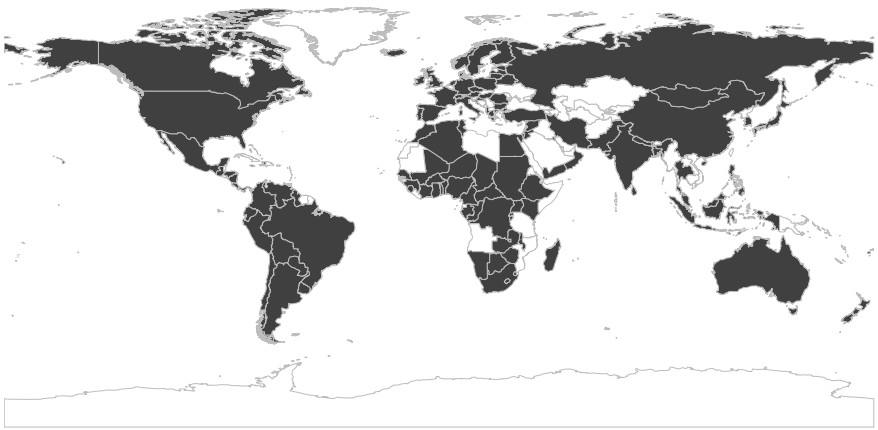

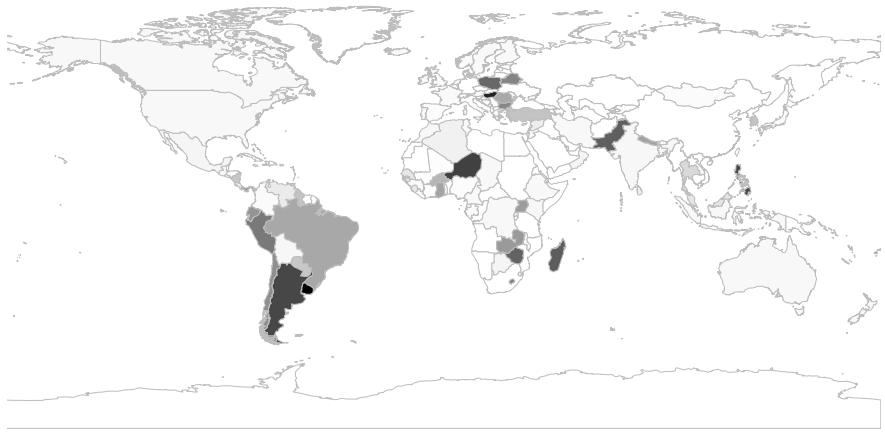

Over 50% of the weight goes to just 12 (out of 114) countries.

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- When a treatment is "as-if" randomly assigned conditional on covariates, regression distorts the sample by implicitly applying weights.
- The effective sample (upon which causal effects are estimated) may have radically different properties than the nominal sample.
- When there is an underlying natural experiment in the data, a properly specified regression model may reproduce the internally valid estimate associated with the natural experiment.

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- More mathematically, look like this:

$$\tau_{imp} = \frac{1}{N} \sum_i \hat{\mu}_1(X_i) - \hat{\mu}_0(X_i)$$

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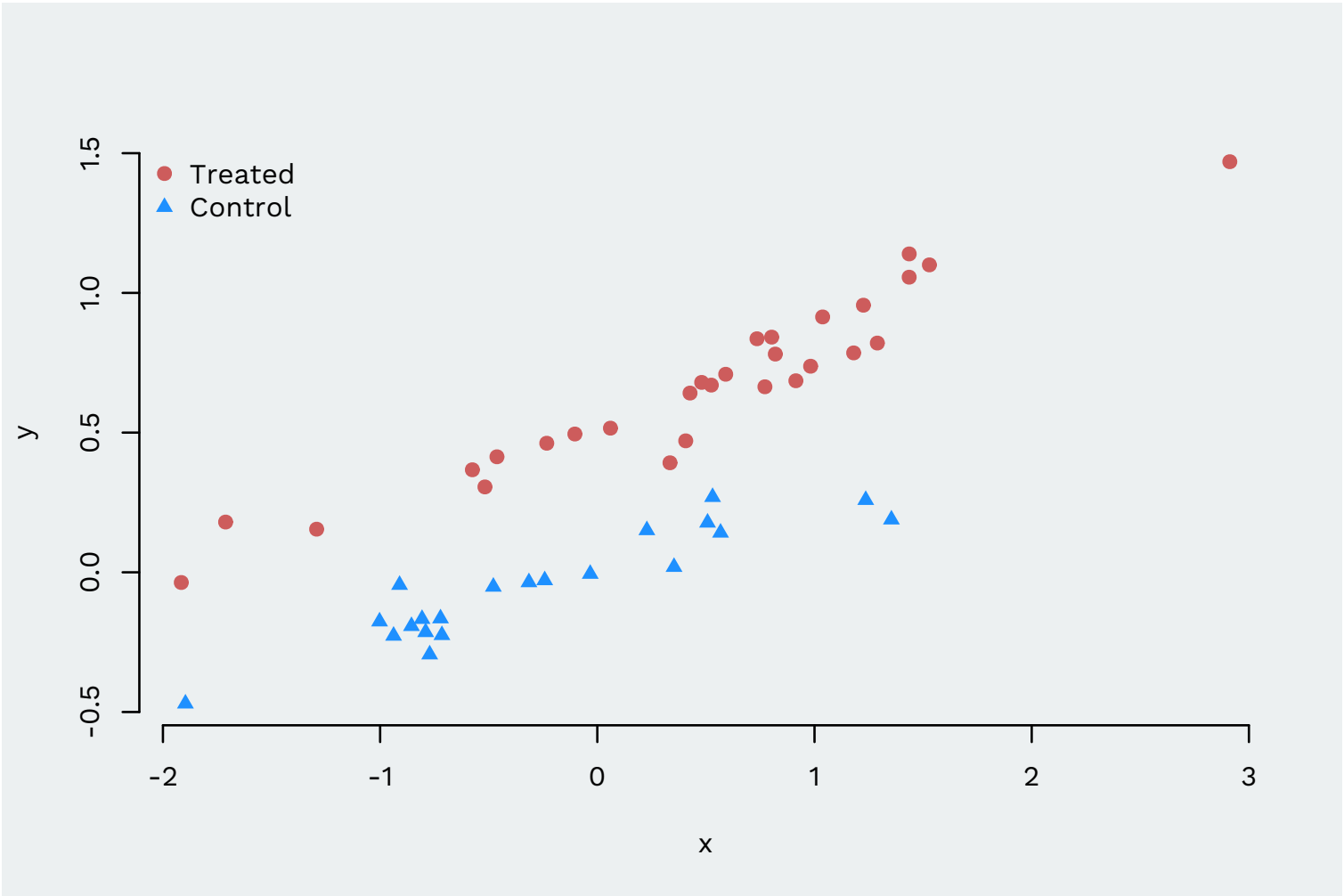
  - Regress Y_i on X_i in the control group and get predicted values for all units (treated or control) using a flexible model.
  - Take the average difference between these predicted values (uncertainty via bootstrap)
- More mathematically, look like this:

$$\tau_{imp} = \frac{1}{N} \sum_i \hat{\mu}_1(X_i) - \hat{\mu}_0(X_i)$$

- If the models are consistent for the conditional expectation (and identification assumptions hold) will be consistent for the treatment effect!

Imputation estimator visualization



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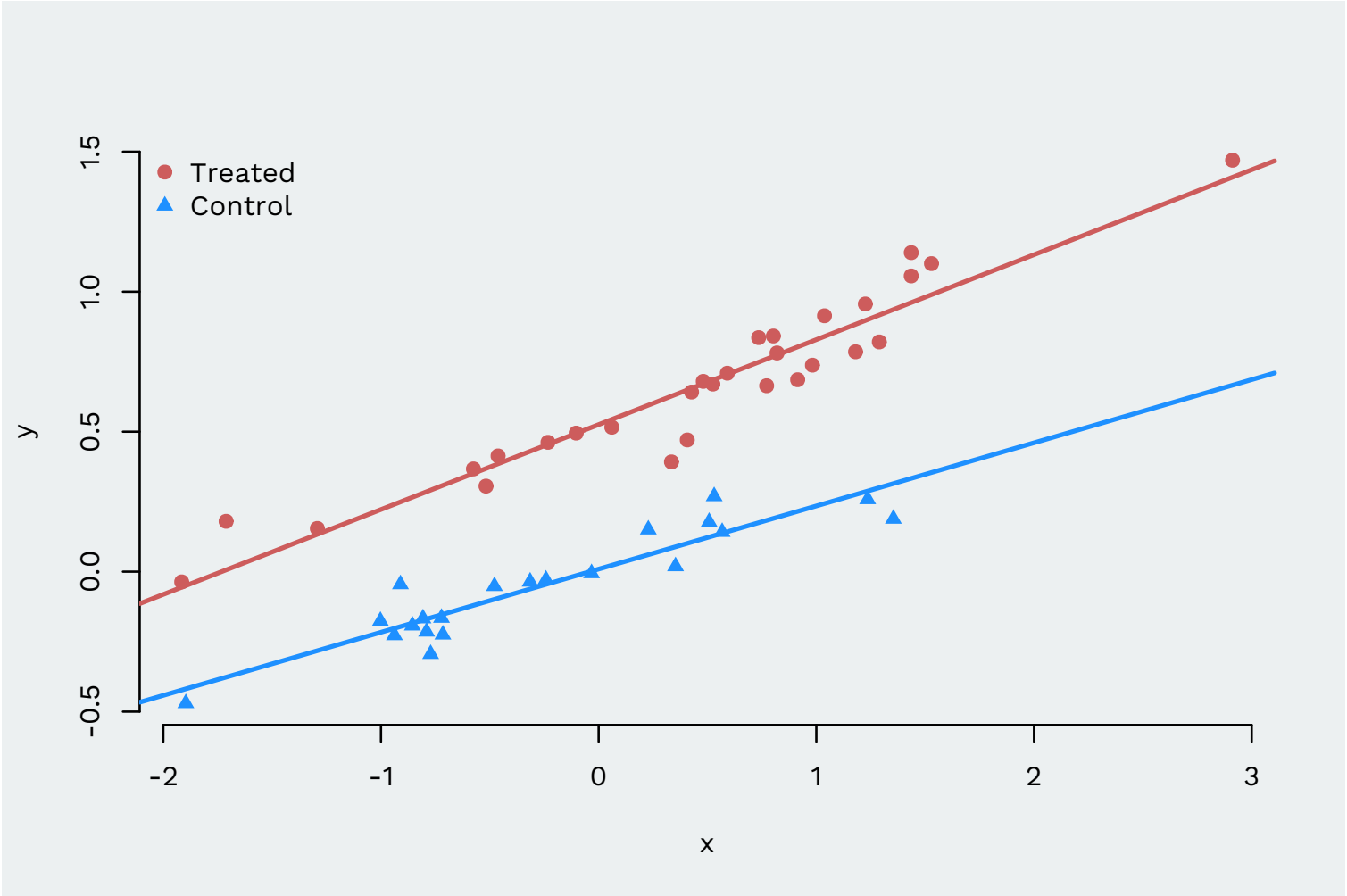

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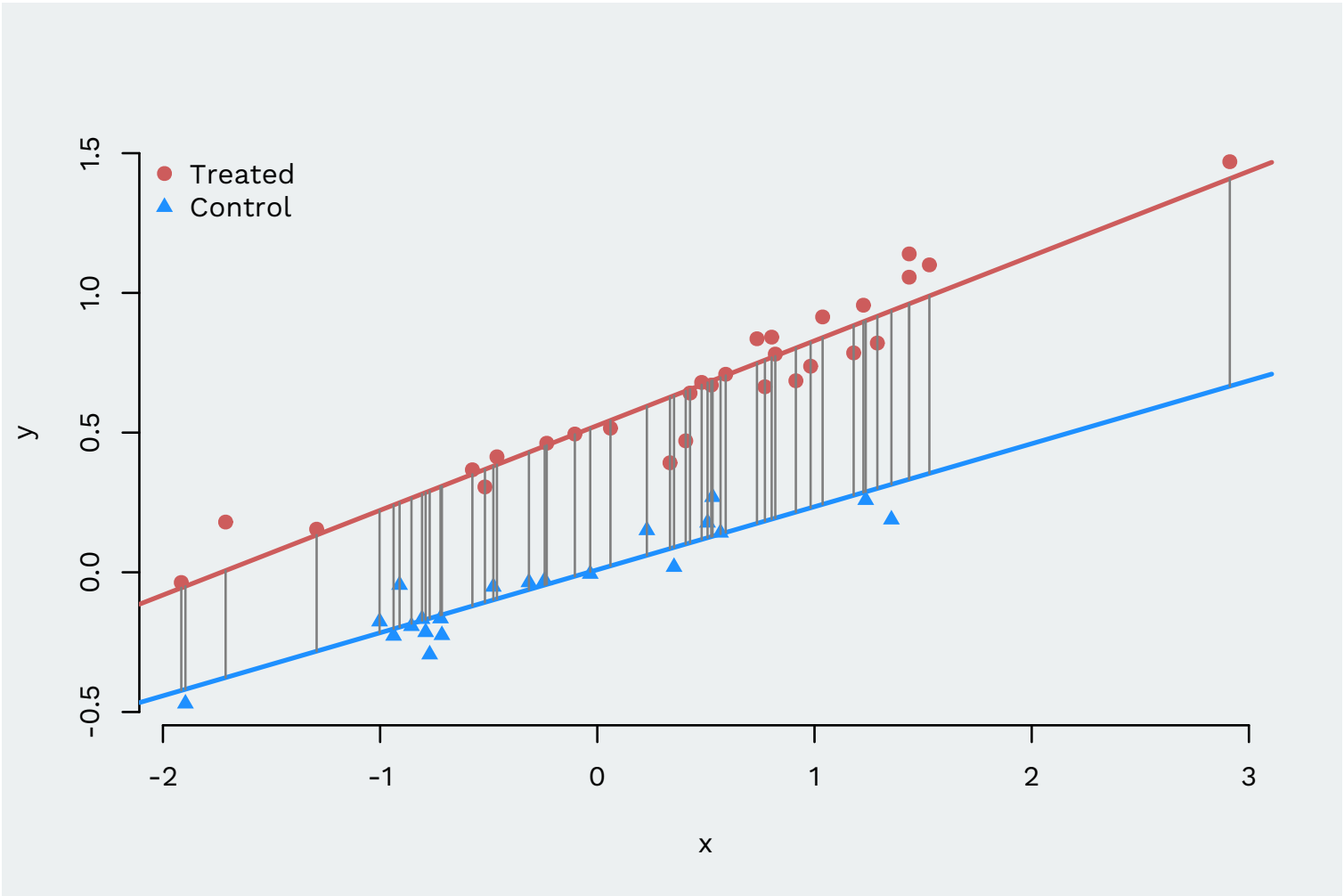

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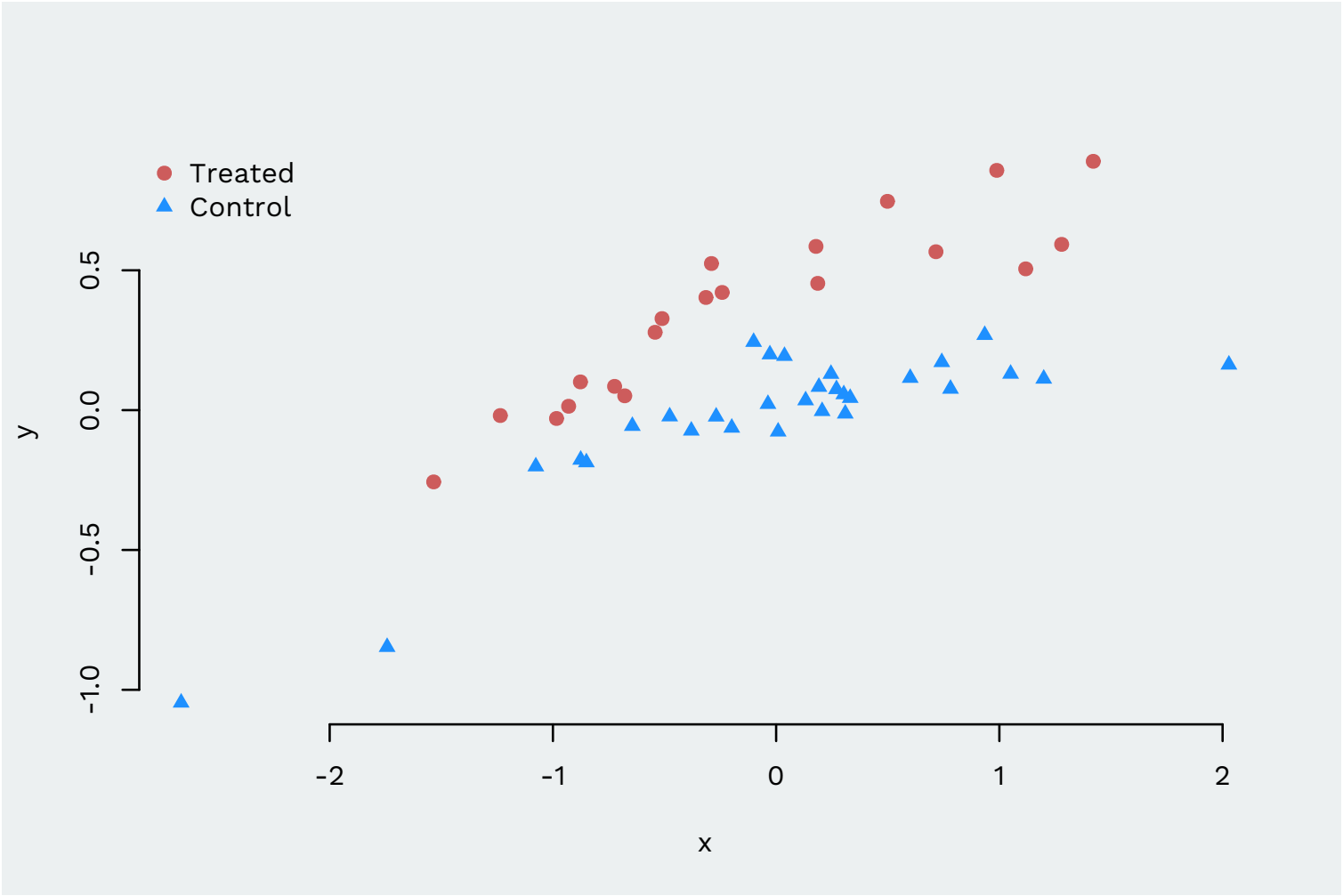

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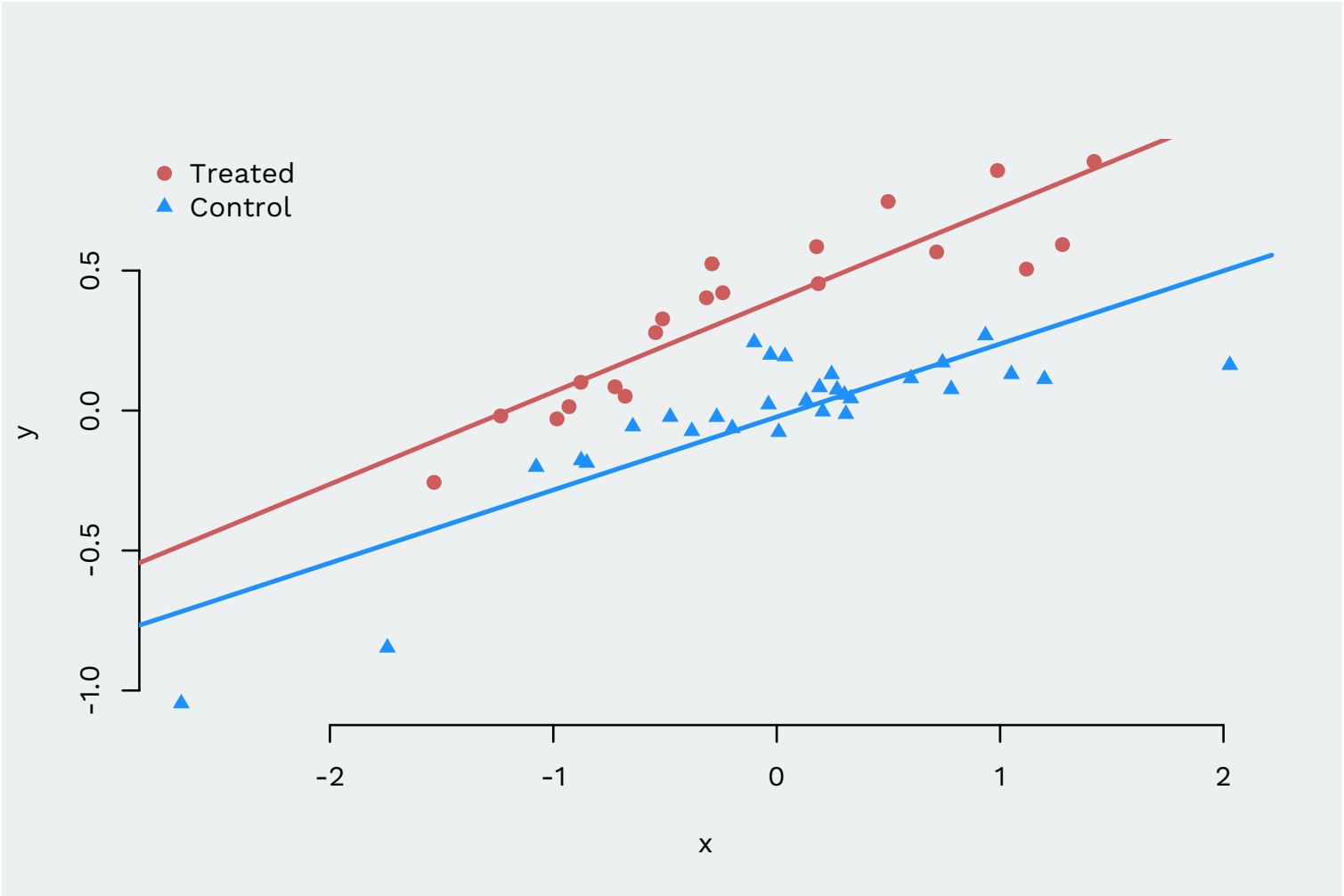

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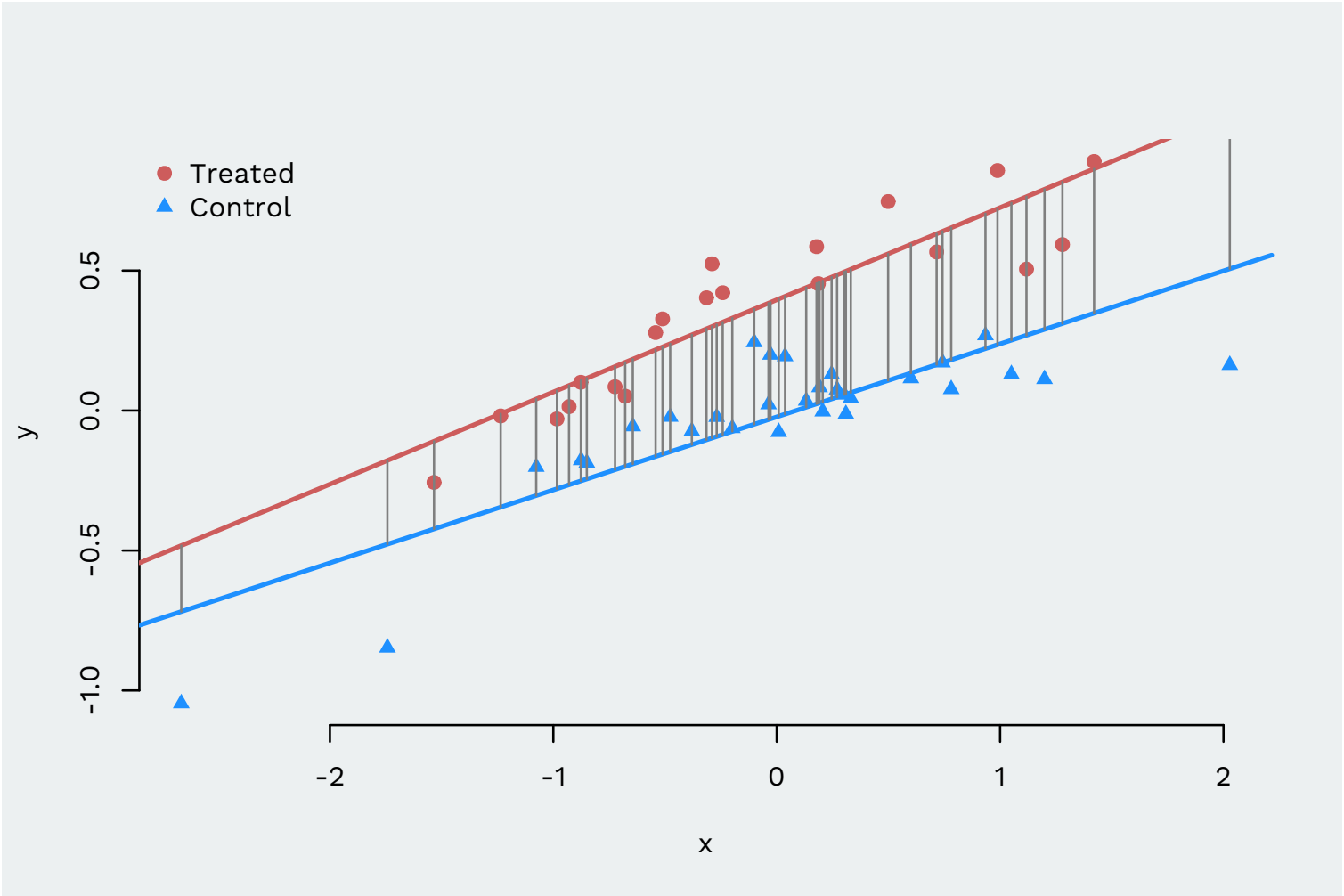

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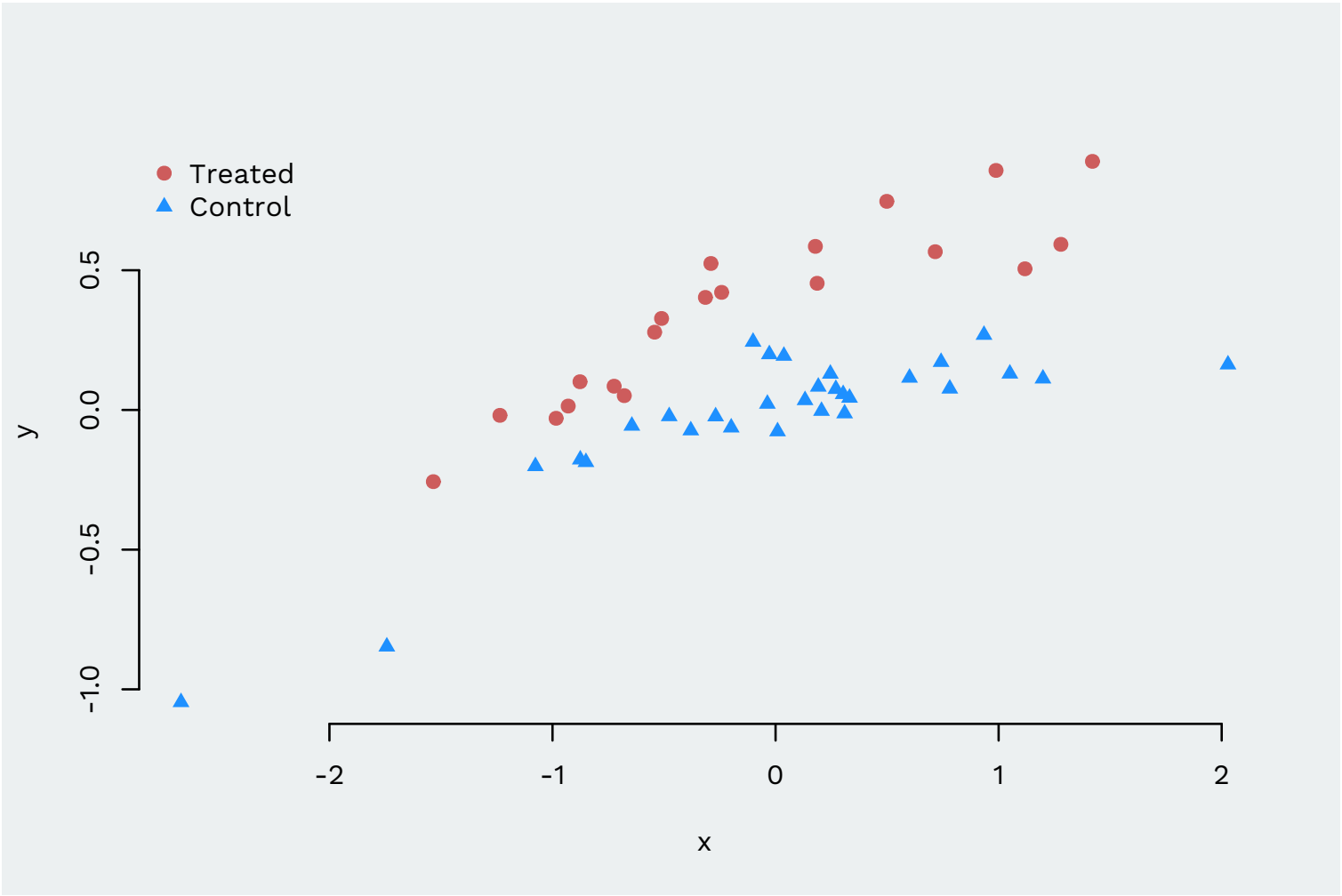
- We can use Generalized Additive Models (GAMs) from the `mgcv` package for flexible estimate (a kind of machine learning).
- More generally we can use general machine learning estimators for this kind of adjustment (see e.g. Double Machine Learning from Chernozhukov et. al.)

Using GAMs



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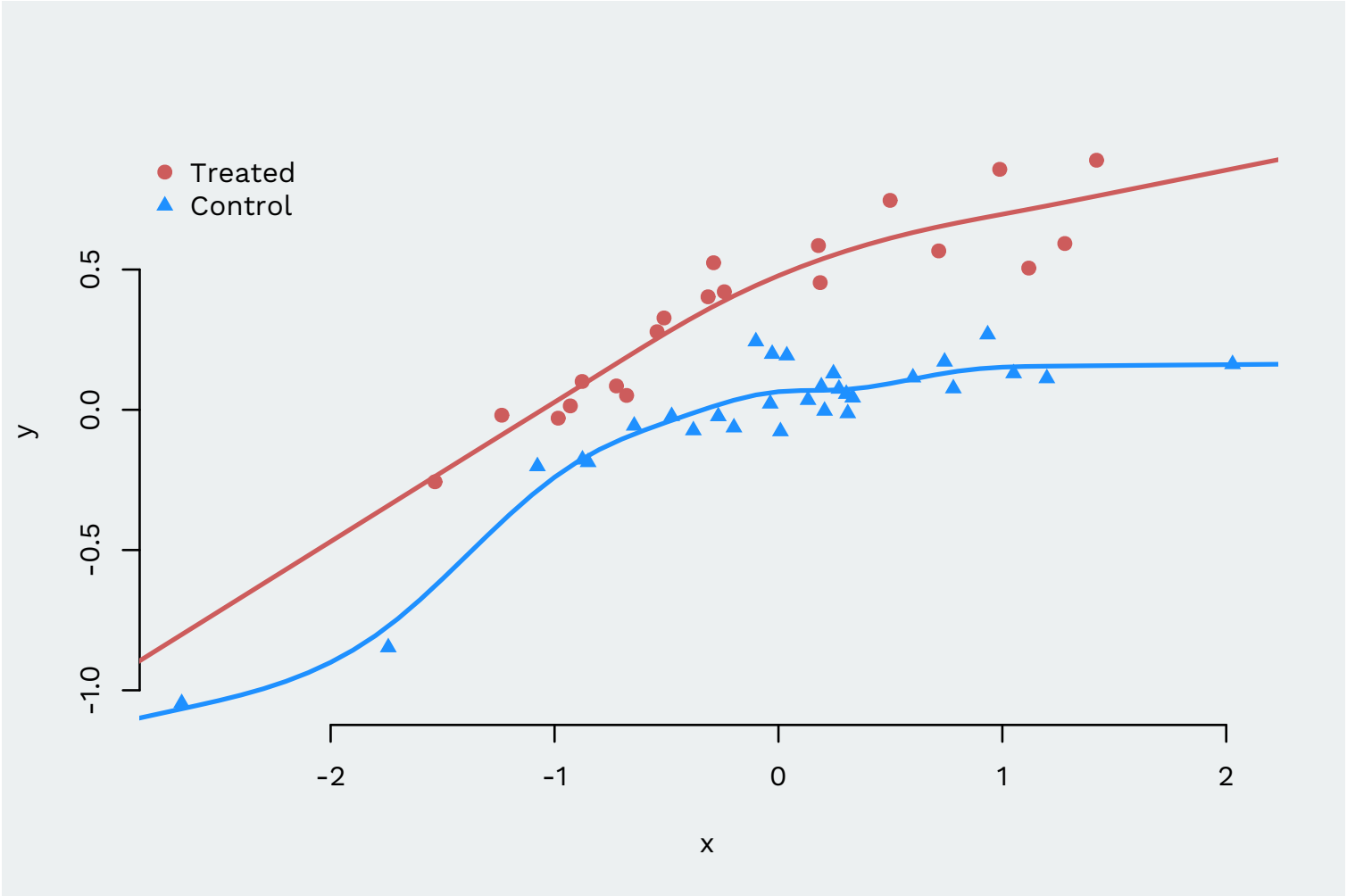

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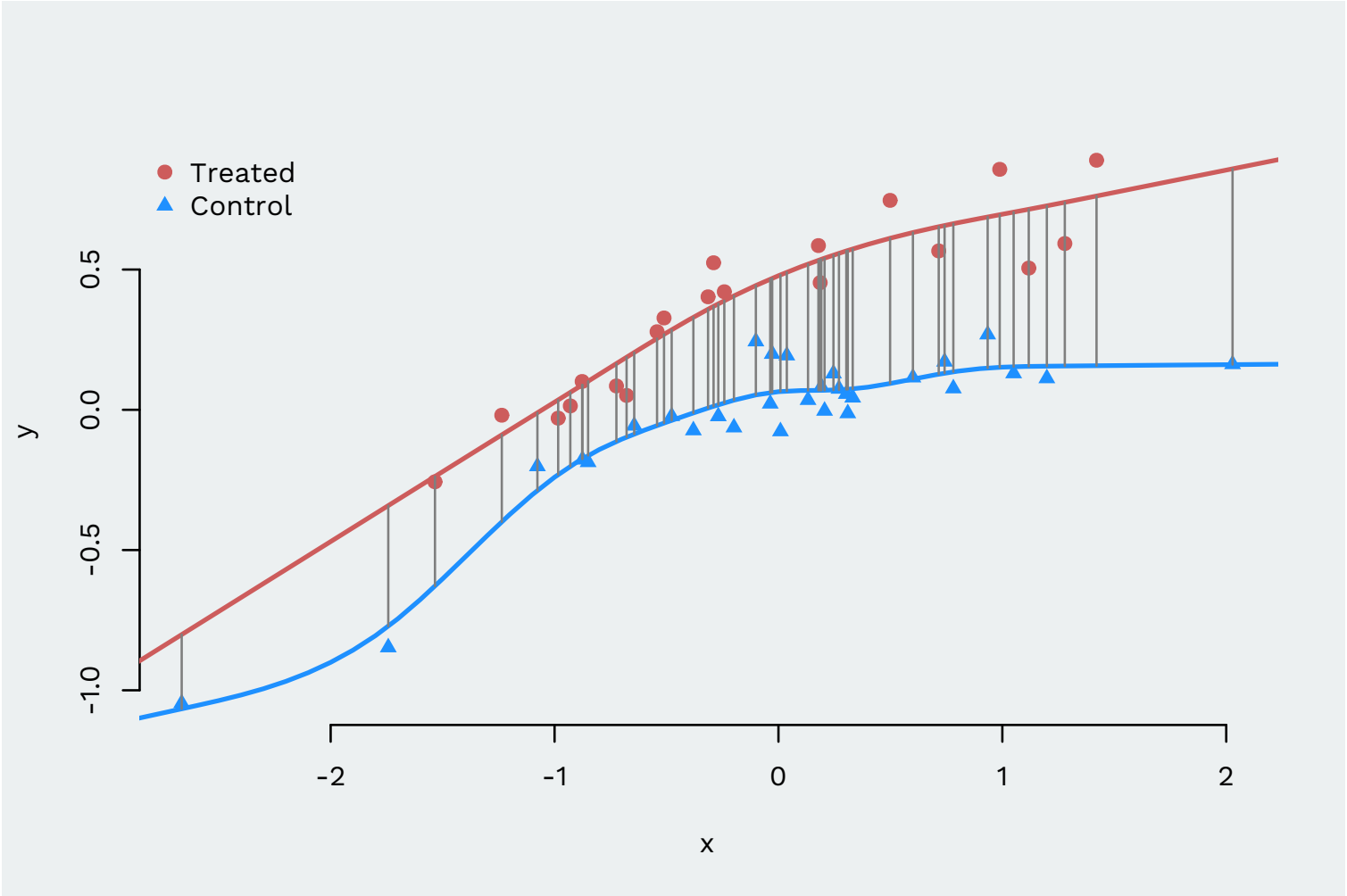

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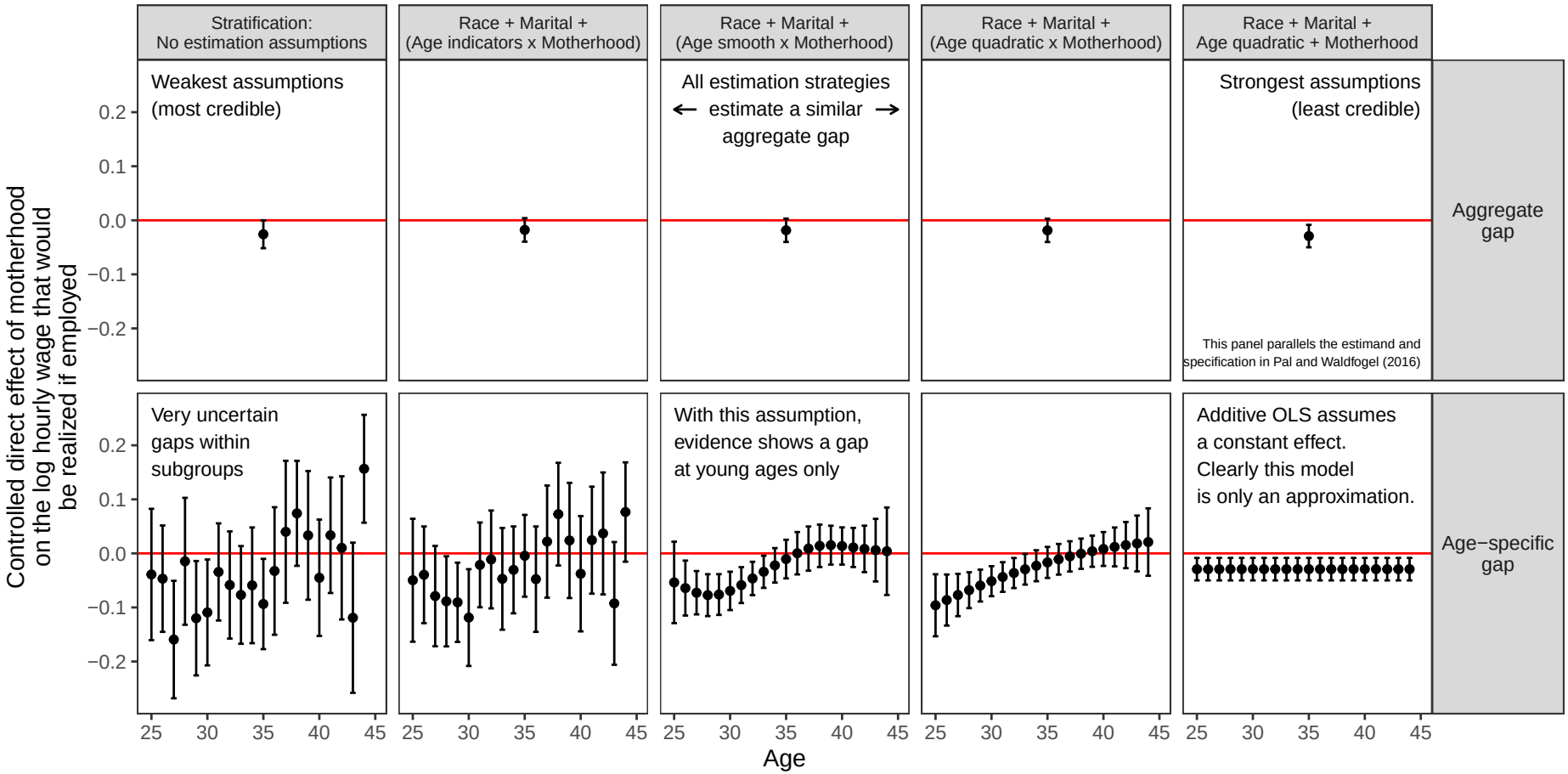
- As we just saw, GAMs are a nice trade-off of the ease vs. flexibility side.
- These kinds of things will tend to matter a lot more for conditional treatment effects than the overall aggregate treatment effect, but you also don't know for sure until you try.

Example from Lundberg, Johnson and Stewart



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- Remember a fancier regression only makes the estimation assumptions more credible, not the identification assumption!

1. Regression with Measured Confounding
   - Regression with Constant Effects
   - Additive Regression with Heterogeneous Effects
   - Imputation Estimators
2. Matching
   - Fundamentals of Matching
   - Two Approaches to Matching
3. Fixed Effects
   - Non-parametric Identification and Fixed Effects

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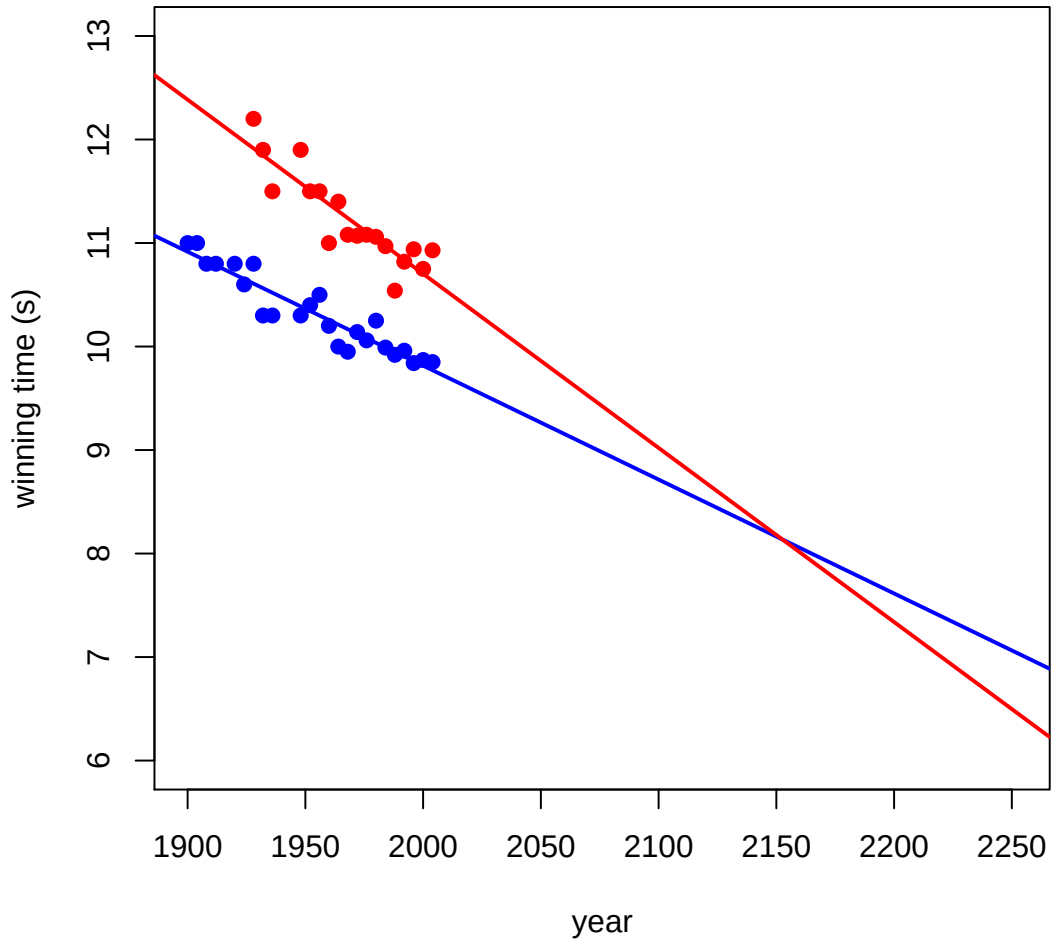

Tatem et al.'s predictions of sprint times with alternate models. Men's times are in blue, women's times are in red.

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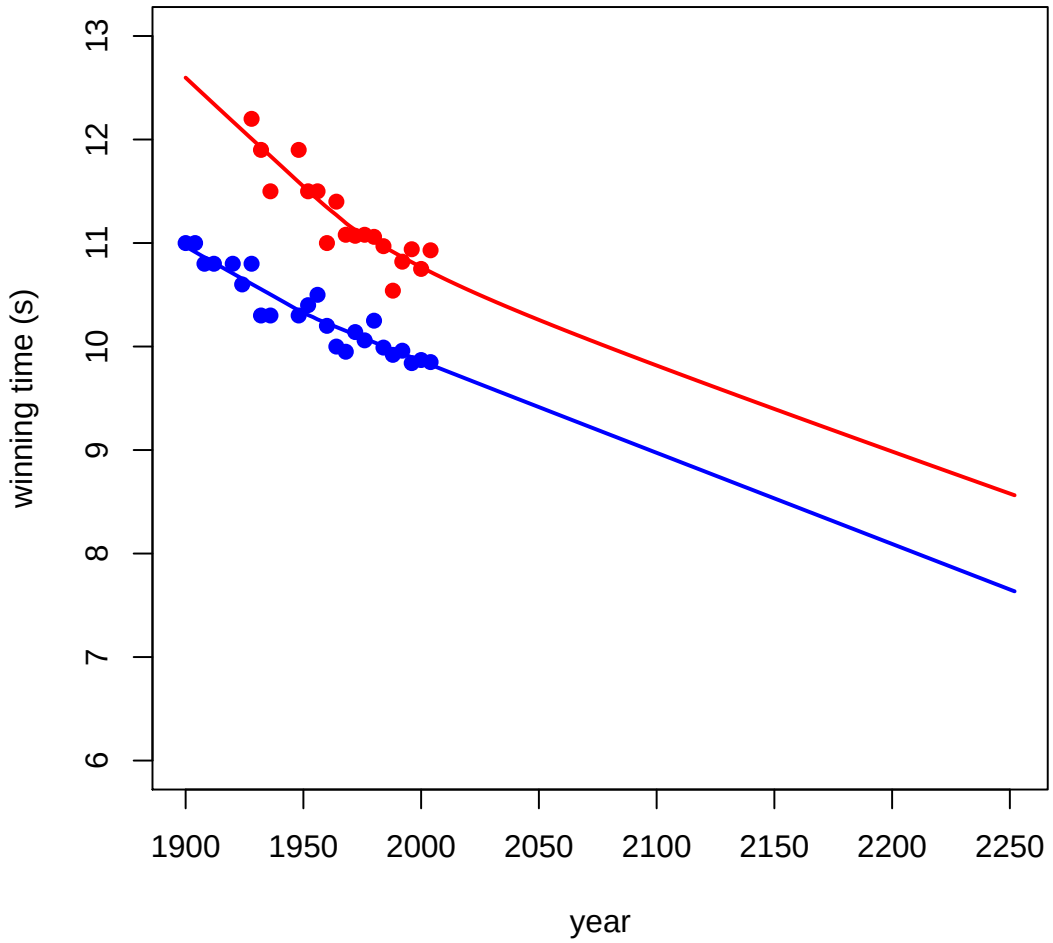

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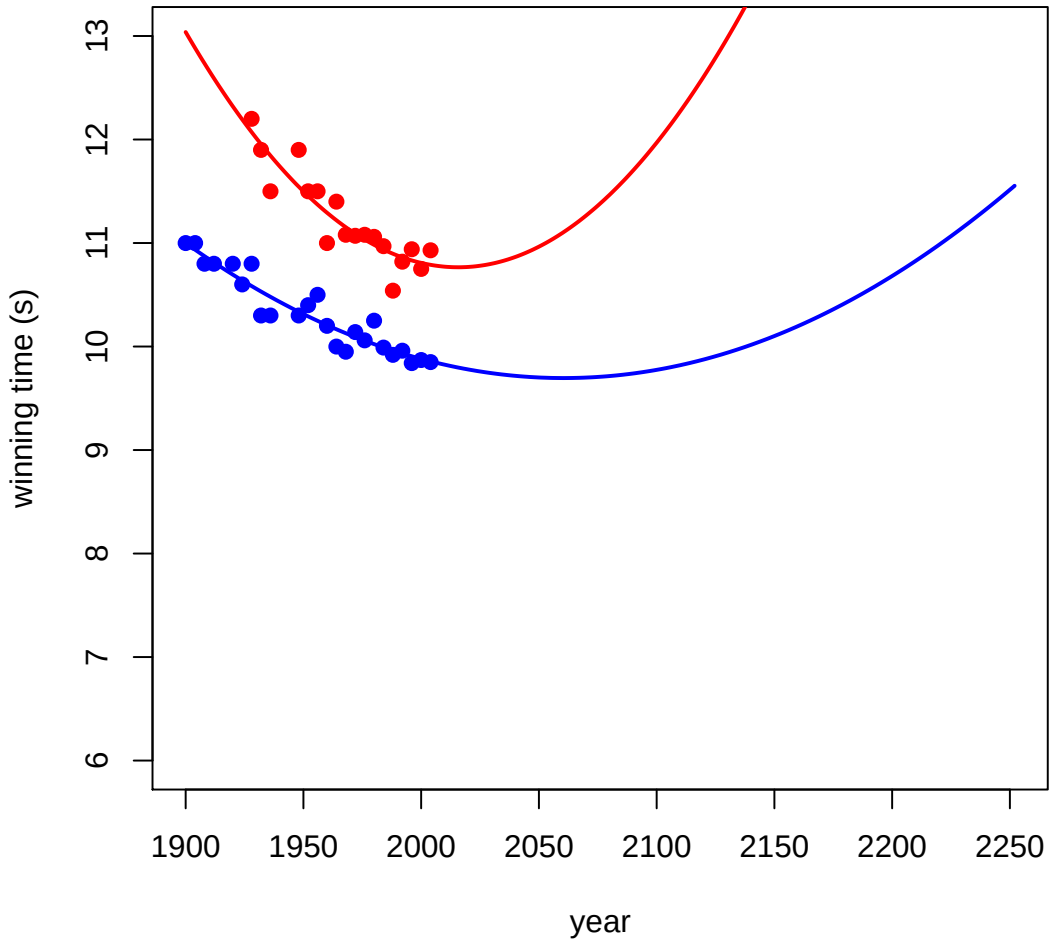

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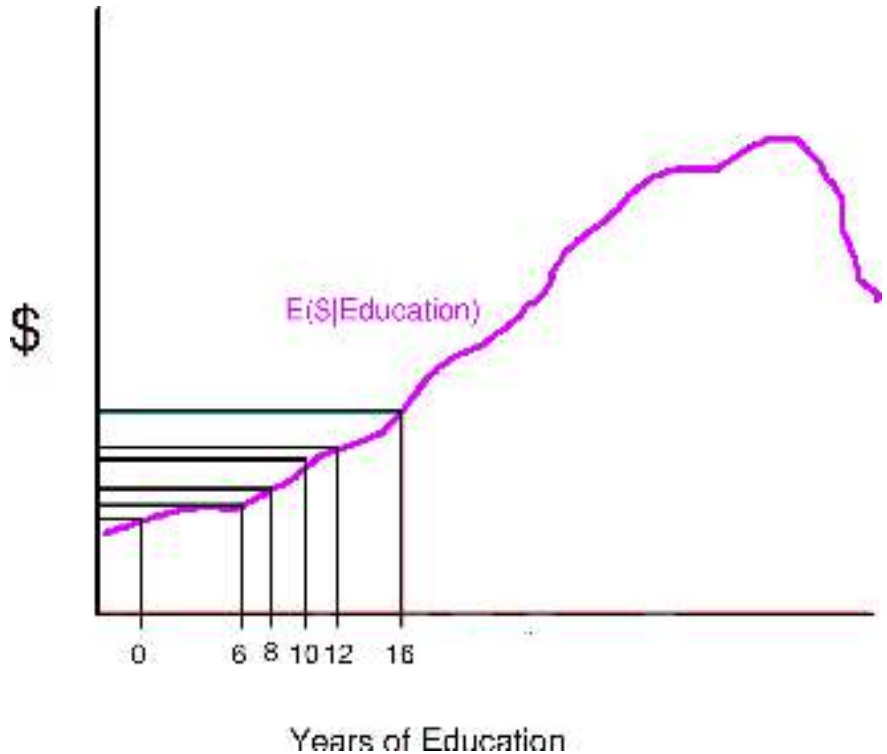
Result

The maximum degree of model dependence: solely a function of the distance from the counterfactual to the data

What Inferences Would You Be Willing to Make?



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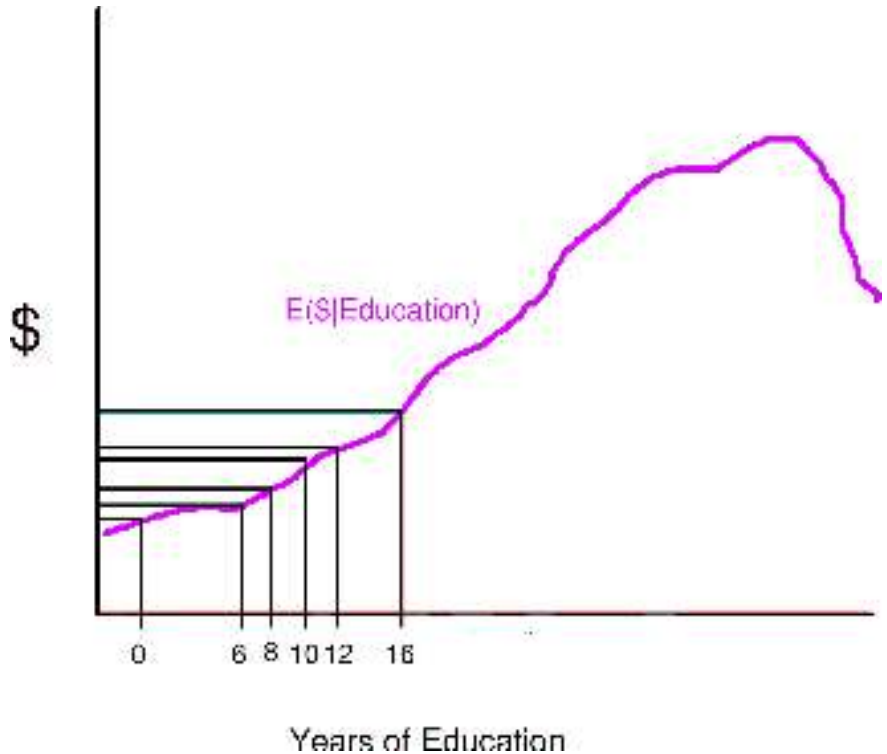

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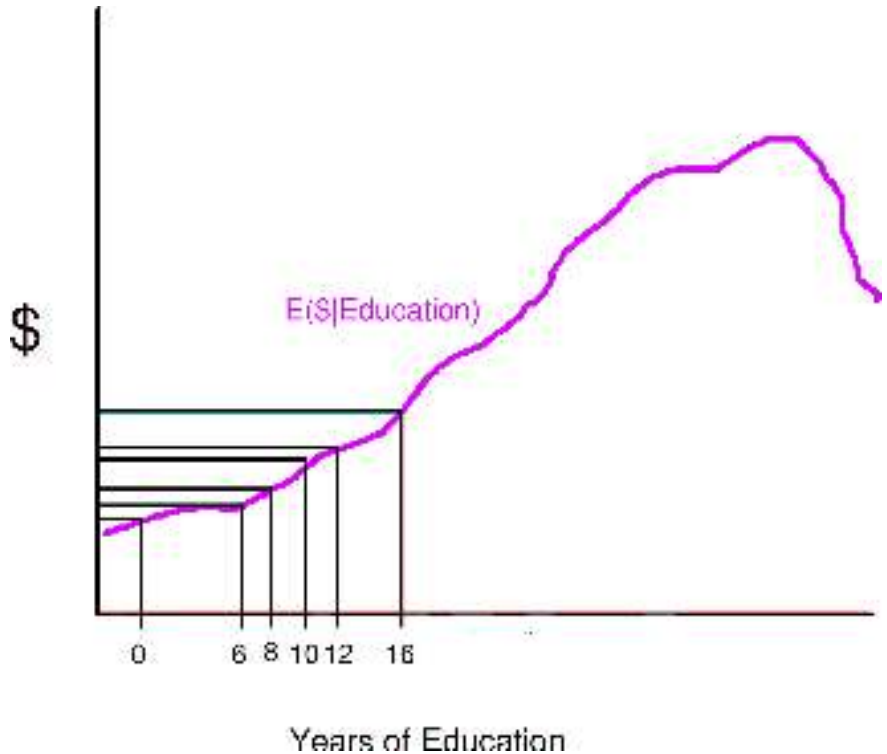

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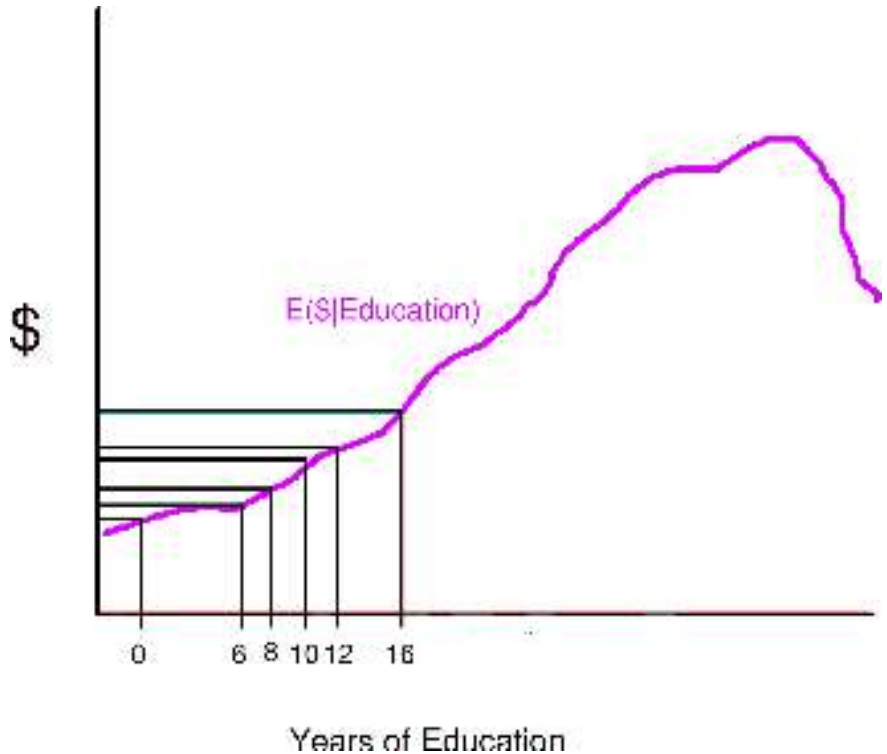

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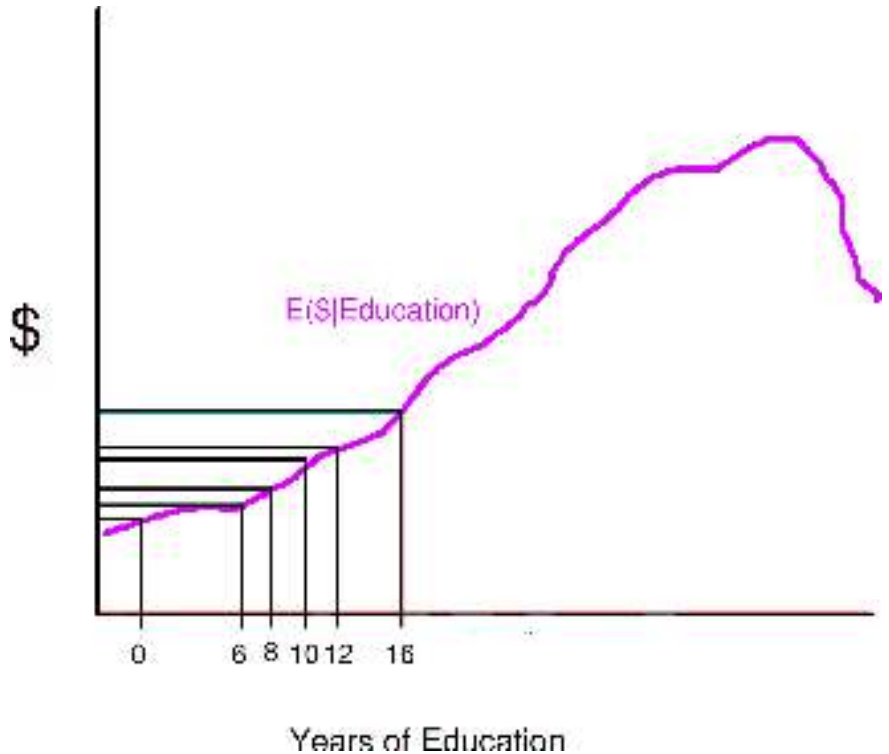
Counterfactual Inferences with Interpolation



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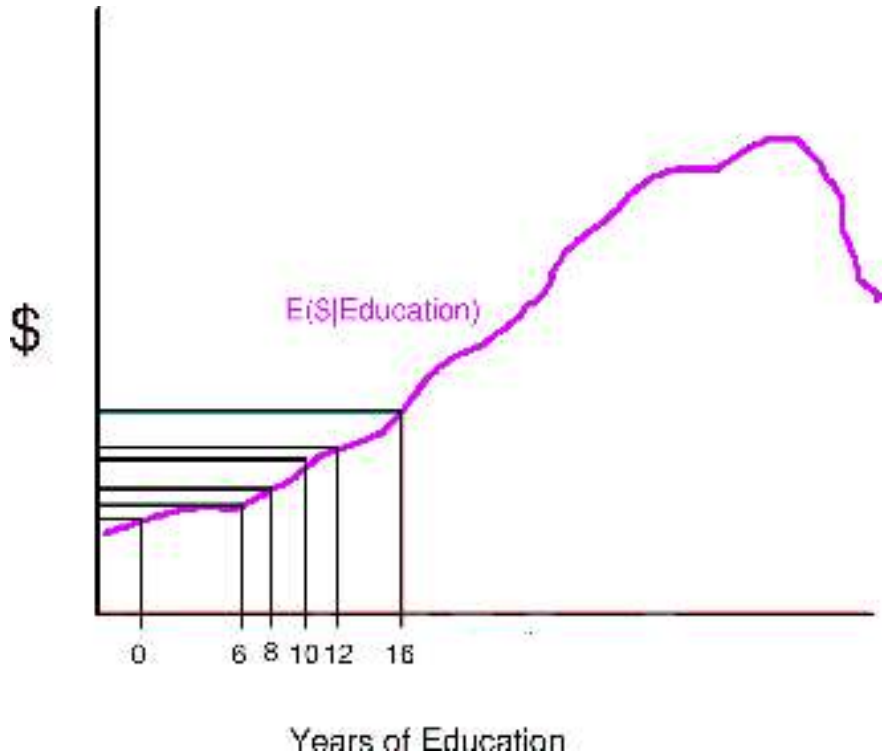

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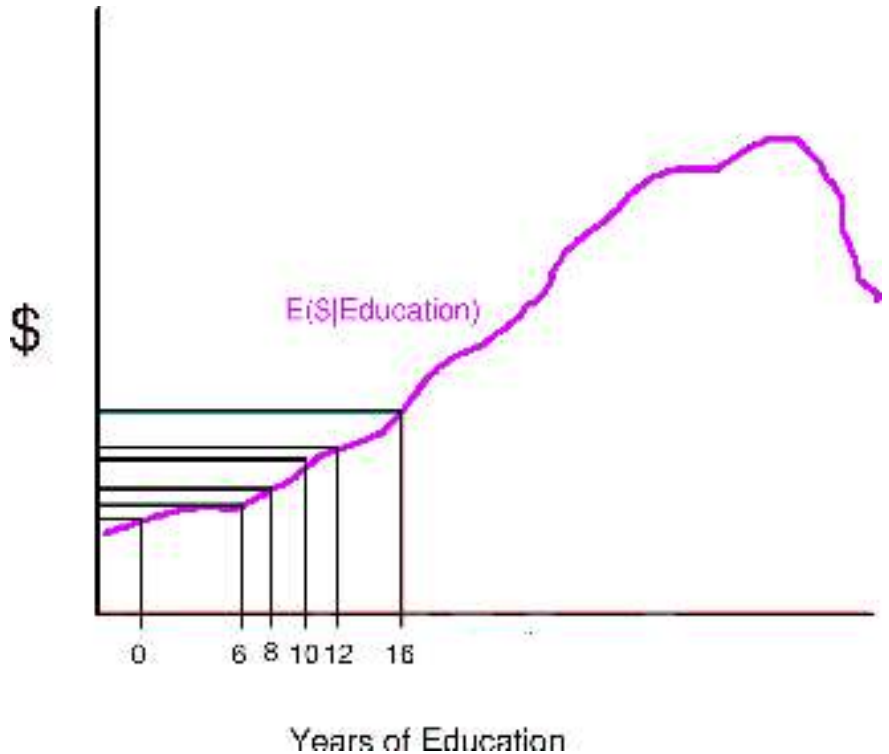

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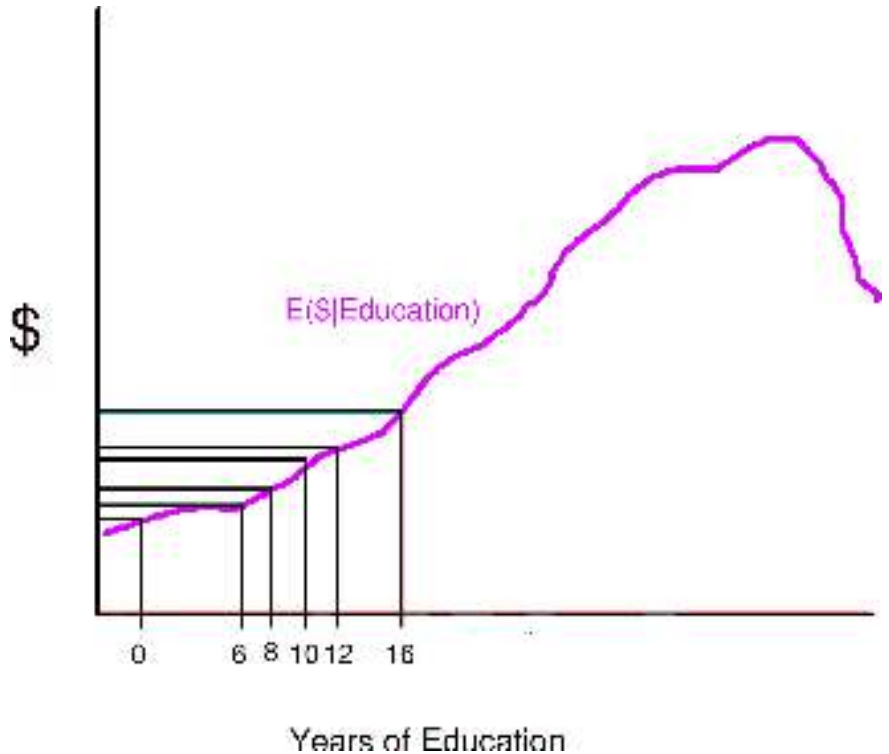

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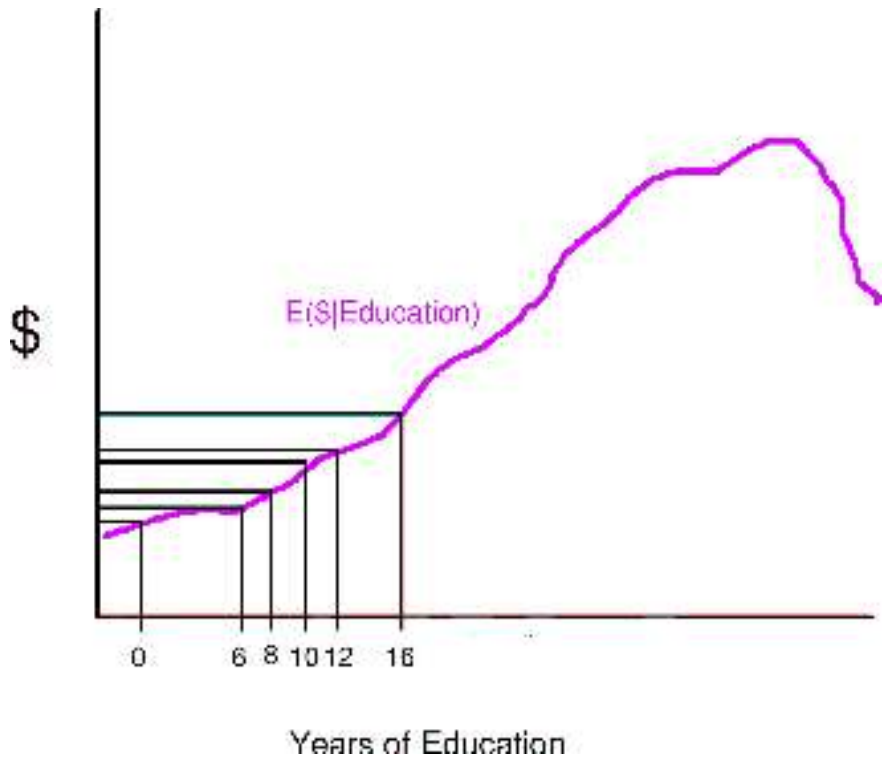

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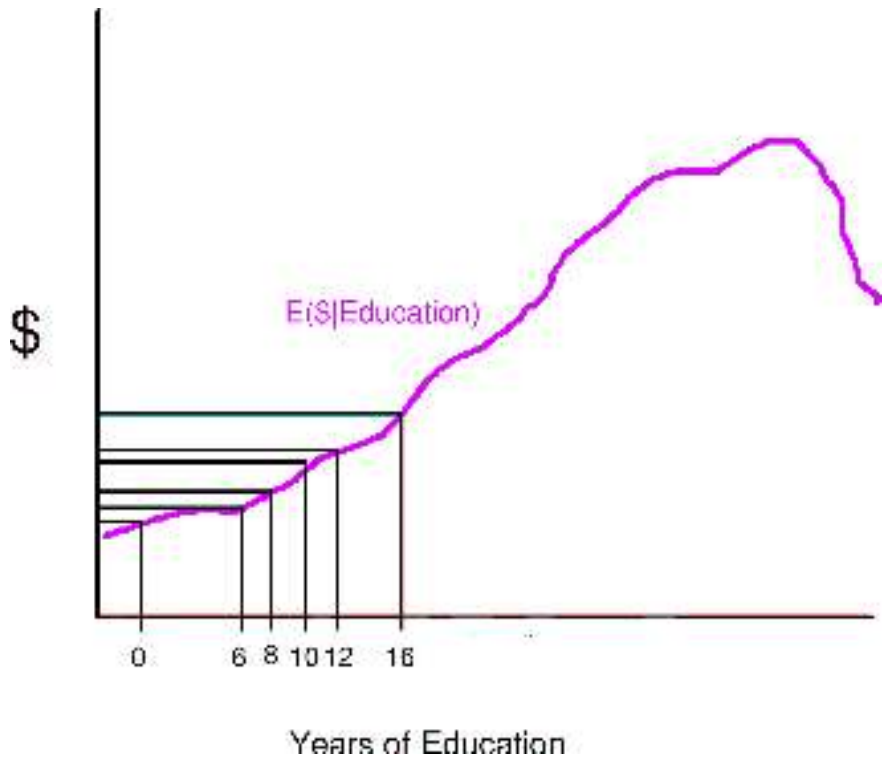

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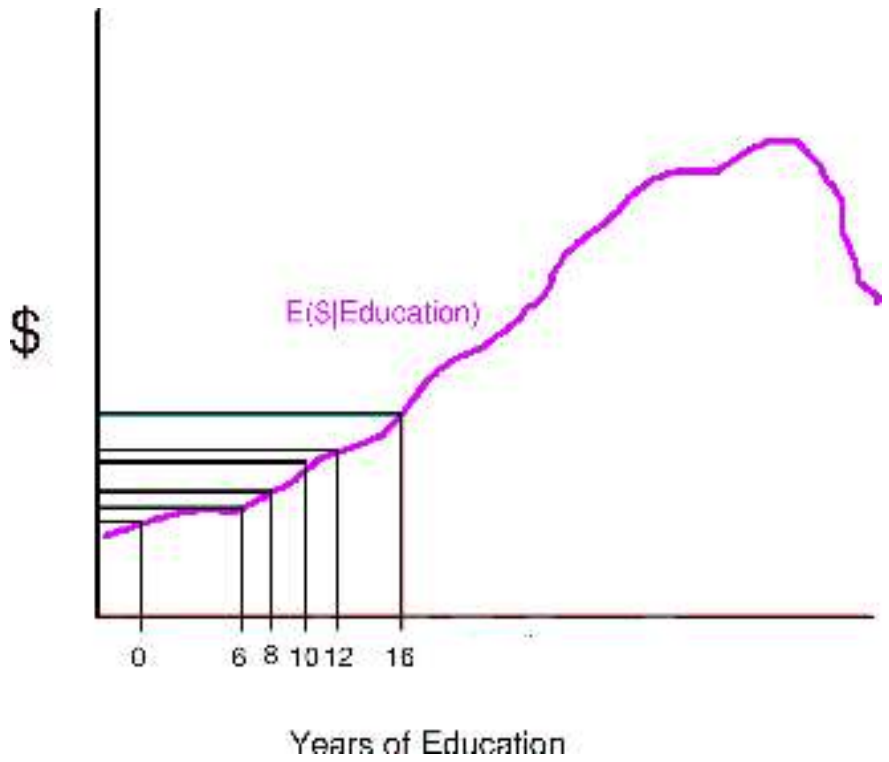

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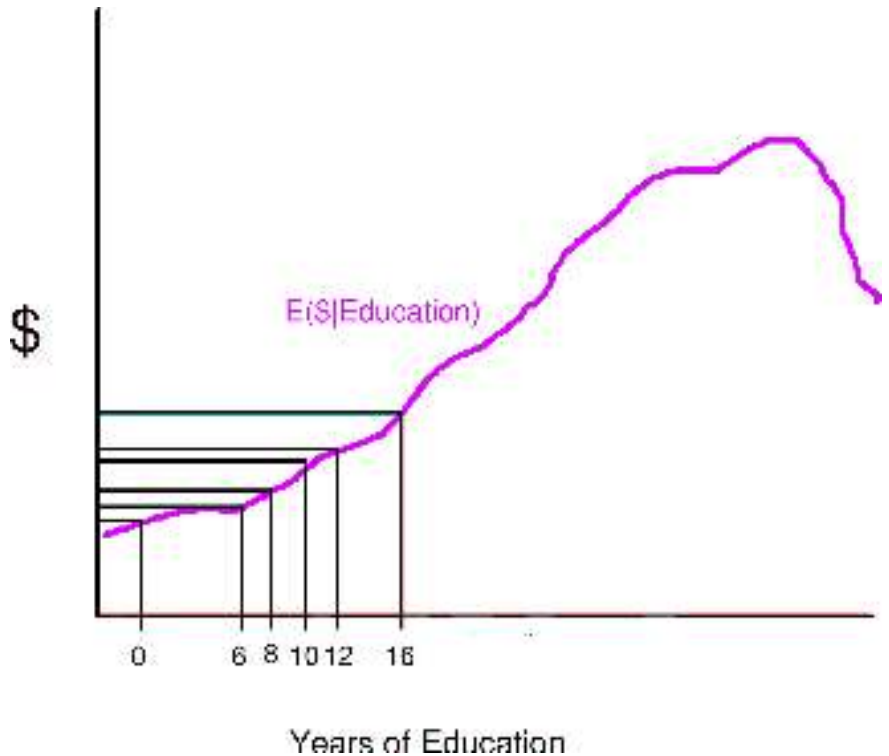
Another Counterfactual Inference with Extrapolation



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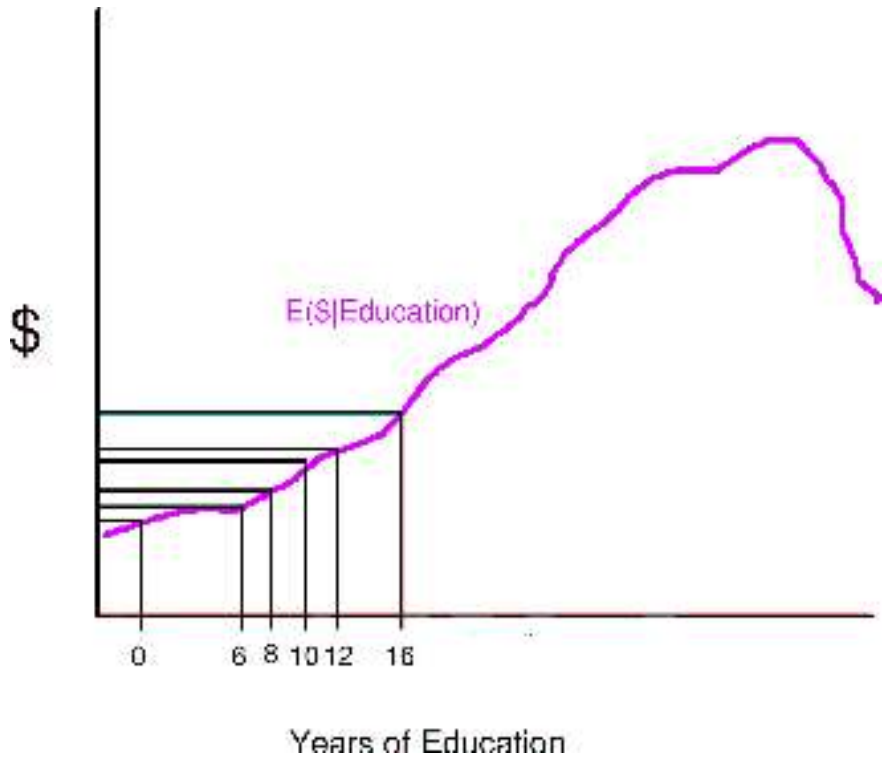

- How much salary would someone receive with 53 years of education?

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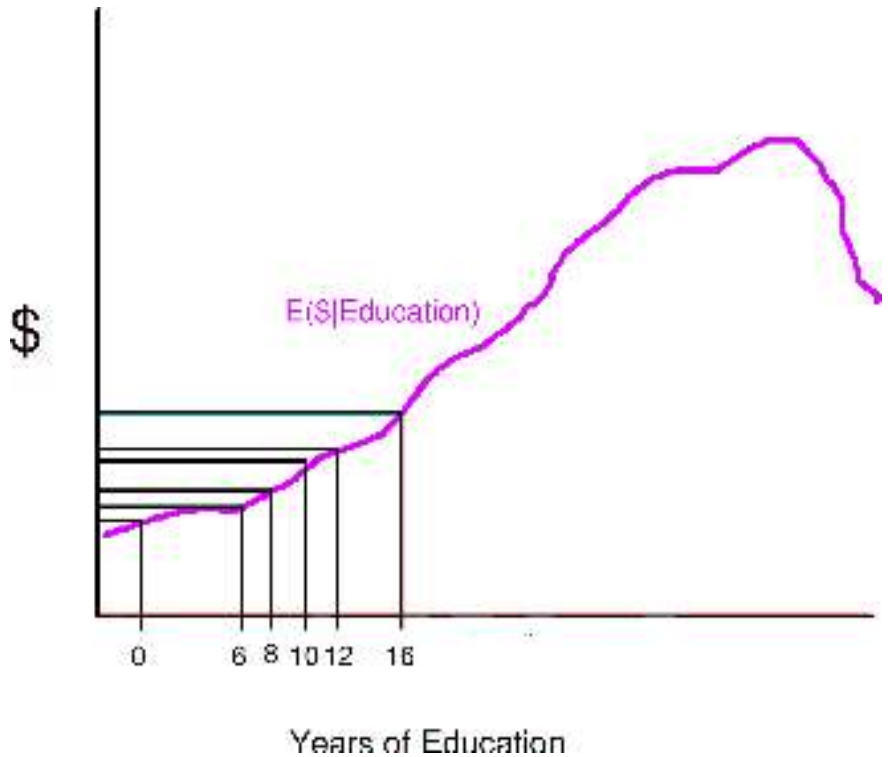

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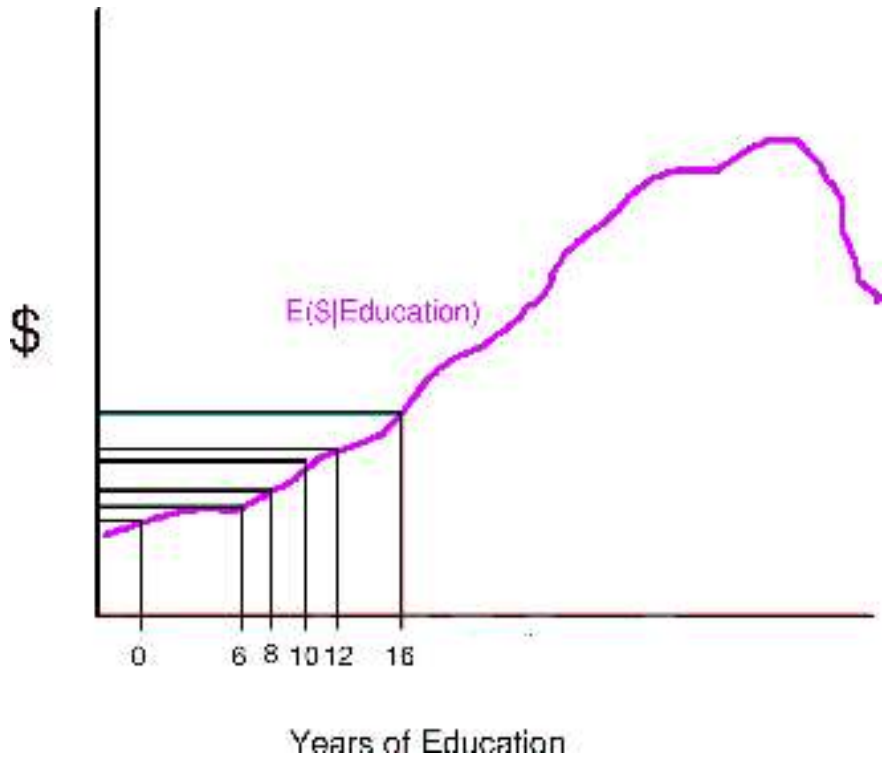

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- What's changed? How would we recognize it when the example is less extreme or multidimensional?

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- The difference: an enormous assumption based on convenience, not evidence or theory.

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- The curse of dimensionality introduces huge assumptions, often unrecognized.

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  - If treated and control groups are better balanced than when you started, due to pruning, model dependence is reduced

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(Warning: Pruning nonmatches can change your feasible estimand.)

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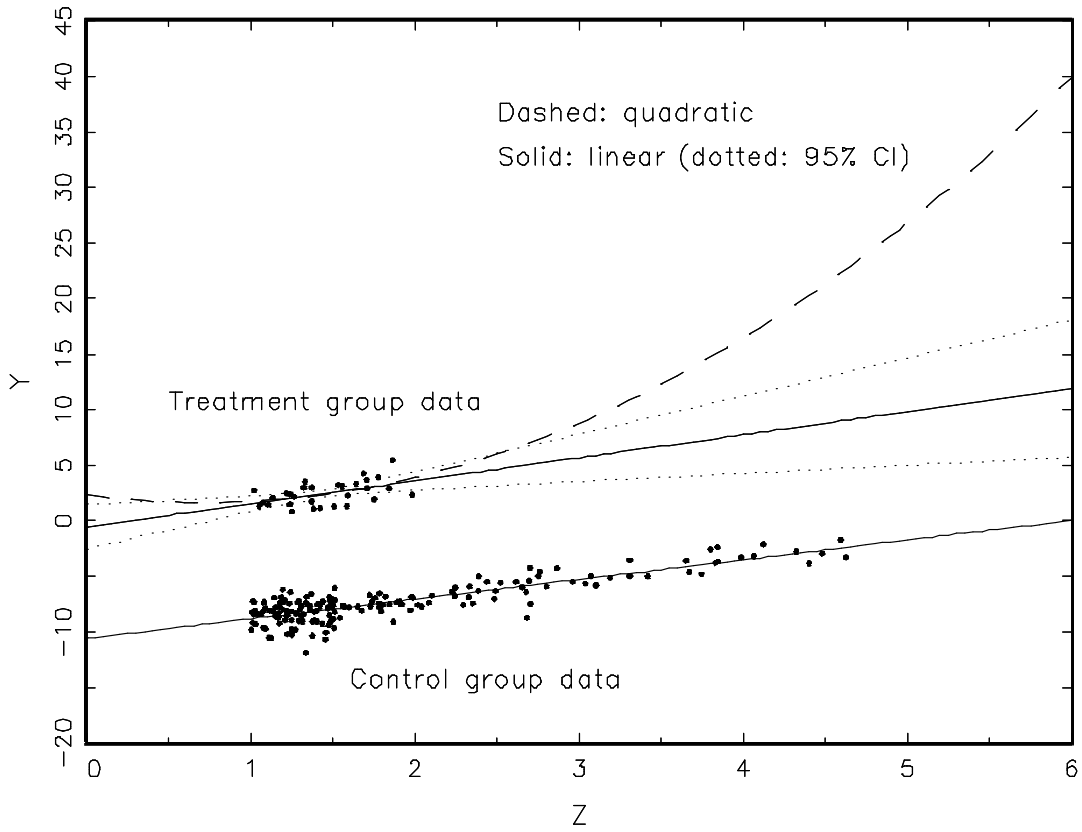

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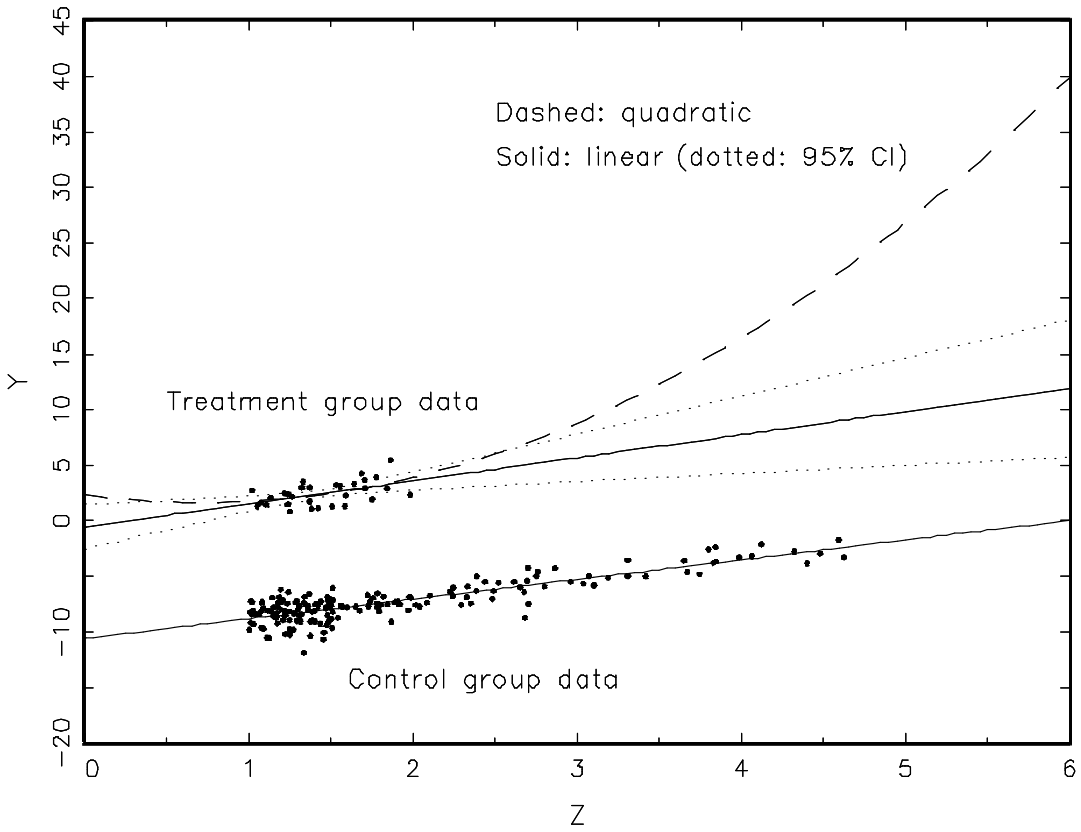

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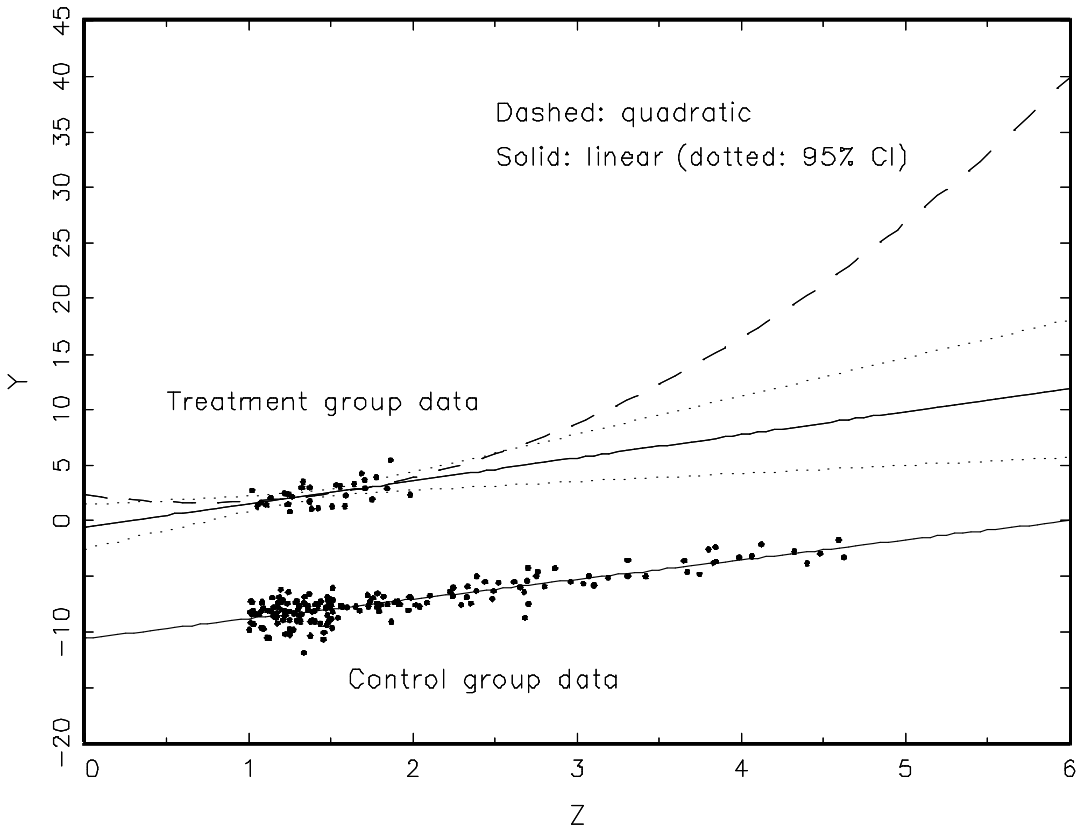

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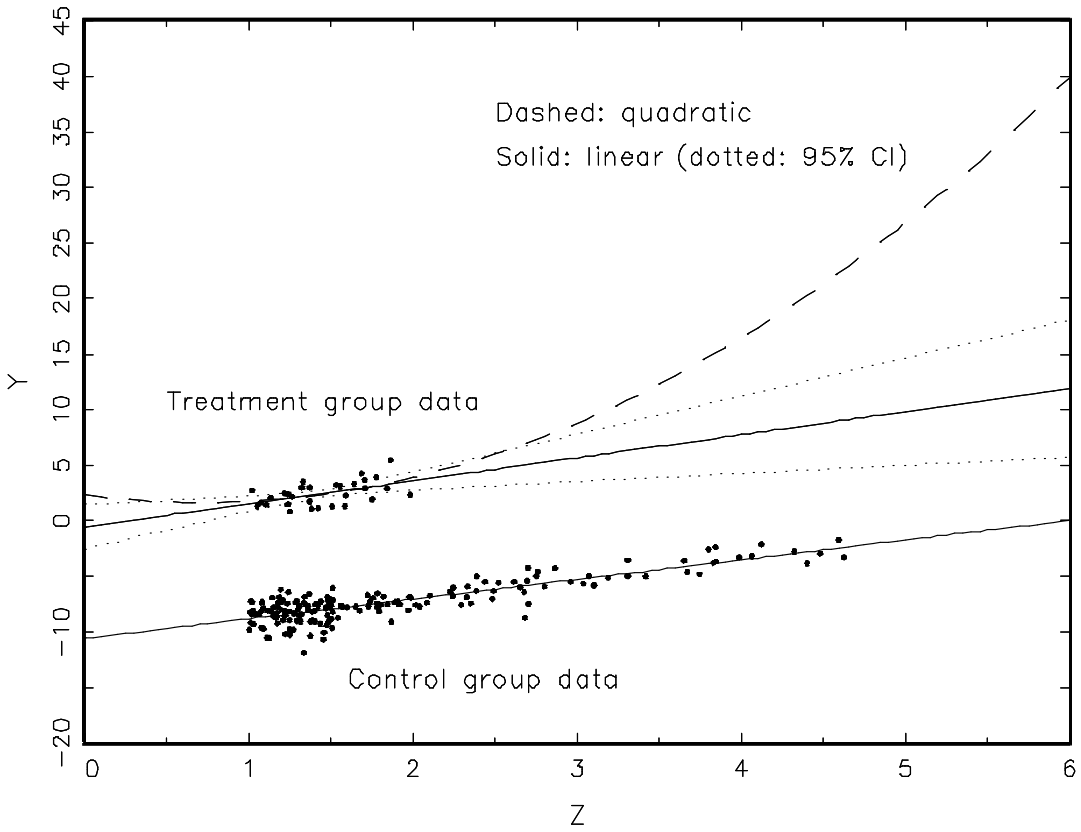

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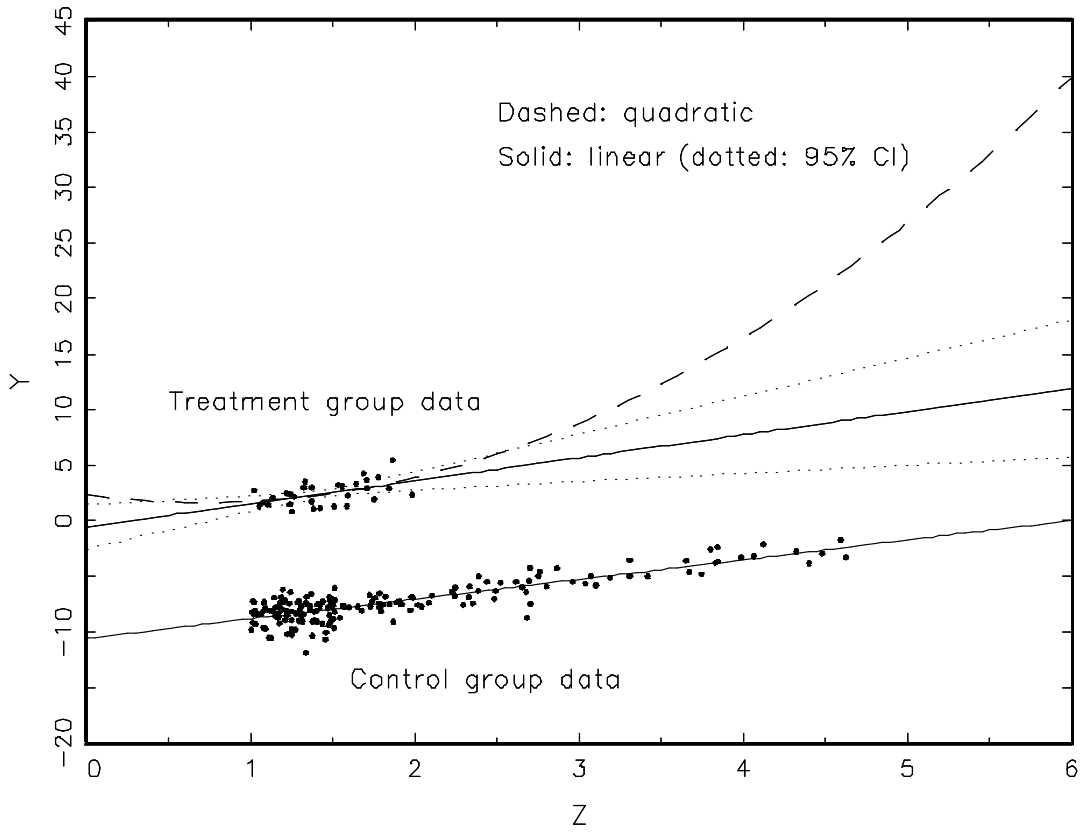

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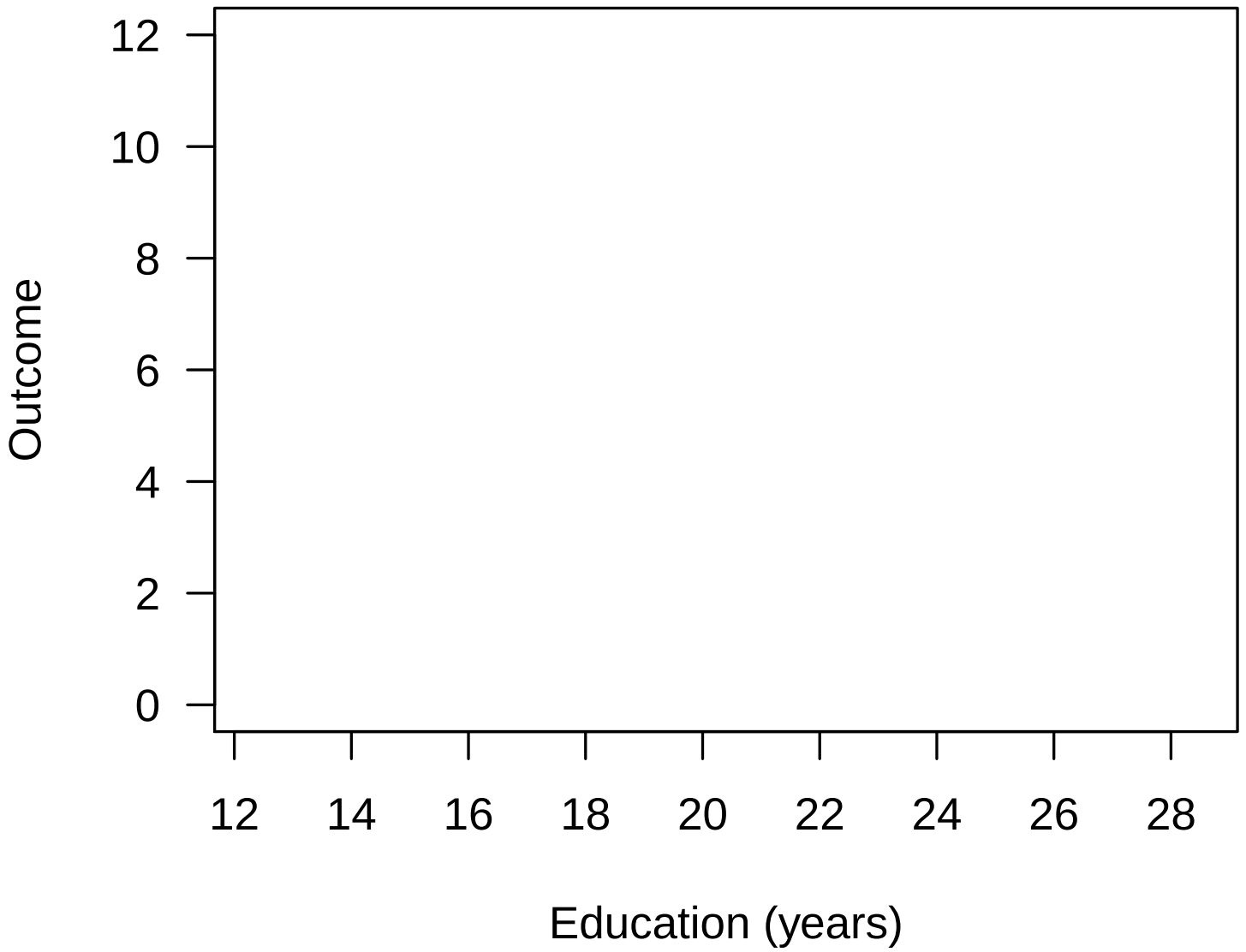

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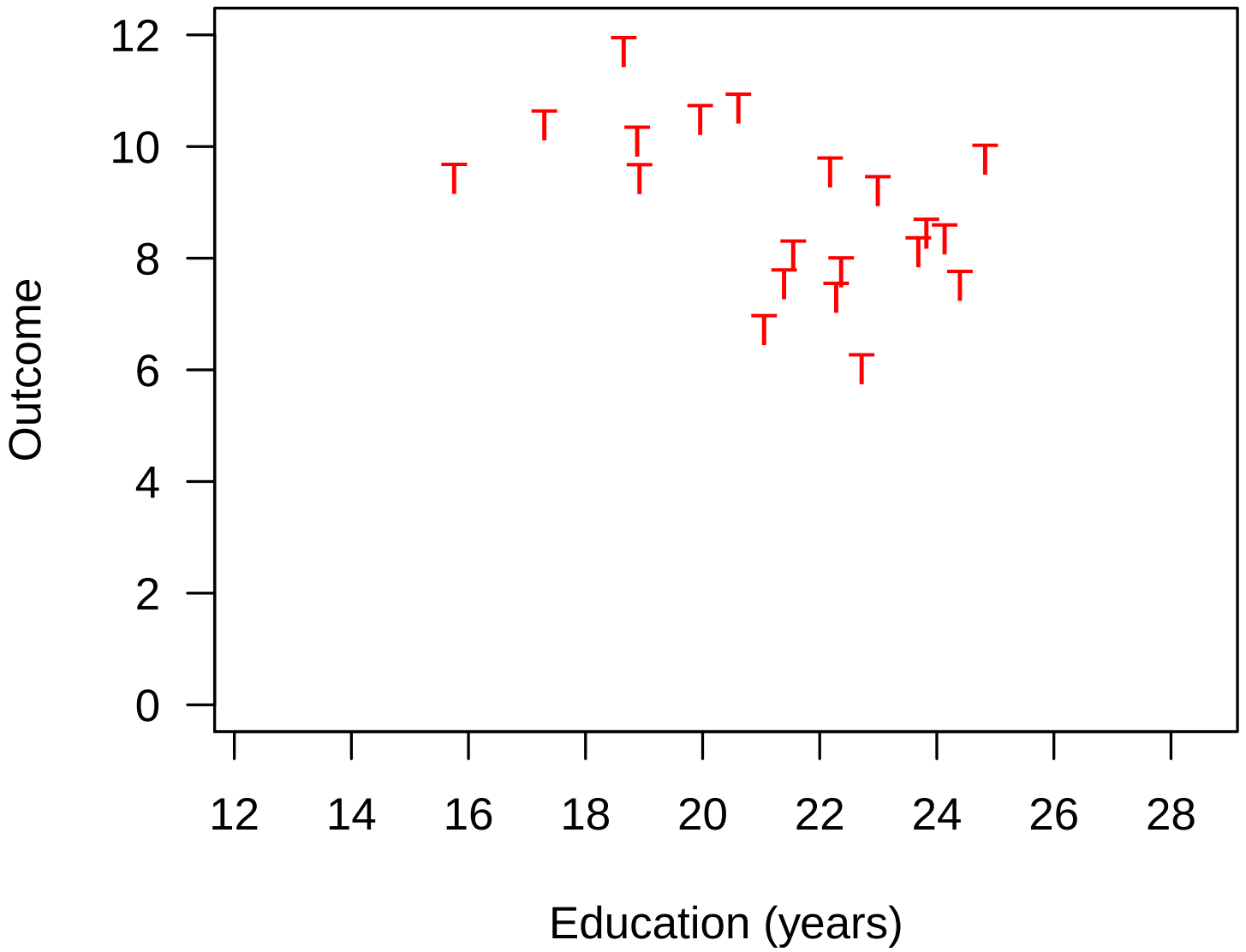

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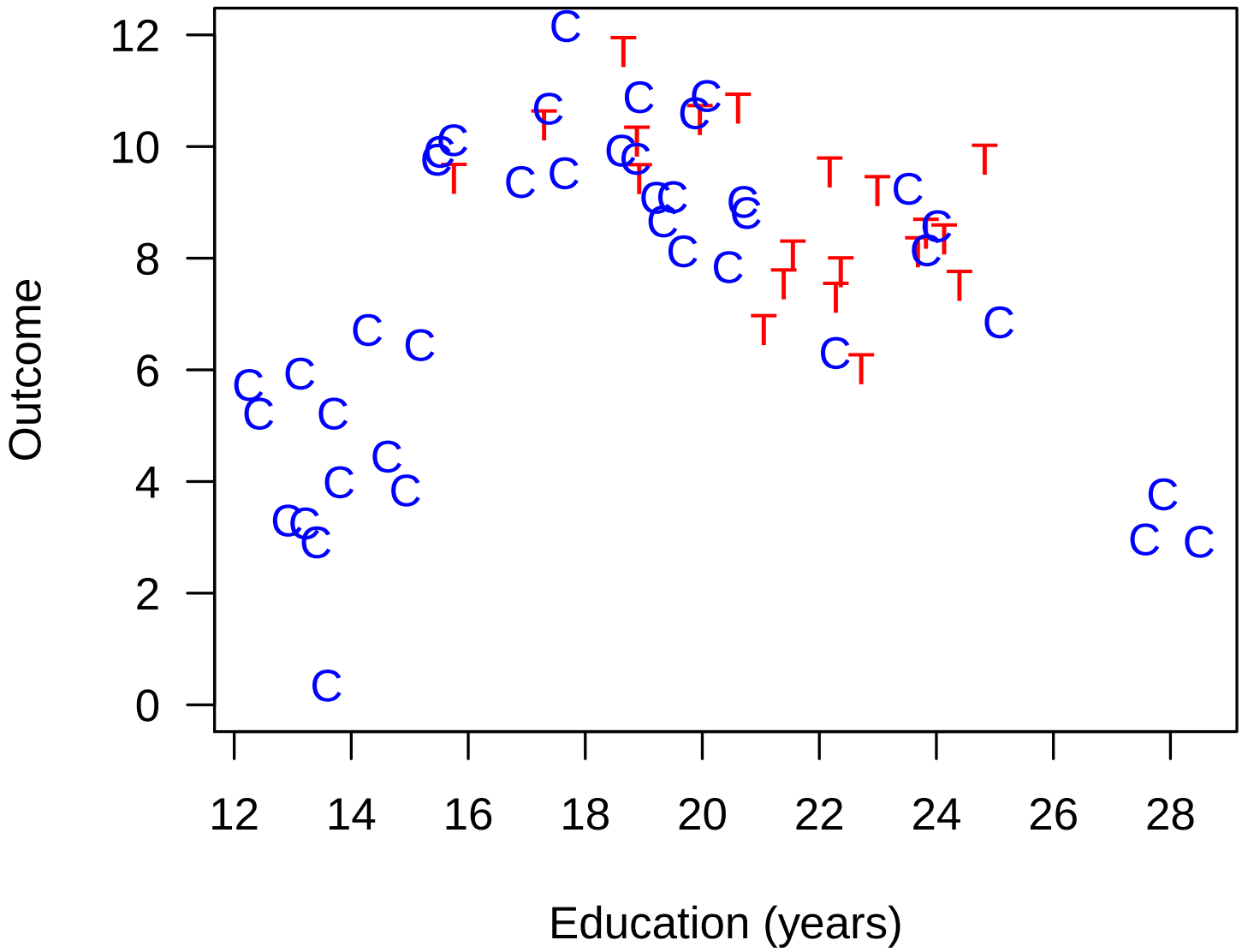

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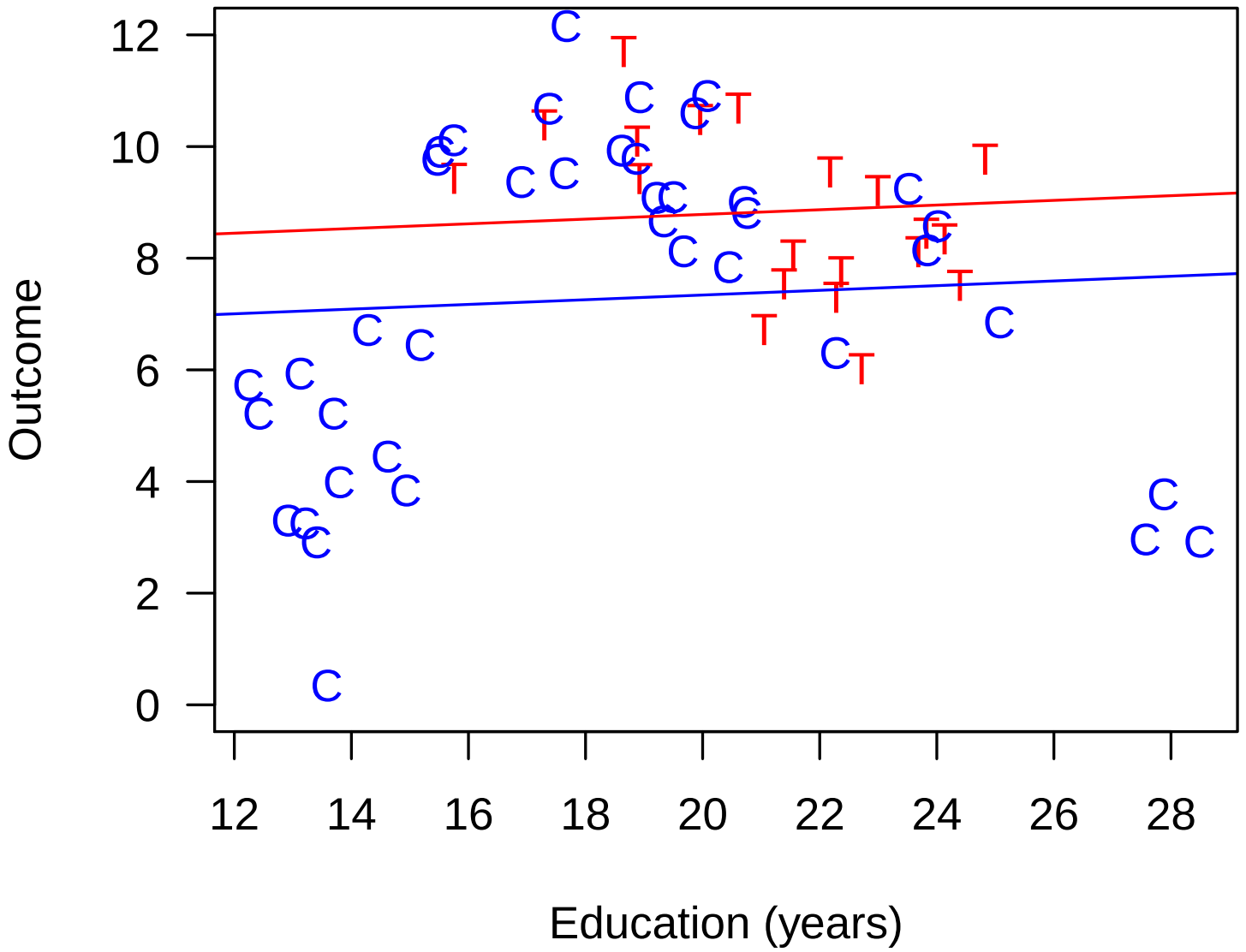

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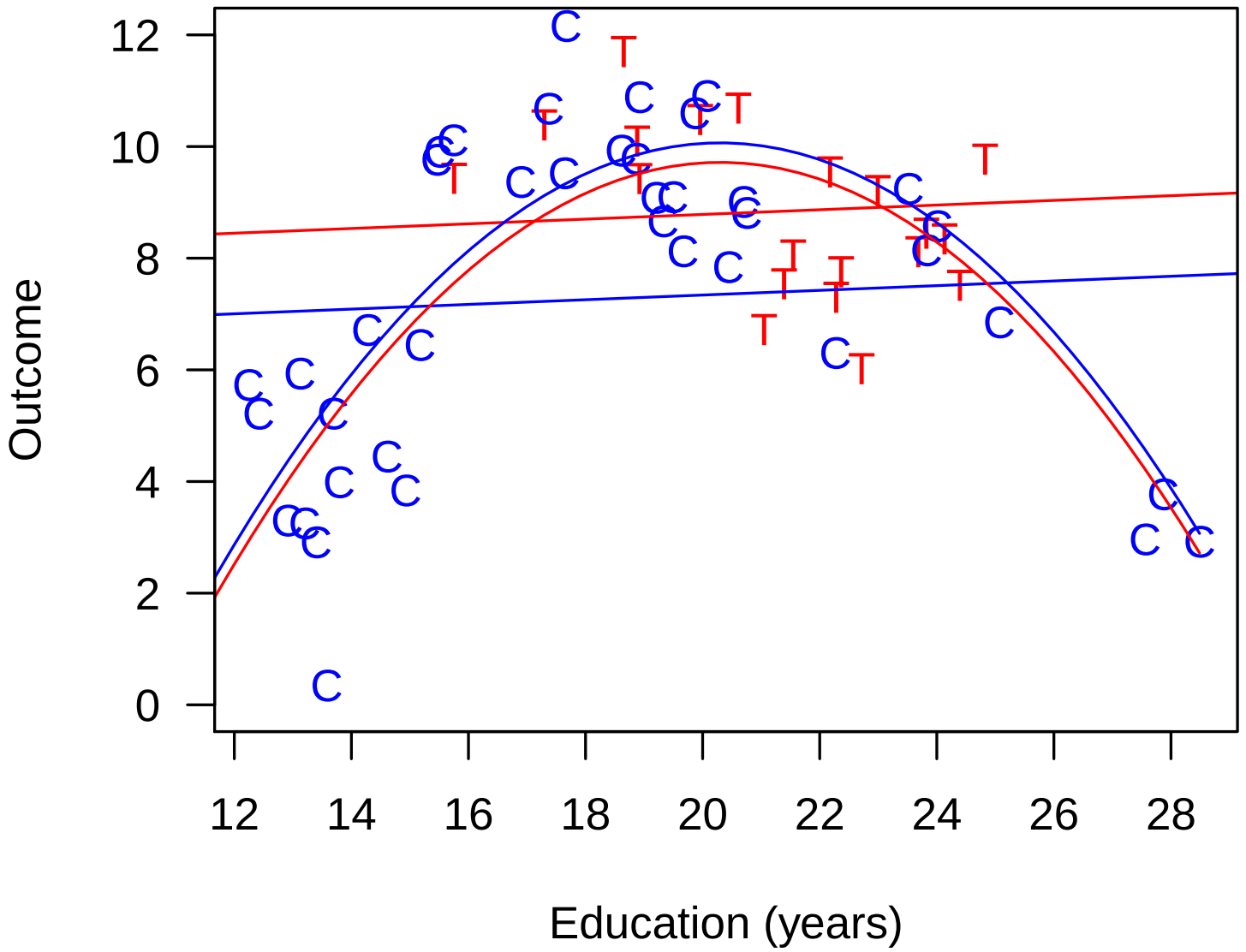

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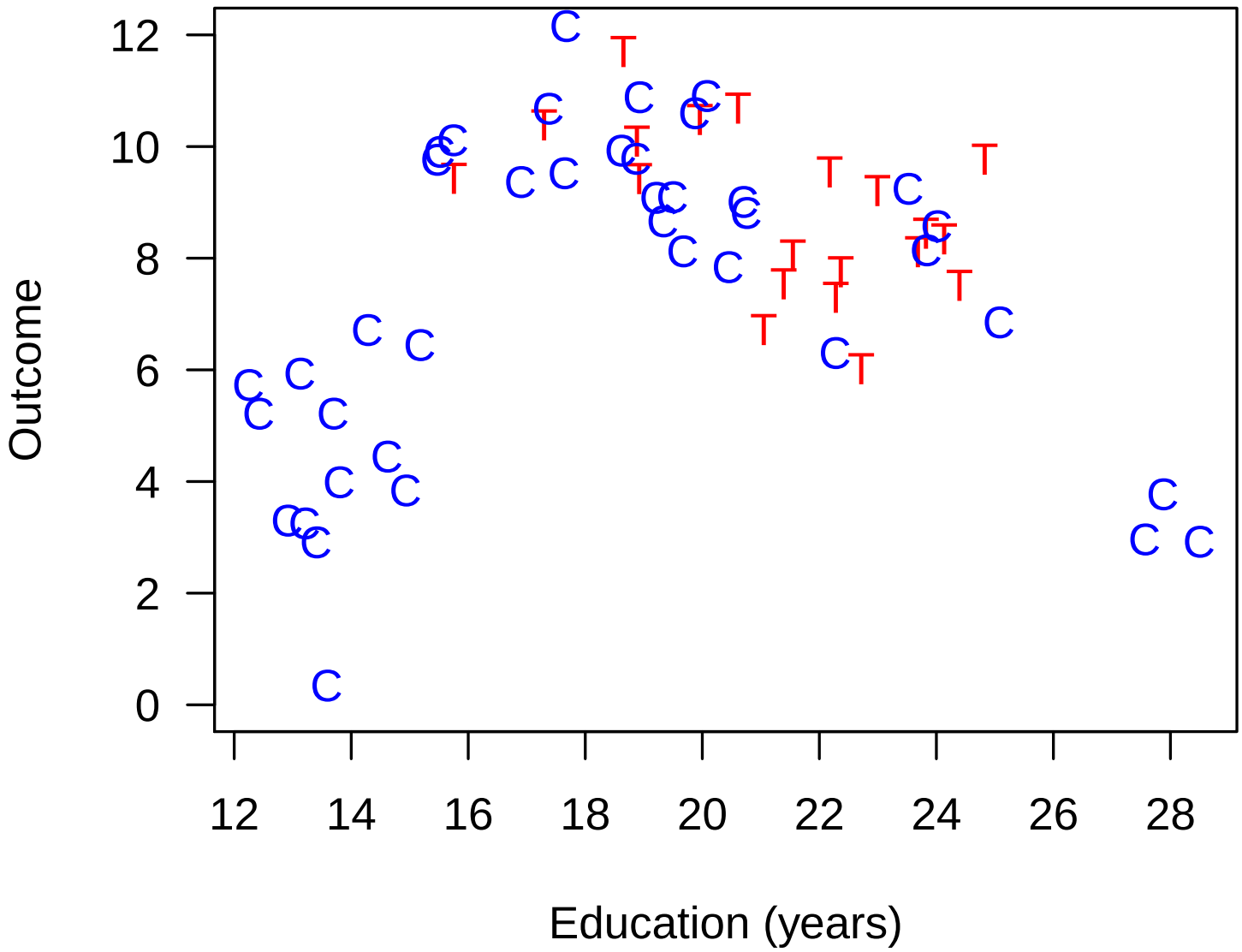

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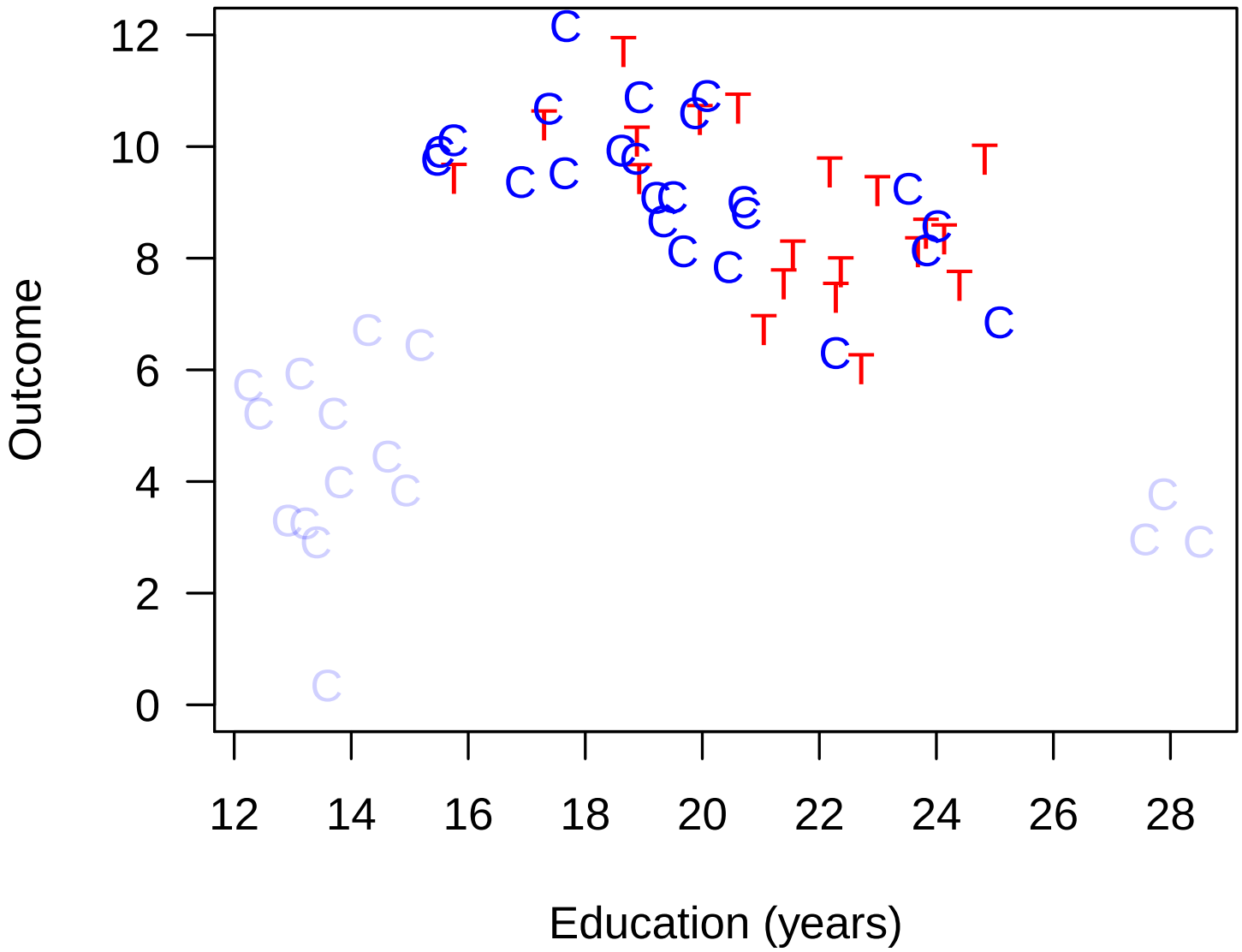

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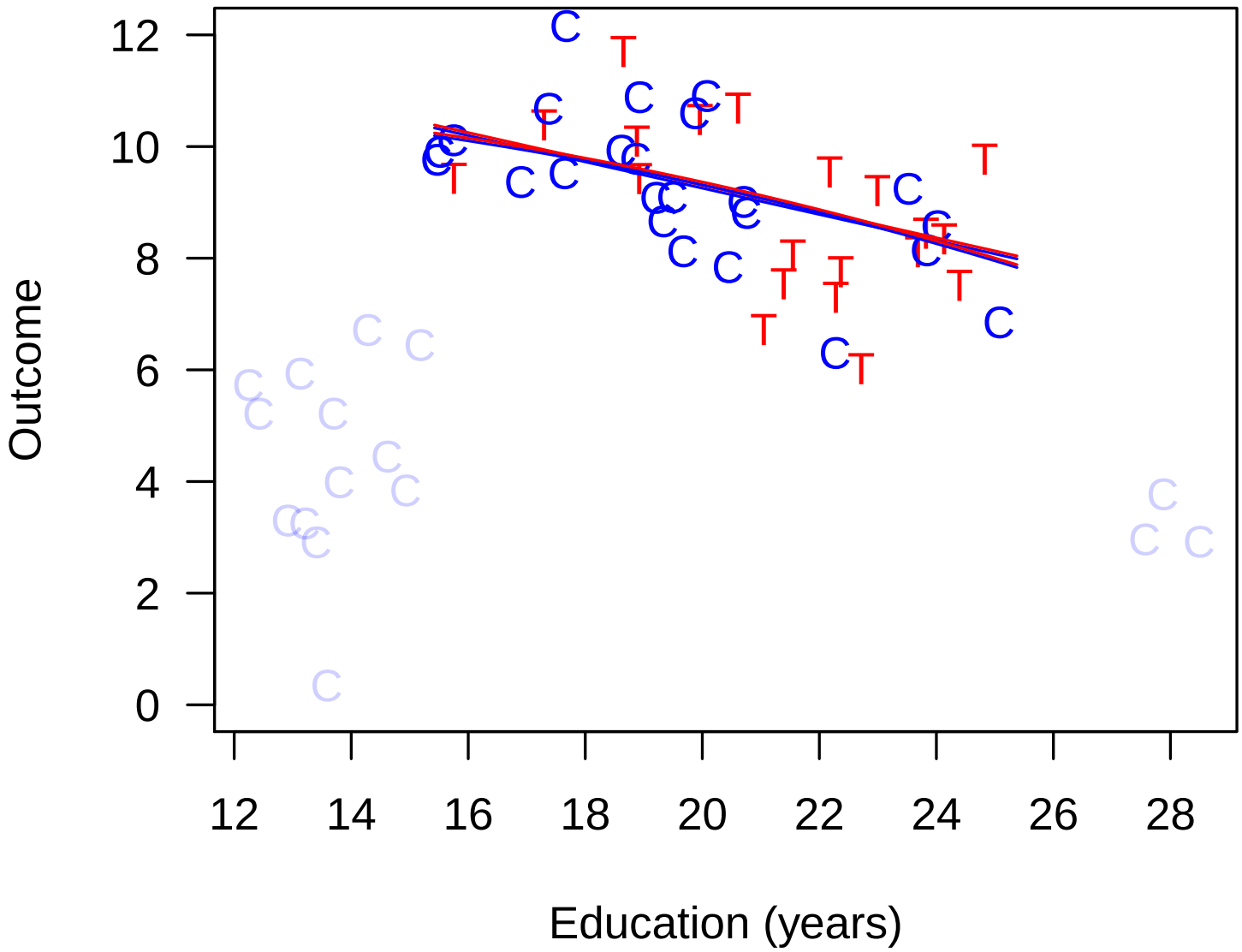

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- 18 control variables (clinical factors, firm characteristics, media variables, etc.)

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- (Normal applications would only use one or a few specifications.)

Reducing Model Dependence

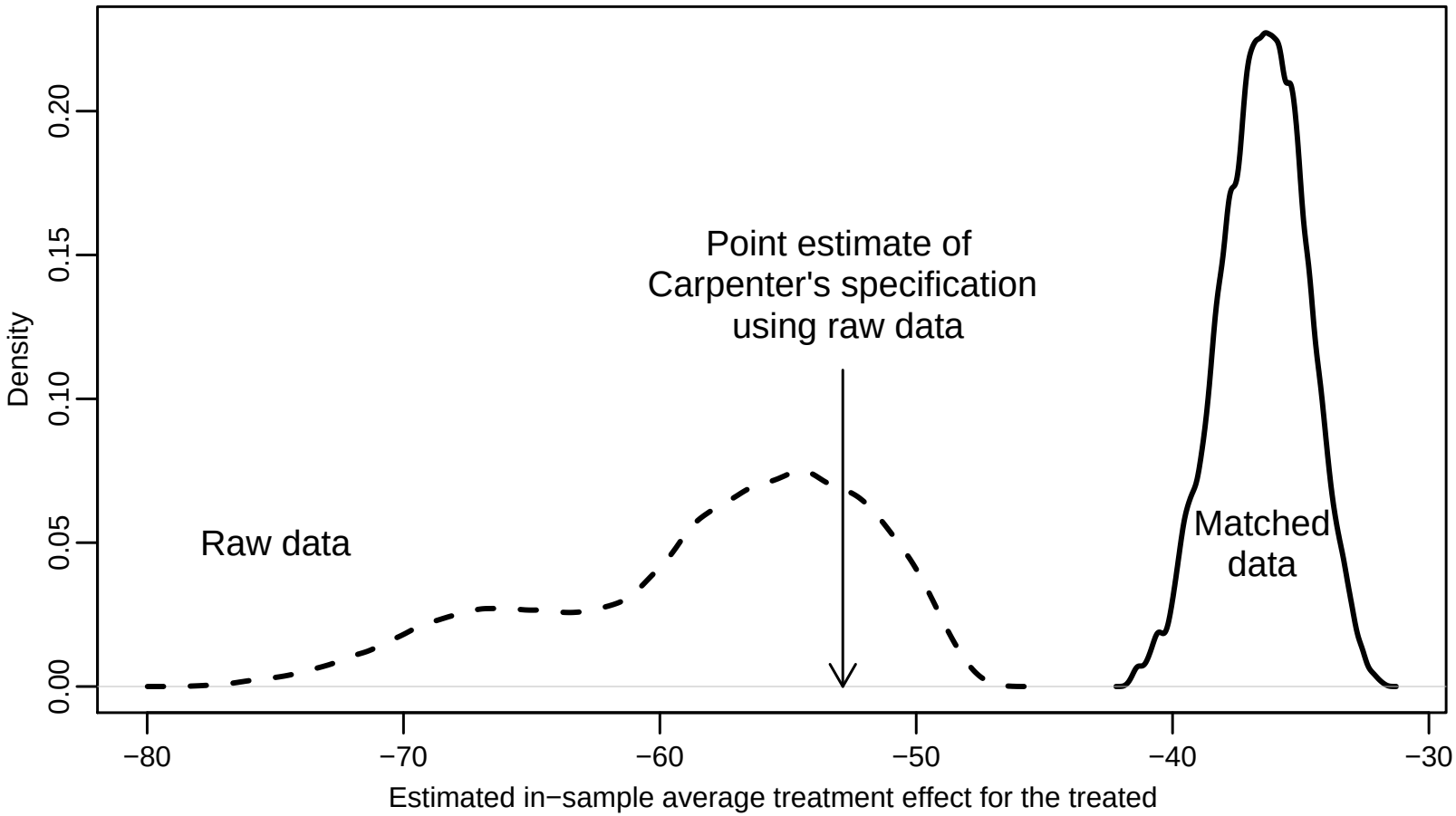

Figure: SATT Histogram: Effect of Democratic Senate majority on FDA drug approval time, across 262,143 specifications.

1. Regression with Measured Confounding
   - Regression with Constant Effects
   - Additive Regression with Heterogeneous Effects
   - Imputation Estimators
2. Matching
   - Fundamentals of Matching
   - Two Approaches to Matching
3. Fixed Effects
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- Last, discard all unmatched control units.
- The distribution of X_i will be exactly the same for treated and matched control:

$$P(X_i = x | T_i = 1) = P(X_i = x | T_i = 0, \mathbb{I}_c)$$

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- $\rightsquigarrow$ average of the within matched-pair differences.

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- Balance is checkable $\rightsquigarrow$ are T_i and X_i related in the matched data?

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6. Calculate the effect of the treatment on the outcome in the matched dataset.

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$$D_{ij} = \begin{cases} 0 & \text{if } X_i = X_j \\ \infty & \text{if } X_i \neq X_j \end{cases}$$

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- Here, $\widehat{\sigma}_k^2$ is the variance of the kth variable:

$$\widehat{\sigma}_k^2 = \frac{1}{N-1} \sum_{i=1}^{N} (X_{ik} - \bar{X}_k)$$

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- $\widehat{\Sigma}$ is the estimated variance-covariance matrix of the observations:

$$\widehat{\Sigma} = \frac{1}{N} \sum_{i=1}^{N} (X_i - \bar{X})(X_i - \bar{X})^T$$

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- Implementation: a caliper, which is the maximum distance we would accept

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- Estimated:

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- Note: in nearest neighbor without replacement the order matters!

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1. Regression with Measured Confounding
   - Regression with Constant Effects
   - Additive Regression with Heterogeneous Effects
   - Imputation Estimators
2. Matching
   - Fundamentals of Matching
   - Two Approaches to Matching
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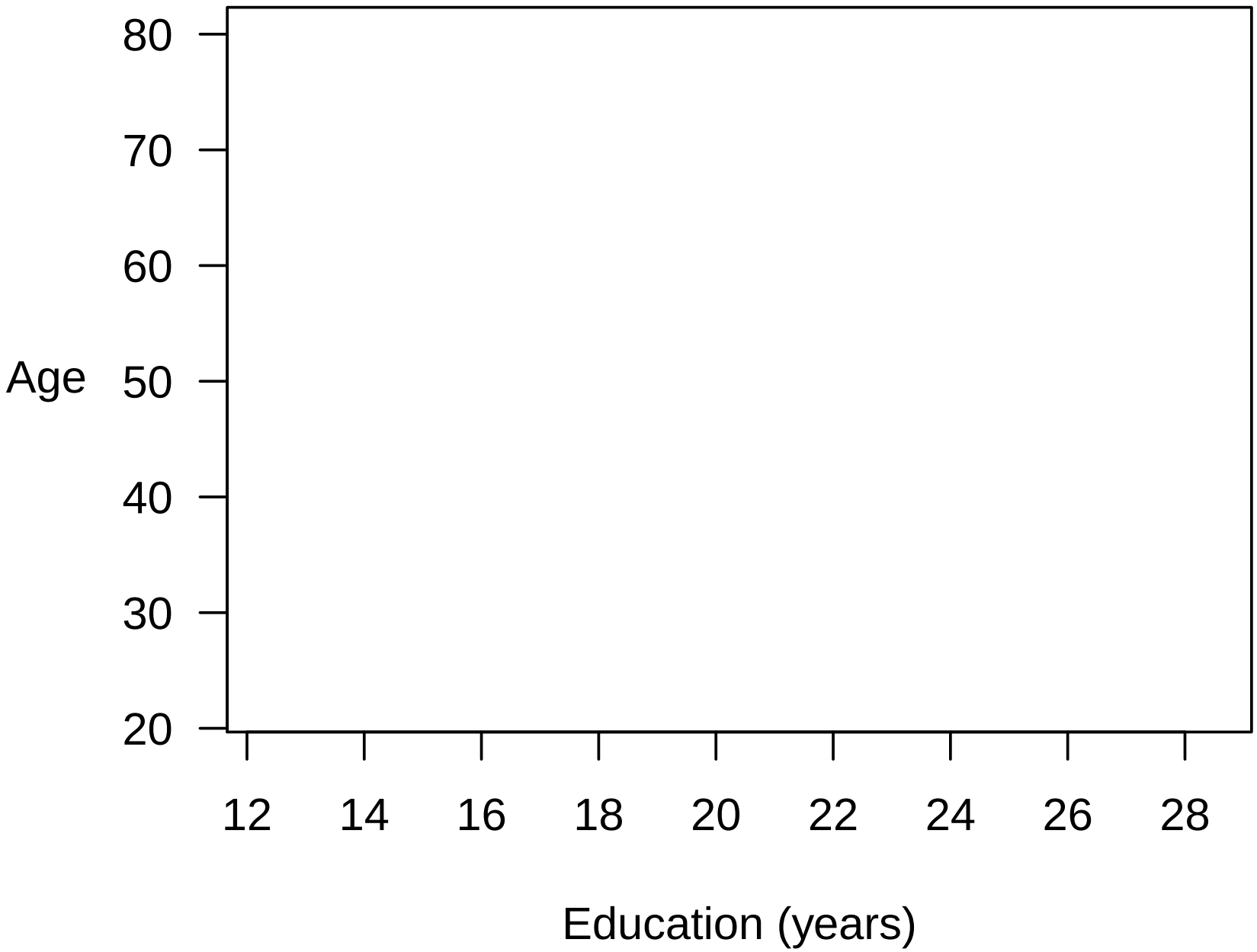

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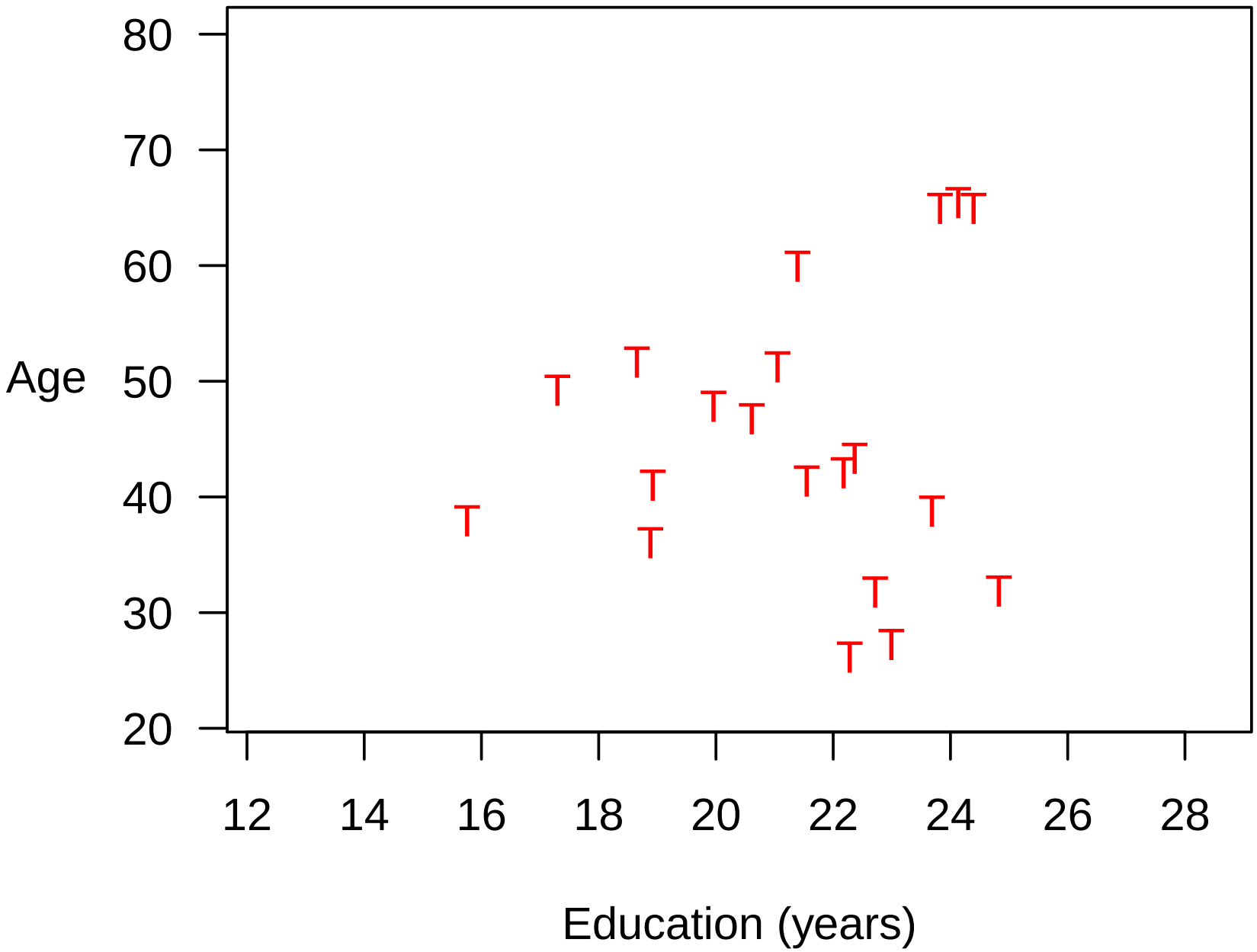

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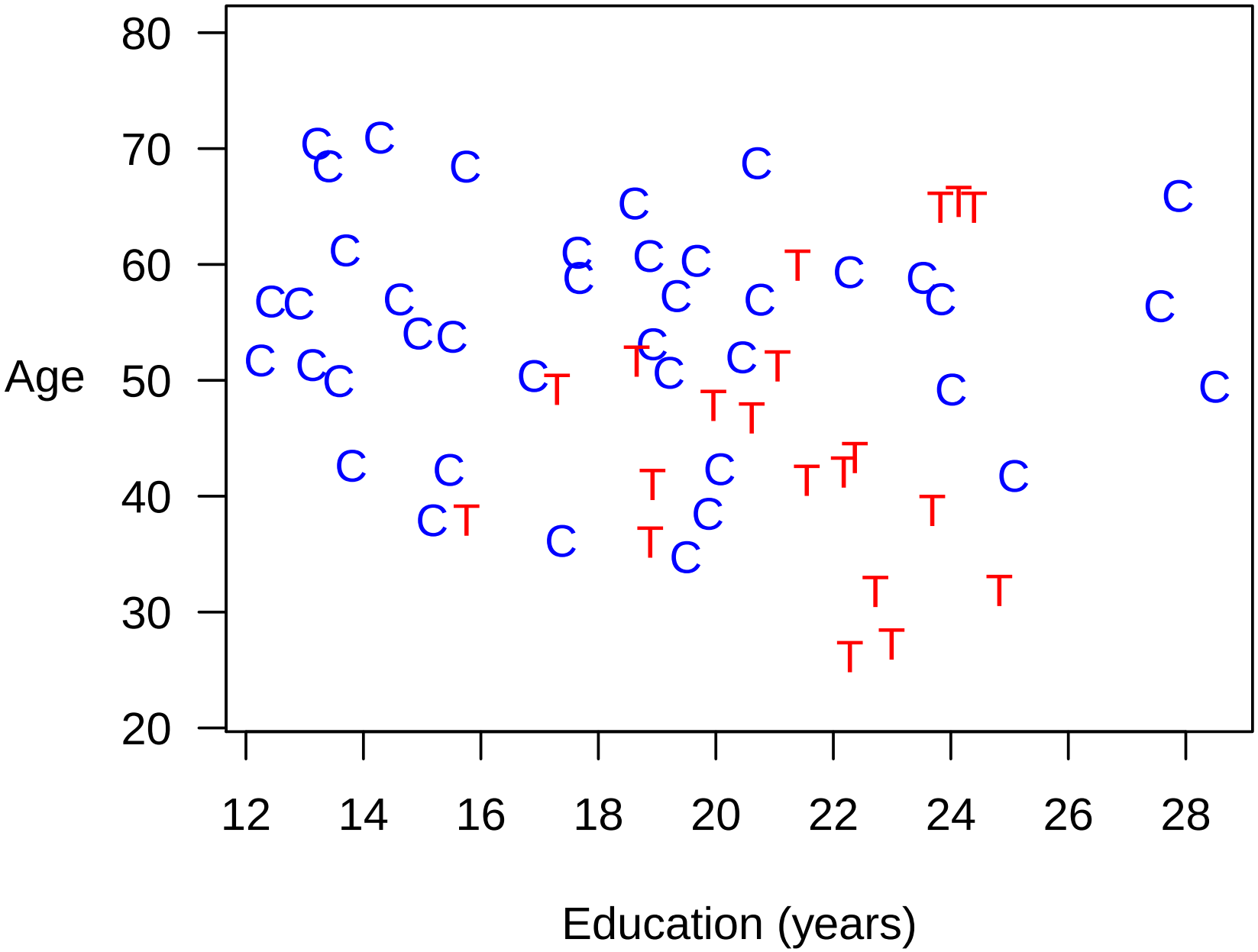

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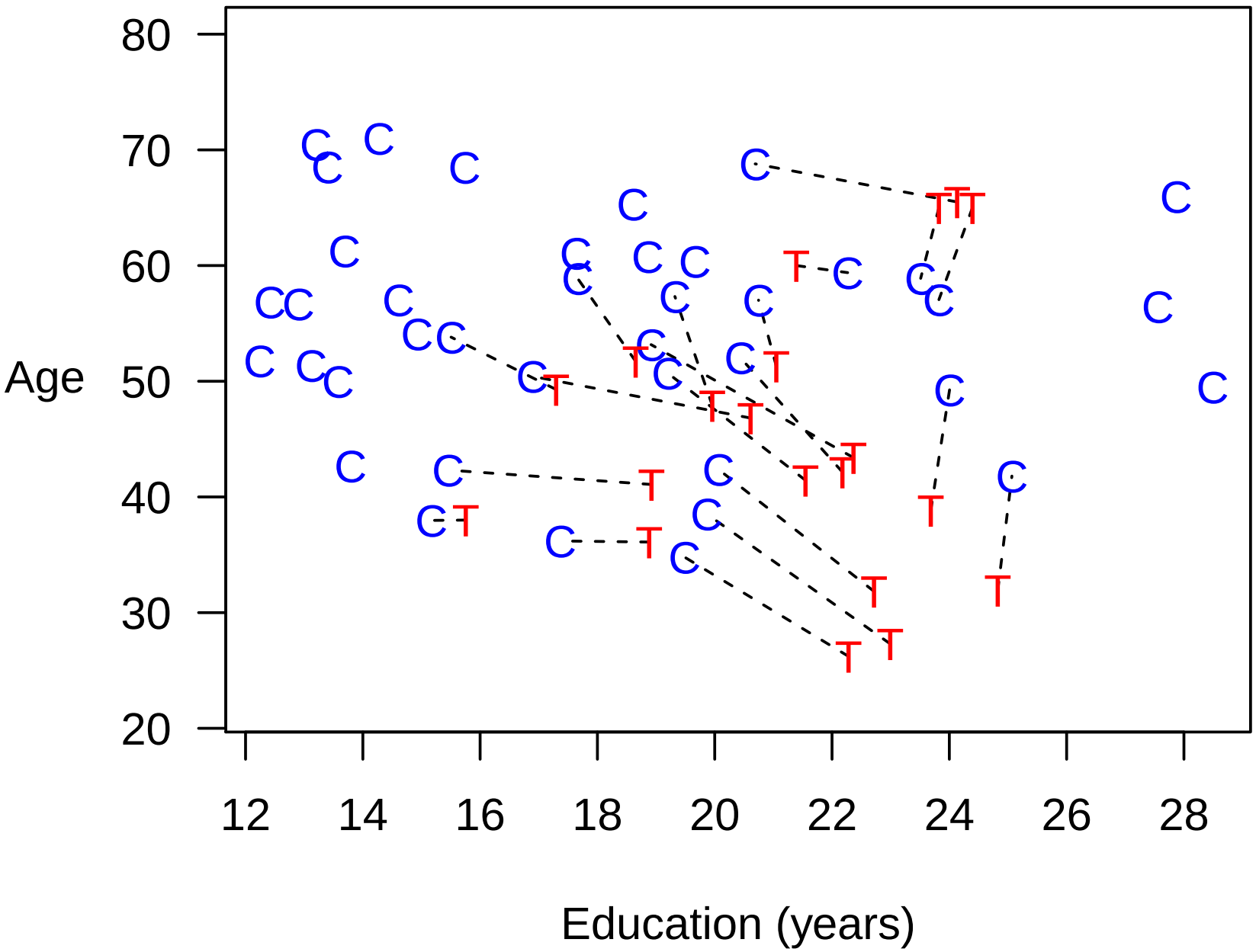

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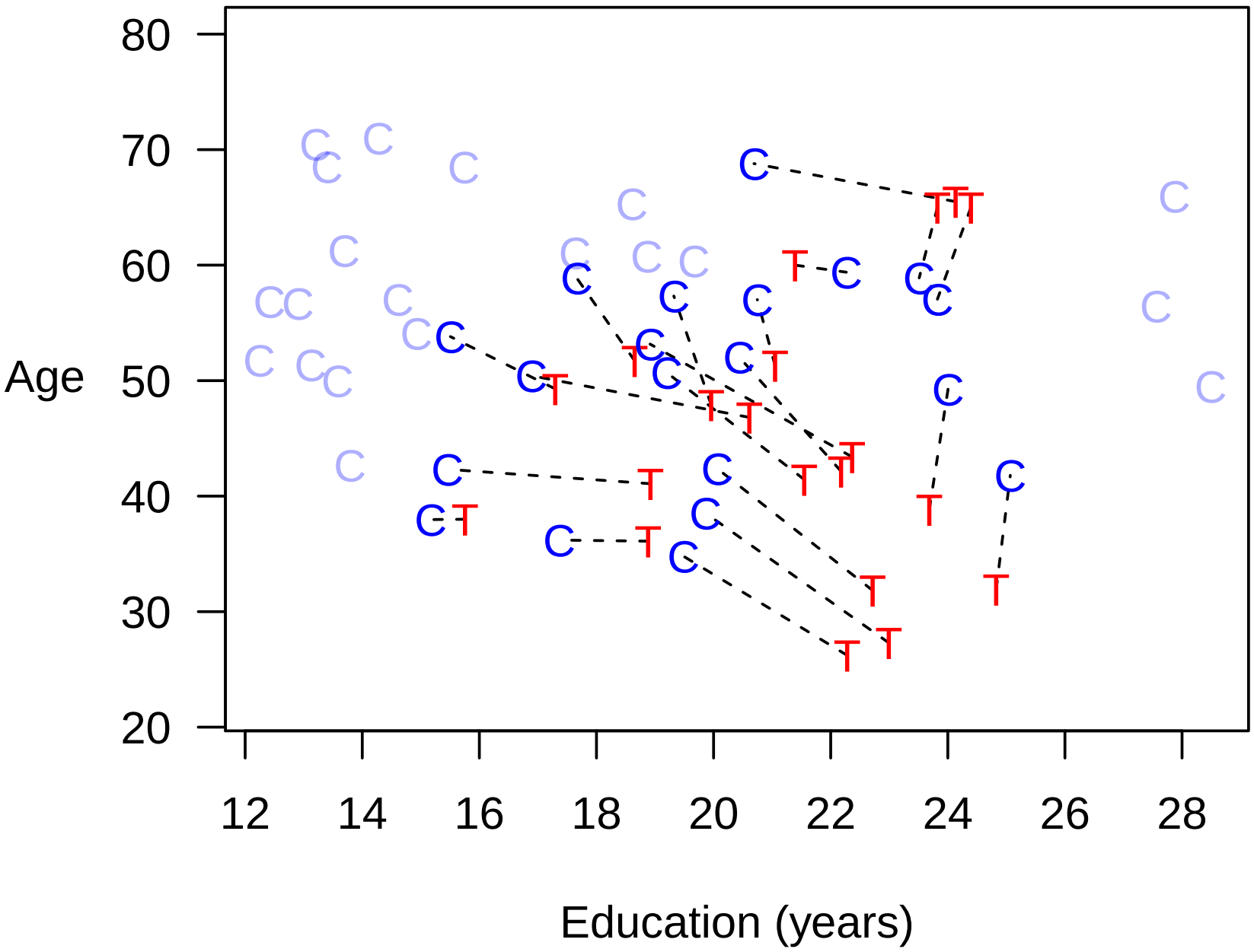

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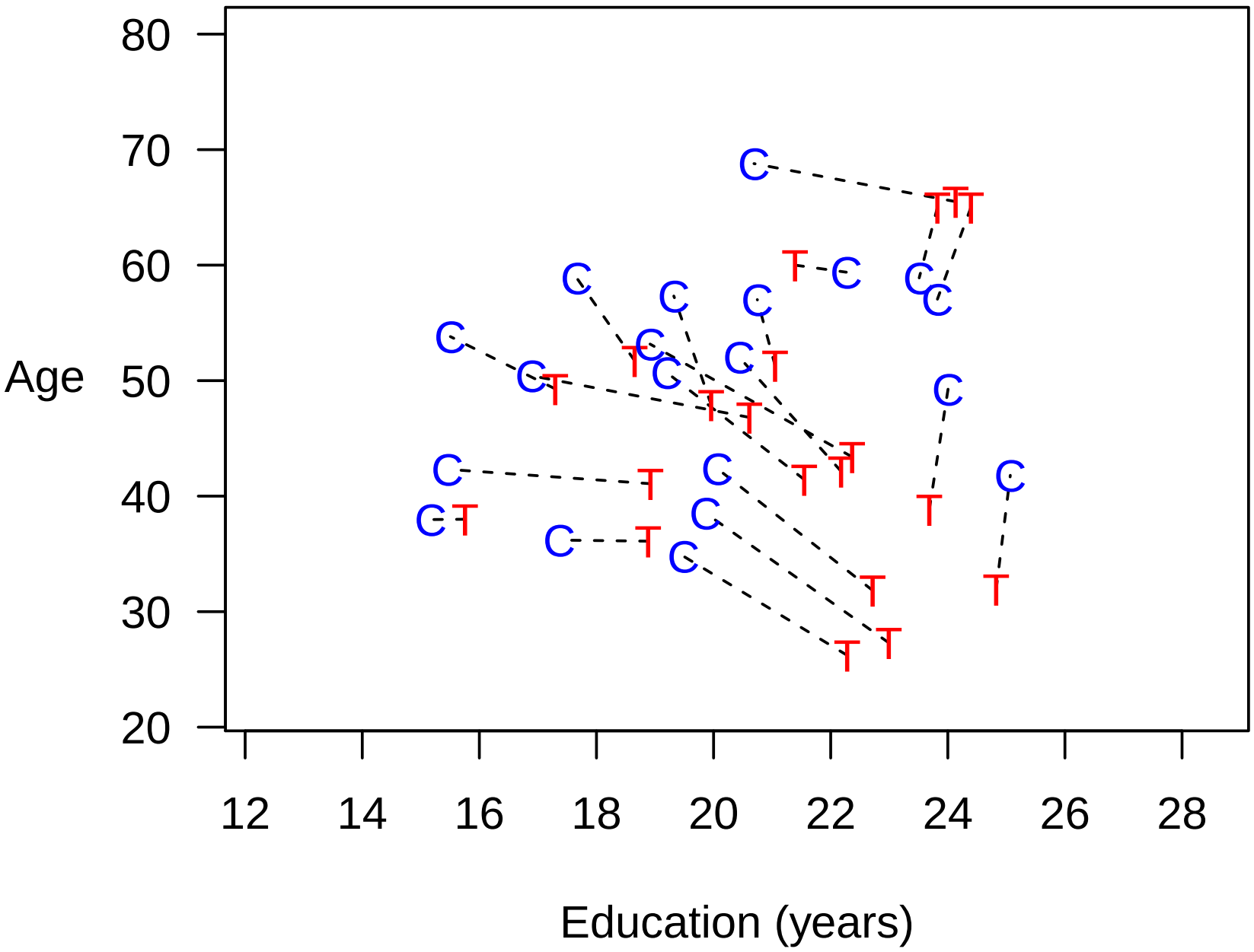

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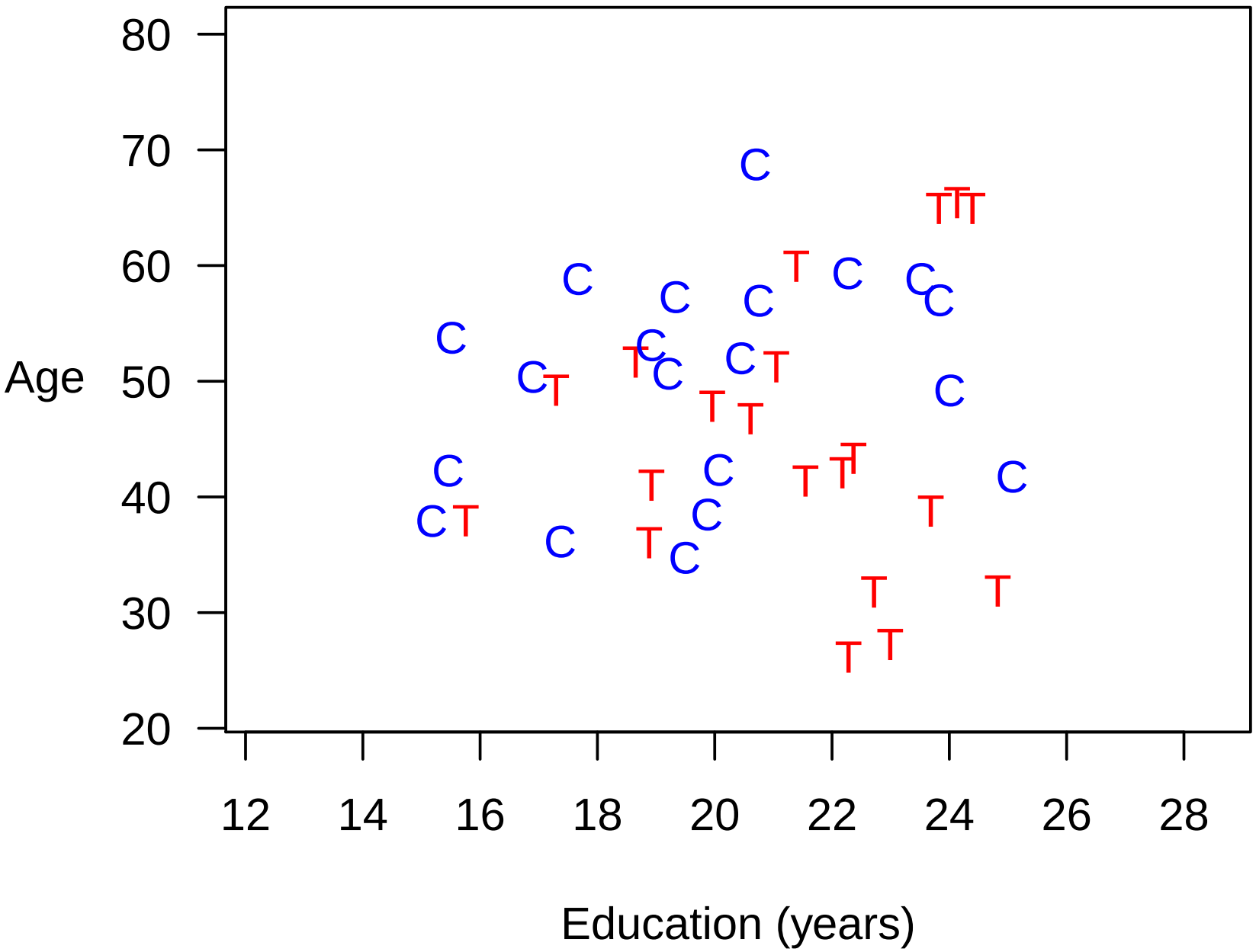

Method 2: Coarsened Exact Matching

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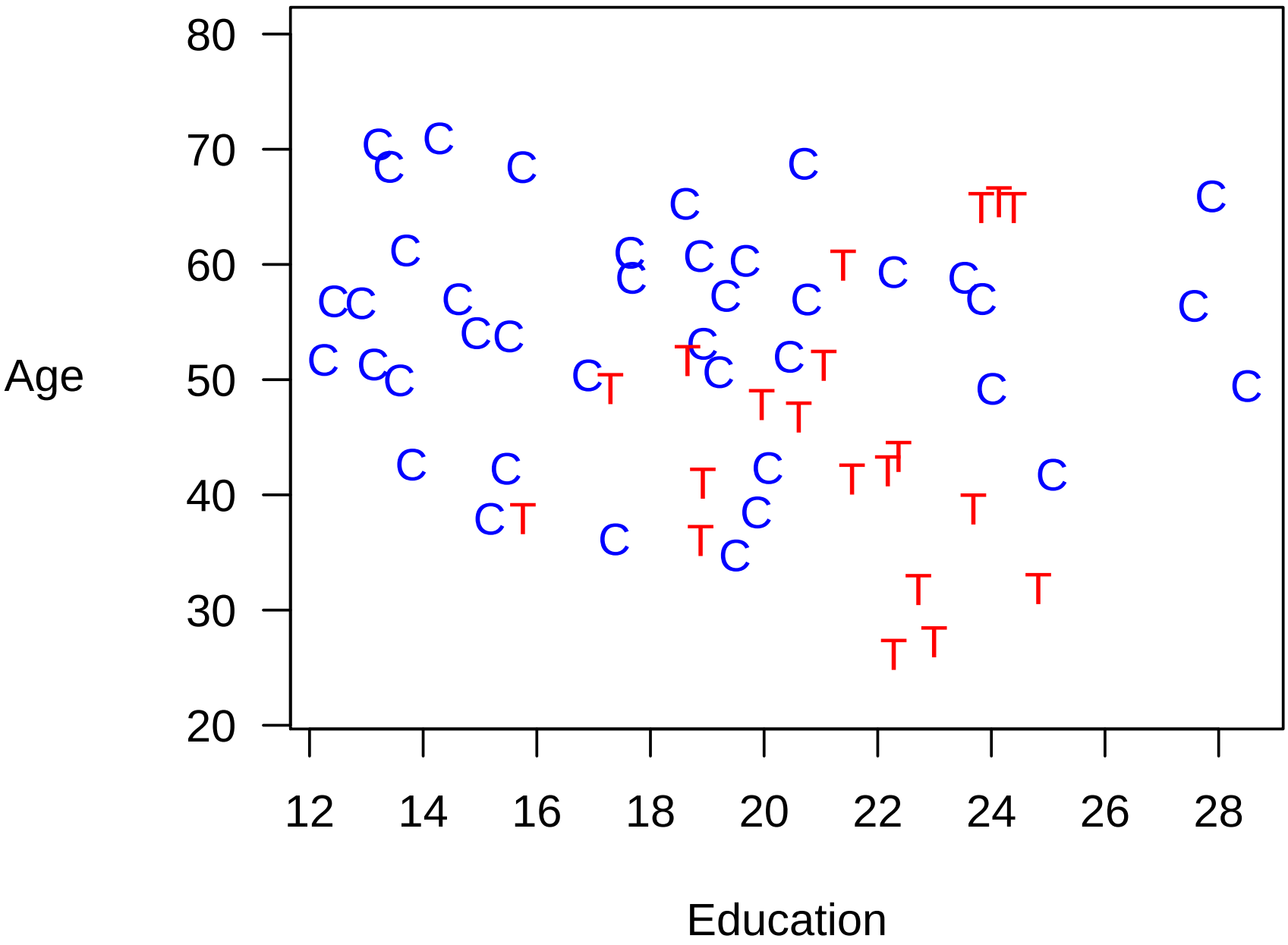

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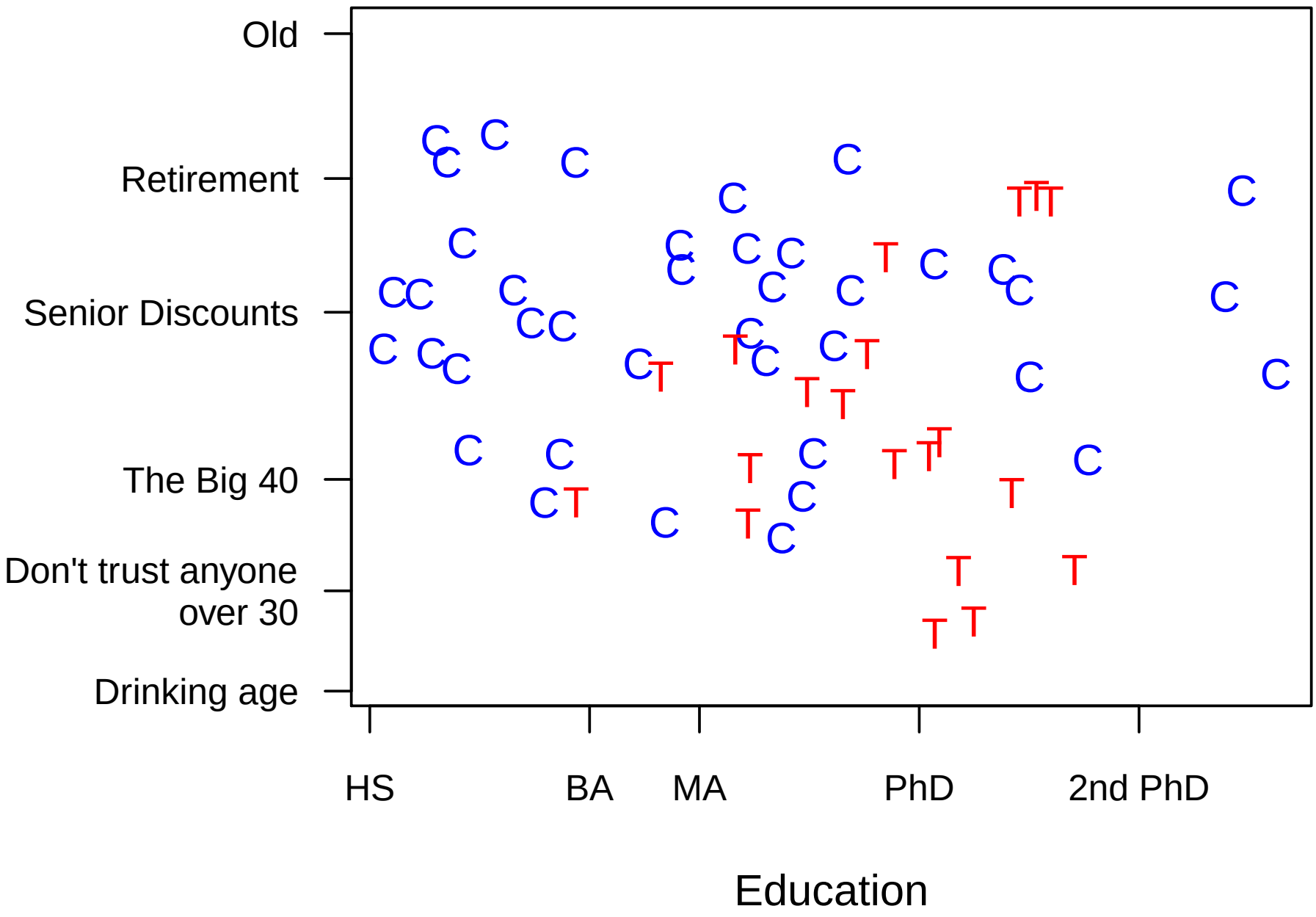

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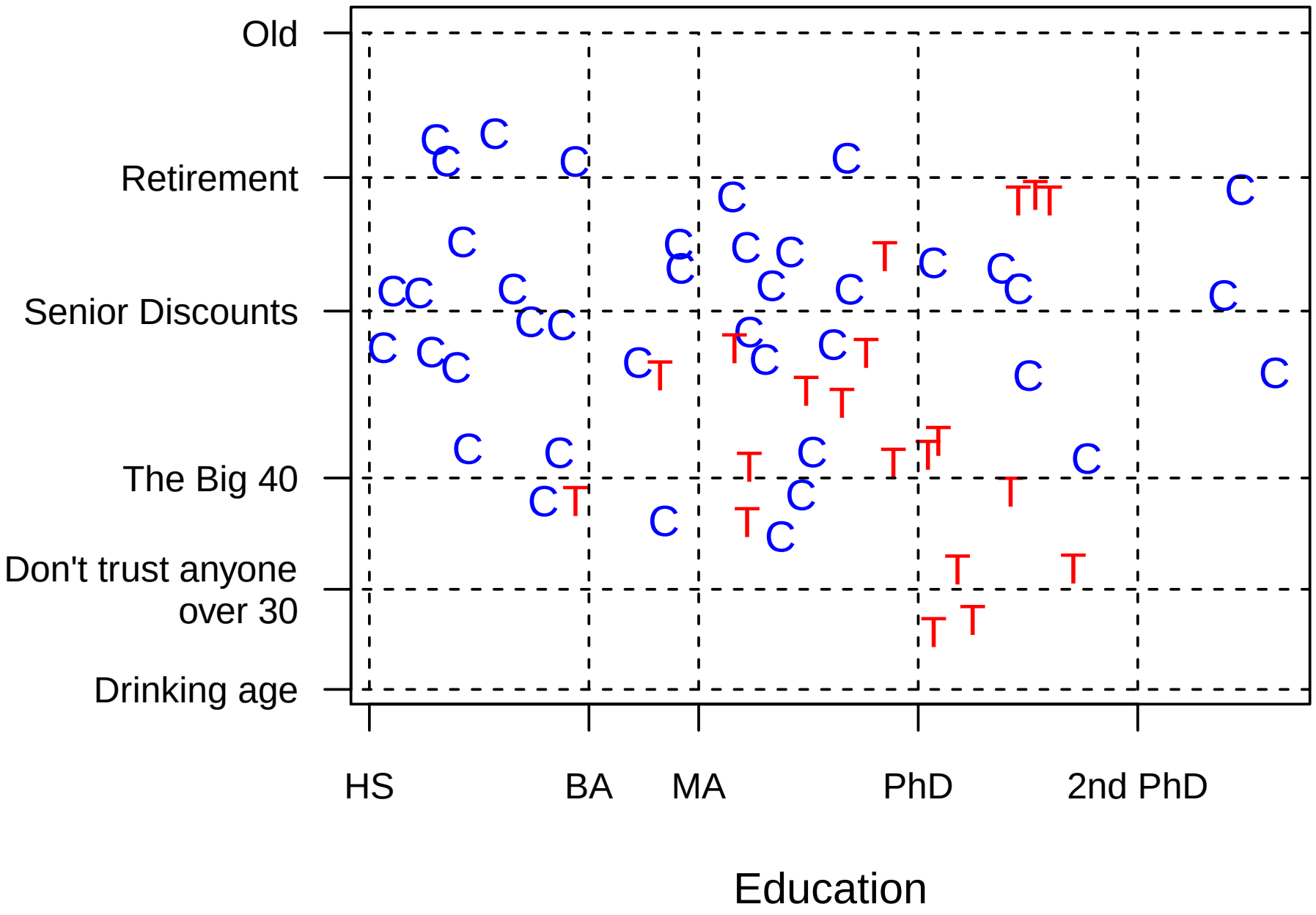

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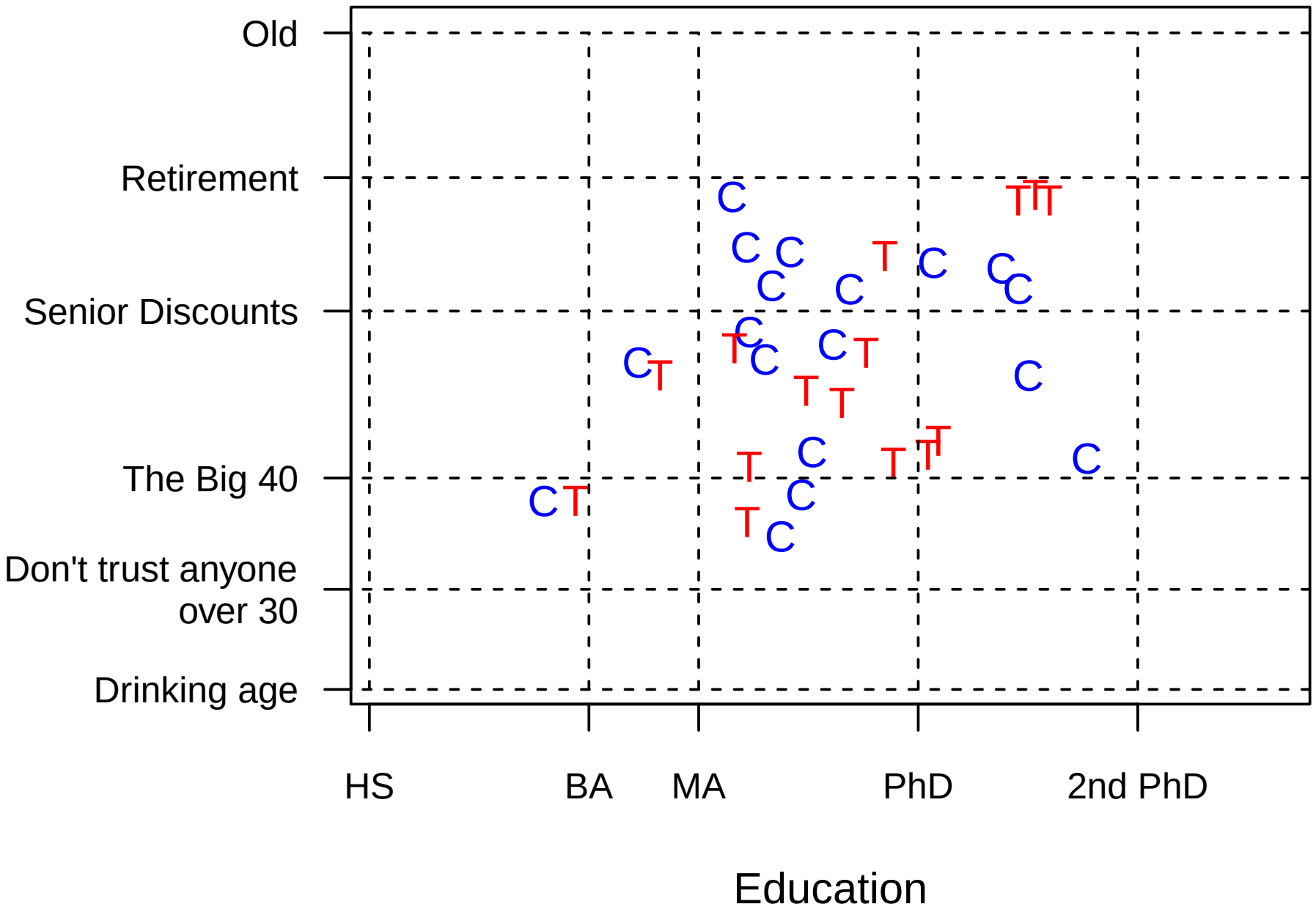

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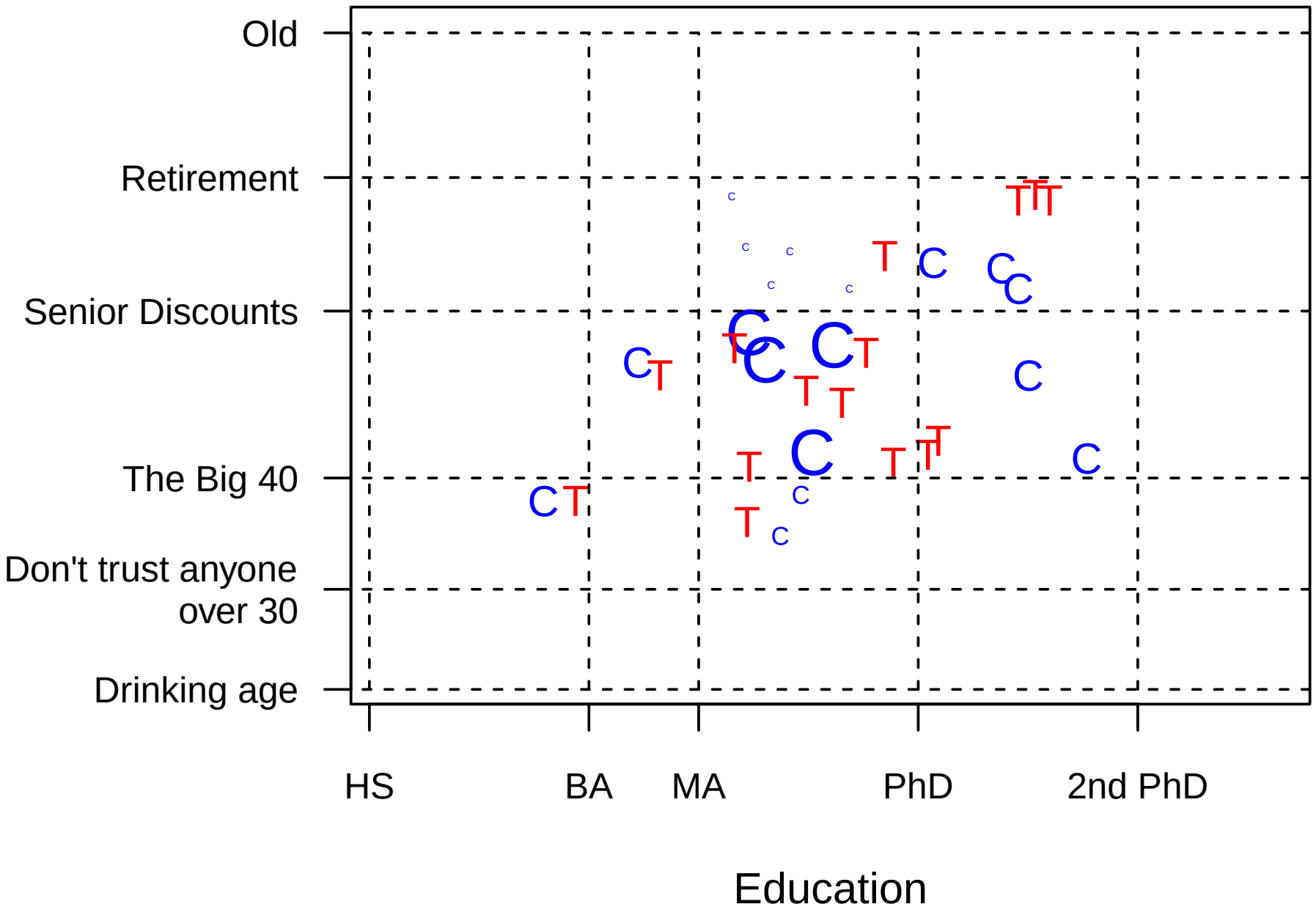

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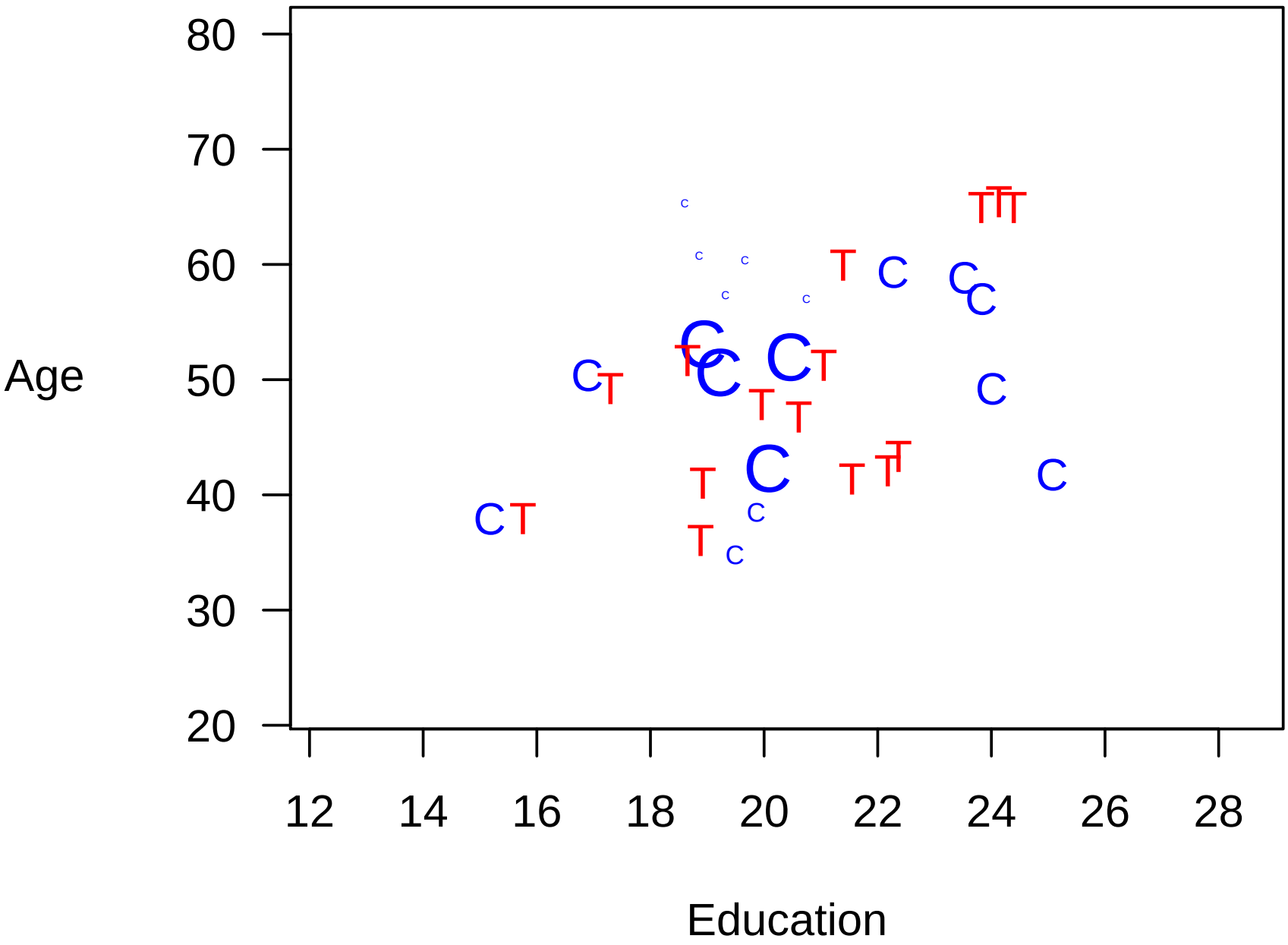

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- Technically the thing it is buying you relative to regression methods we will talk about next is that it limits extrapolation.
- But there is no need for these techniques to compete—we can match and then use regression!
- Importantly, there is nothing magic about matching, it is just another way of conditioning.

1. Regression with Measured Confounding
   - Regression with Constant Effects
   - Additive Regression with Heterogeneous Effects
   - Imputation Estimators
2. Matching
   - Fundamentals of Matching
   - Two Approaches to Matching
3. Fixed Effects
   - Non-parametric Identification and Fixed Effects

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- We will start by assuming the model and discussing properties and in the next section, we will consider non-parametric identification.

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- This regression is sometimes called the "between regression"

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- NB: you must demean the X variables not just the Y variables.

Fixed Effects Example with data from Ross

```
fe.mod <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur), data = ross, index = c("id", "year"),
 model = "within")
summary(fe.mod)
```

```
## Oneway (individual) effect Within Model
##
## Call:
## plm(formula = log(kidmort_unicef) ~ democracy + log(GDPcur),
##     data = ross, model = "within", index = c("id", "year"))
##
## Unbalanced Panel: n=166, T=1-7, N=649
##
## Residuals :
##     Min.  1st Qu.   Median  3rd Qu.     Max.
## -0.70500 -0.11700  0.00628  0.12200  0.75700
##
## Coefficients :
##              Estimate Std. Error  t-value  Pr(>|t|)
## democracy   -0.143233   0.033500  -4.2756 2.299e-05 ***
## log(GDPcur) -0.375203   0.011328 -33.1226 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Total Sum of Squares:    81.711
## Residual Sum of Squares: 23.012
## R-Squared      :  0.71838
##       Adj. R-Squared :  0.53242
## F-statistic: 613.481 on 2 and 481 DF, p-value: < 2.22e-16
```

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- This is because the composite errors, $\ddot{u}_{it}$ are function of the errors in every time period through the average, $\overline{u}_i$
- This rules out, for instance, lagged dependent variables, since $y_{i,t-1}$ has to be correlated with $u_{i,t-1}$. Thus it can't be a covariate for y_{it}.

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- Basic message: any time-constant variable gets "absorbed" by the fixed effect. It has nothing to contribute because the comparison is within the units.
- Can include interactions between time-constant and time-varying variables, but lower order term of the time-constant variables get absorbed by fixed effects too

Time-constant variables

- Pooled model with a time-constant variable, proportion Islamic:

```
library(lmtest)
p.mod <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur) + islam,
             data = ross, index = c("id", "year"), model = "pooling")
coeftest(p.mod)

##
## t test of coefficients:
##
##                Estimate  Std. Error  t value  Pr(>|t|)
## (Intercept) 10.30607817  0.35951939  28.6663 < 2.2e-16 ***
## democracy   -0.80233845  0.07766814 -10.3303 < 2.2e-16 ***
## log(GDPcur) -0.25497406  0.01607061 -15.8659 < 2.2e-16 ***
## islam        0.00343325  0.00091045   3.7709 0.0001794 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Time-constant variables

- FE model, where the islam variable drops out, along with the intercept:

```
fe.mod2 <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur) + islam,
               data = ross, index = c("id", "year"), model = "within")
coeftest(fe.mod2)

##
## t test of coefficients:
##
##               Estimate Std. Error  t value  Pr(>|t|)
## democracy    -0.129693   0.035865  -3.6162 0.0003332 ***
## log(GDPcur) -0.379997   0.011849 -32.0707 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Alternate Computation: Least Squares Dummy Variable

- As an alternative to the within transformation, we can also include a series of $n-1$ dummy variables for each unit:

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- Why are these equivalent? (remember partialing out strategy and Frisch-Waugh-Lovell theorem)

Example with Ross data

```
library(lmtest)
lsdv.mod <- lm(log(kidmort_unicef) ~ democracy + log(GDPcur) +
                as.factor(id), data = ross)
coeftest(lsdv.mod)[1:6,]
coeftest(fe.mod)[1:2,]

##                     Estimate Std. Error    t value      Pr(>|t|)
## (Intercept)       13.7644887 0.26597312  51.751427 1.008329e-198
## democracy         -0.1432331 0.03349977  -4.275644  2.299393e-05
## log(GDPcur)       -0.3752030 0.01132772 -33.122568 3.494887e-126
## as.factor(id)AGO   0.2997206 0.16767730   1.787485  7.448861e-02
## as.factor(id)ALB  -1.9309618 0.19013955 -10.155498  4.392512e-22
## as.factor(id)ARE  -1.8762909 0.17020738 -11.023558  2.386557e-25

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  - Cluster fixed effects in hierarchical data
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  - strict exogeneity
- We've seen the trouble with constant effects before, it goes back to Lecture 10 and results on regression with heterogenous treatment effects more generally.

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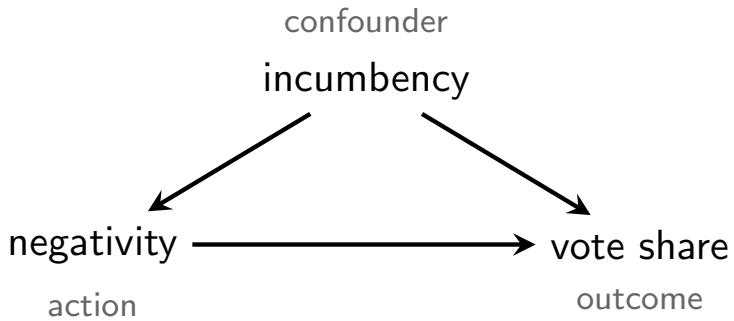
Examples of static and dynamic causal inference problems:



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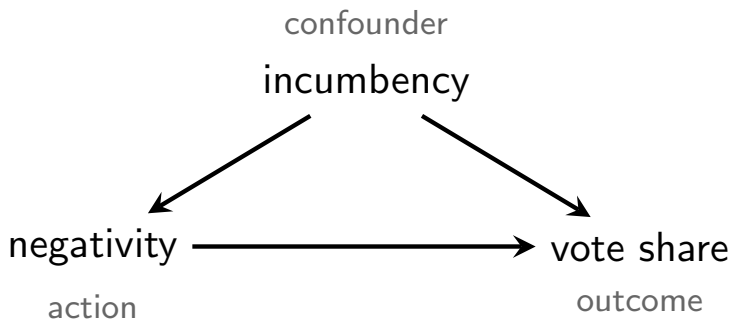

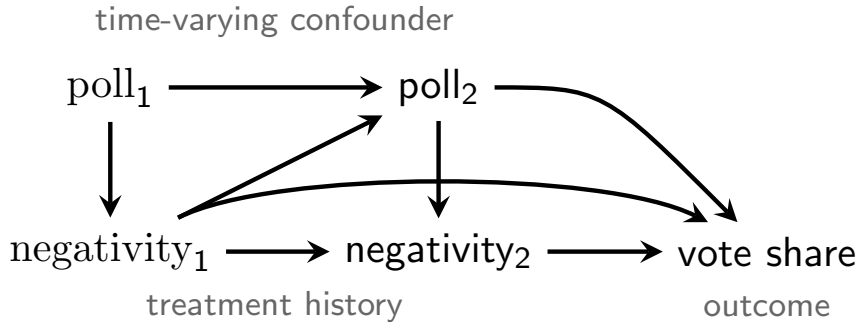

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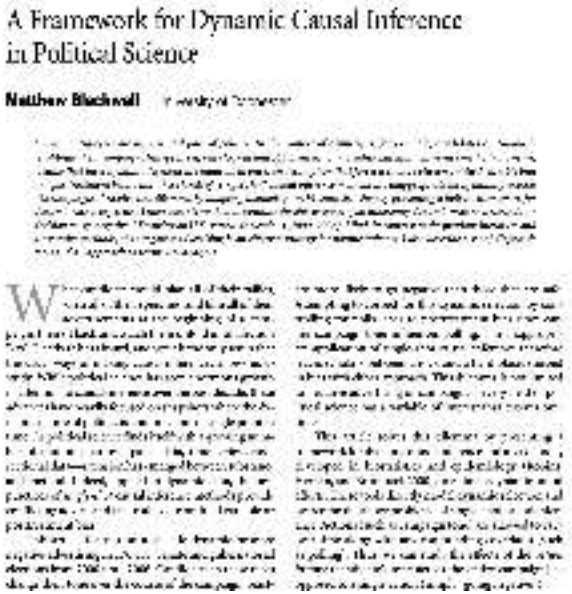
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AJPS

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INTRODUCTION


When Should We Use Unit Fixed Effects Regression Models for Causal Inference with Longitudinal Data?

Kosuke Imai

We are going to focus on addressing unobserved time-invariant confounders using the last paper.

Next several slides are based on slides graciously provided by In Song Kim and Kosuke Imai.

Directed Acyclic Graph (DAG)

Non-parametric identification assumptions for fixed effects:

$$Y_{it} = g(X_{it}, \mathbf{U}_i, \epsilon_{it}) \quad \text{and} \quad \epsilon_{it} \perp\!\!\!\perp \{\mathbf{X}_i, \mathbf{U}_i\}$$

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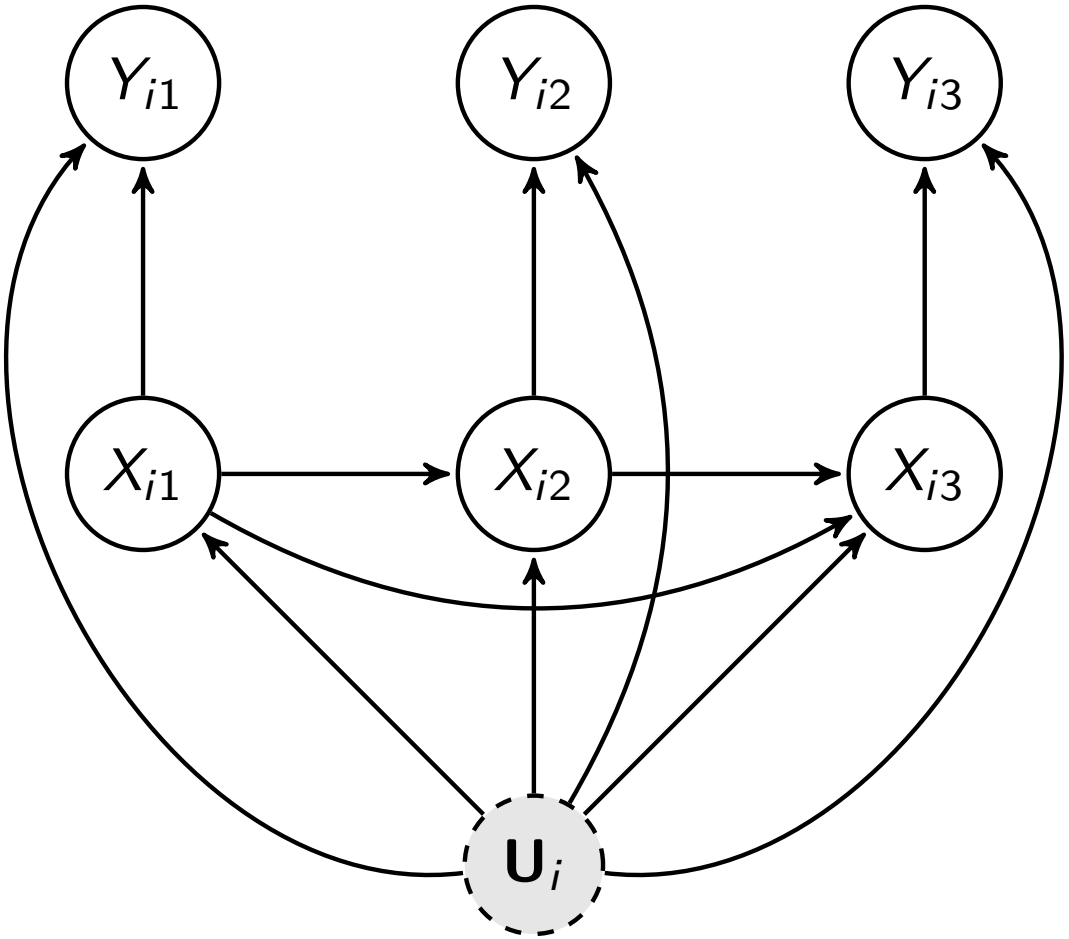

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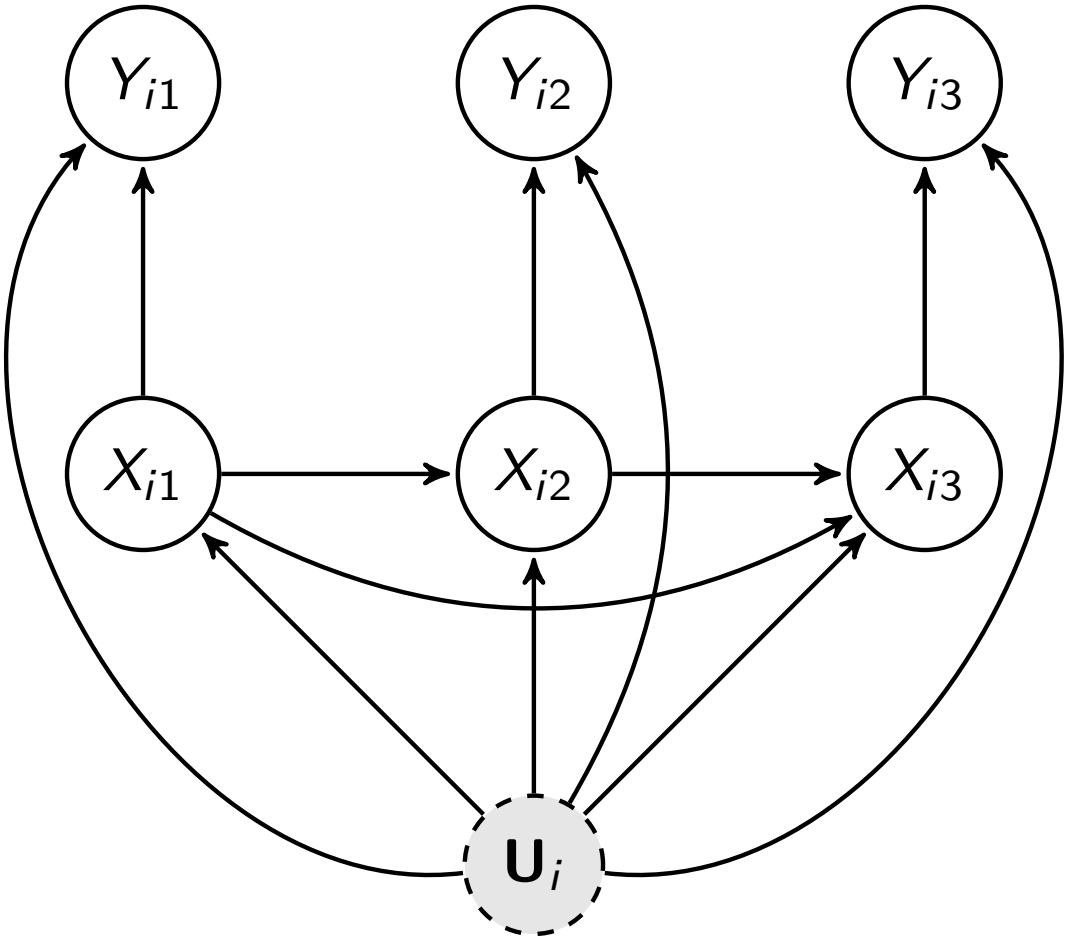

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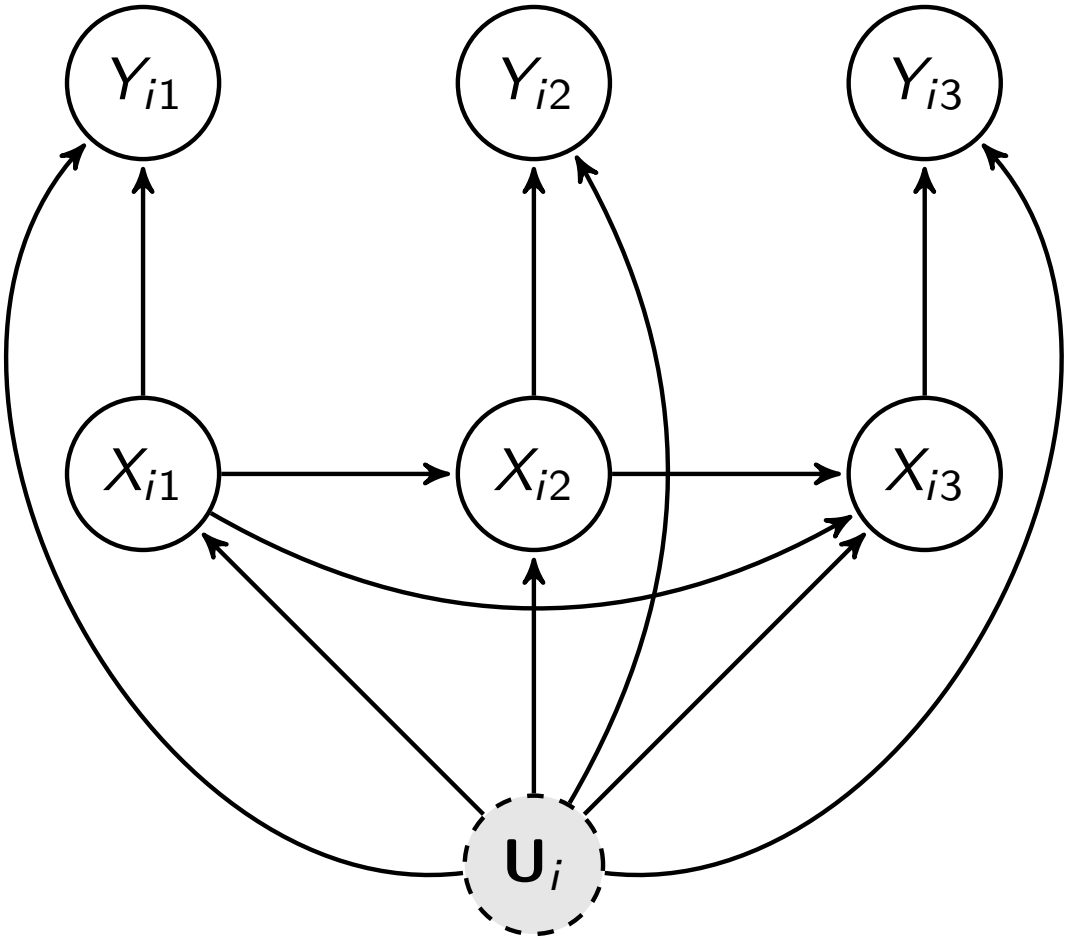

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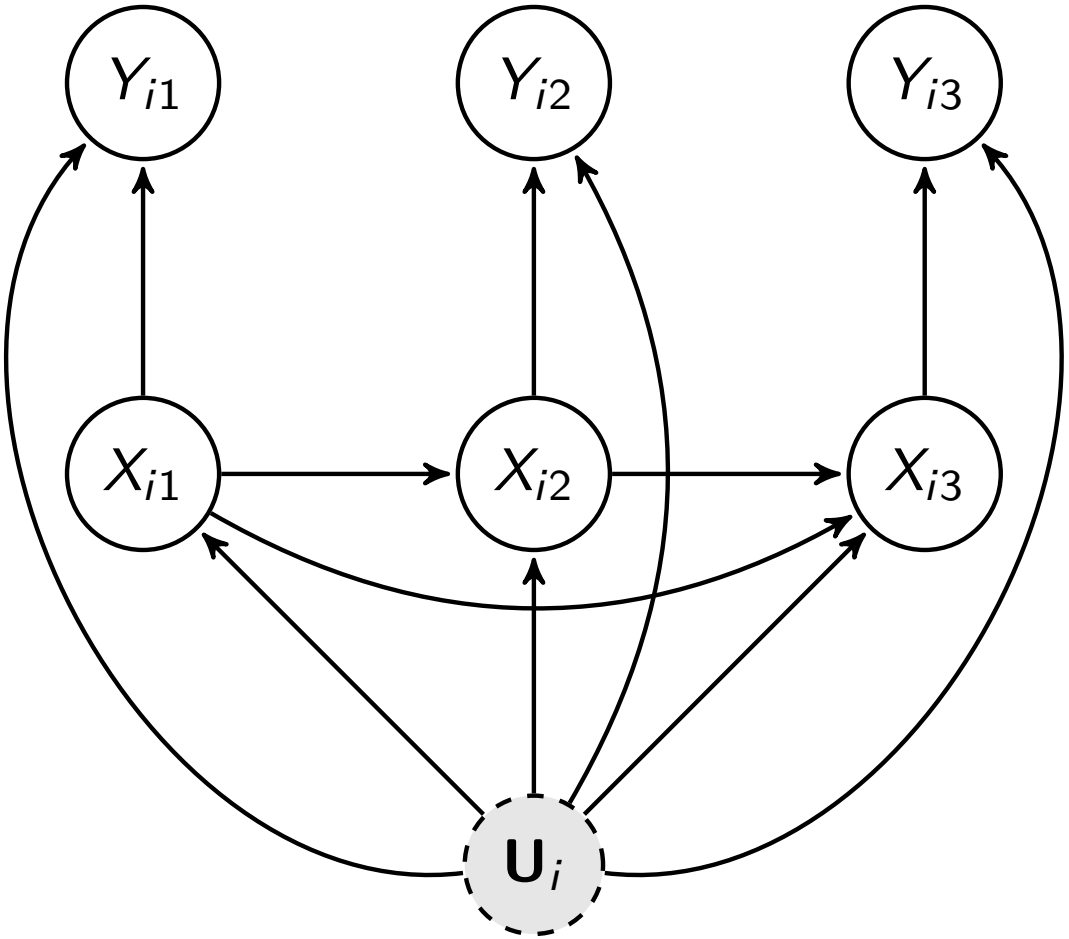

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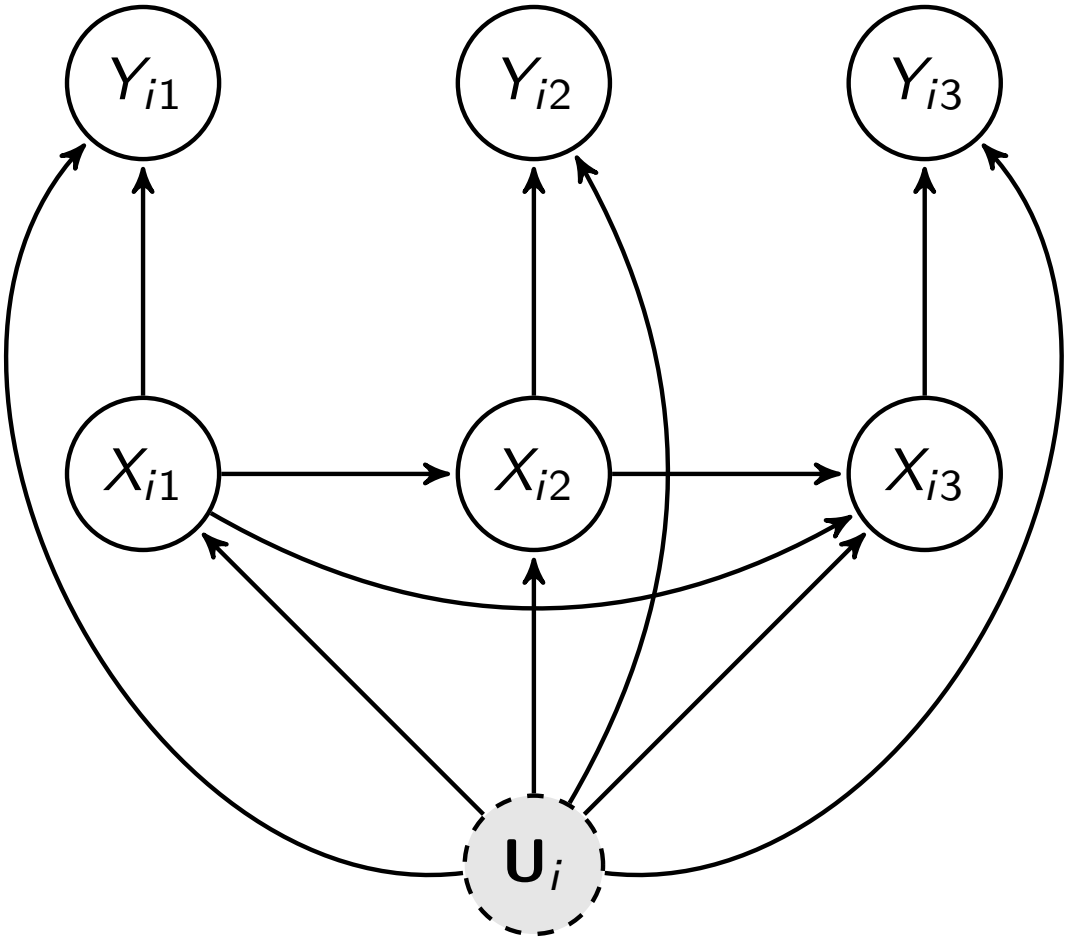

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4. Past treatments do not directly affect current outcome

the result implies that the counterfactual outcome for a treated observation in a given time period is estimated using the observed outcomes of different time periods of the same unit. Since such a comparison is valid only when no causal dynamics exist, this finding underscores the important limitation of linear regression models with unit fixed effects.

- Imai and Kim (2019)

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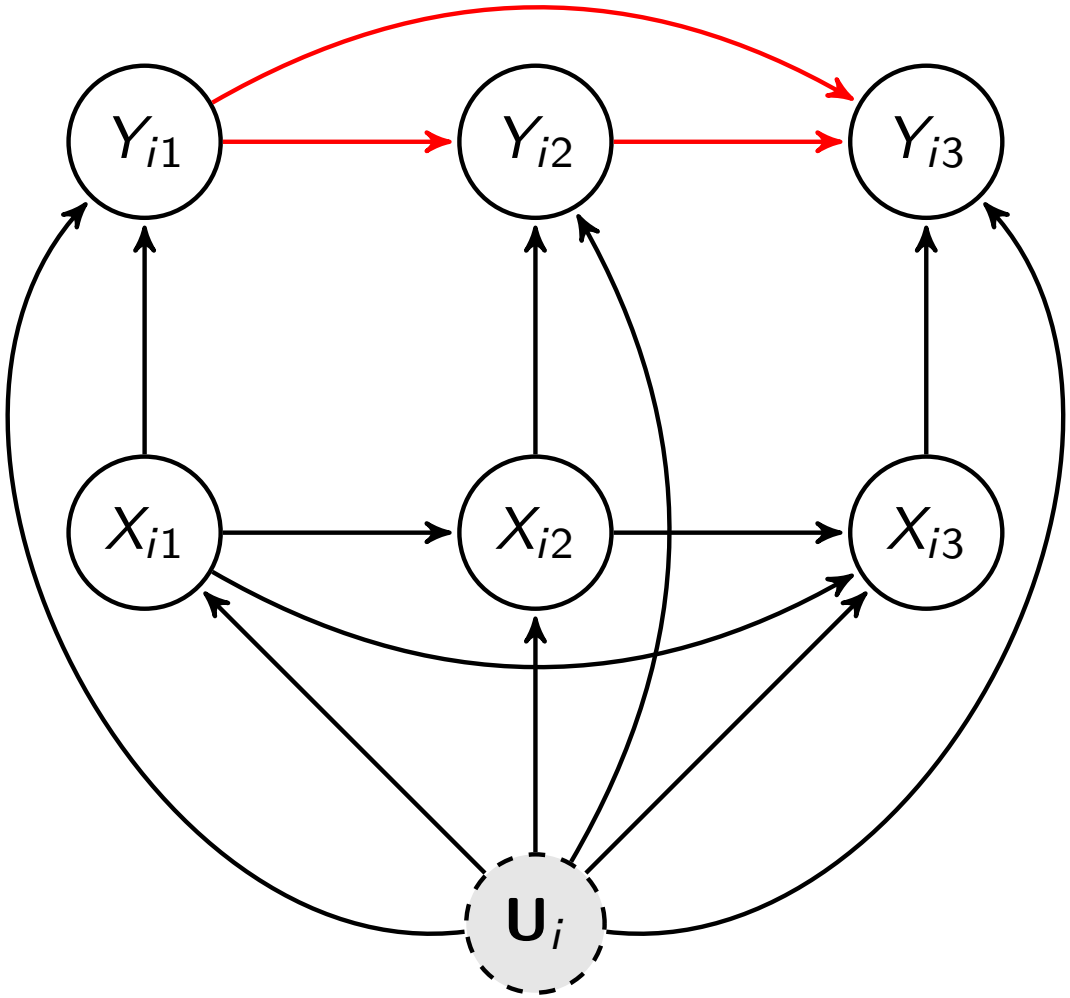
- Now let's consider each assumption in turn.

Past Outcomes Don't Directly Affect Current Outcome (A2)



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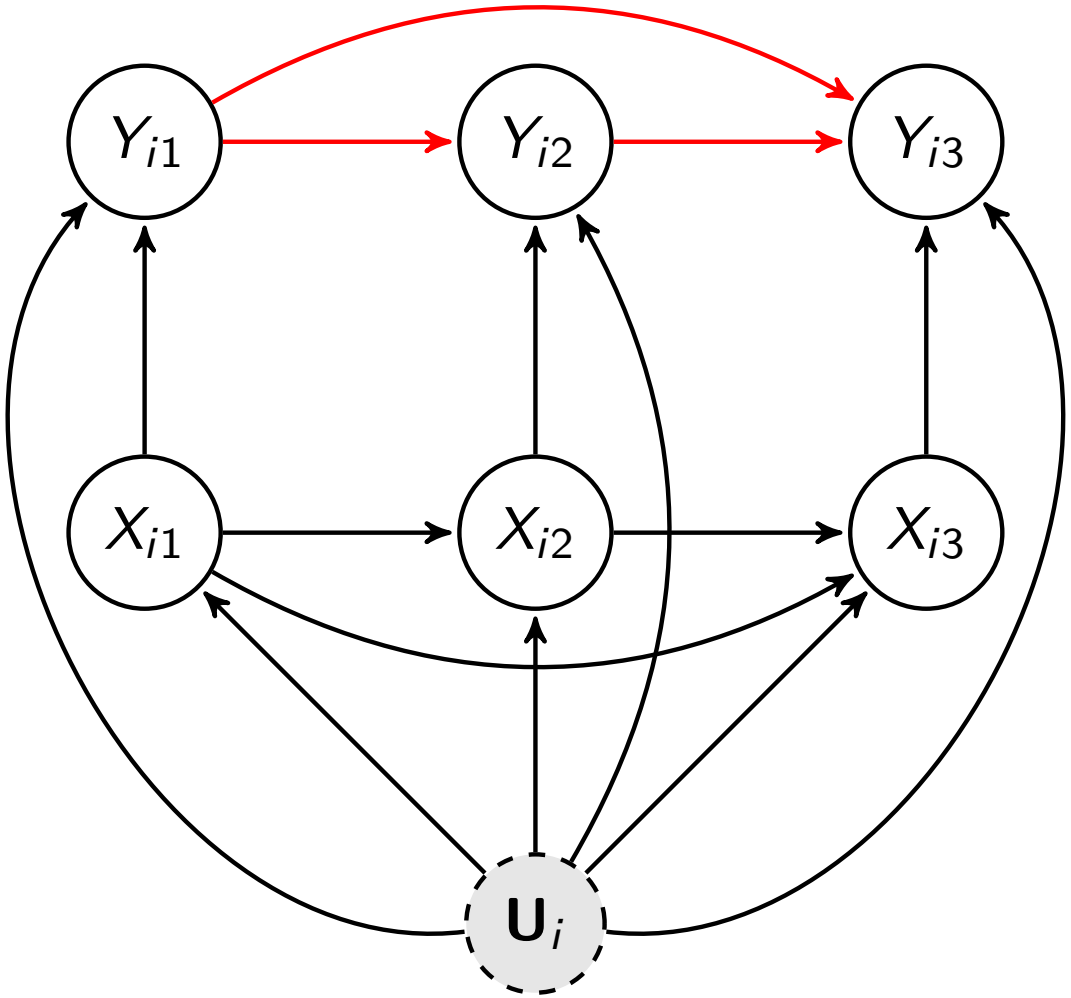

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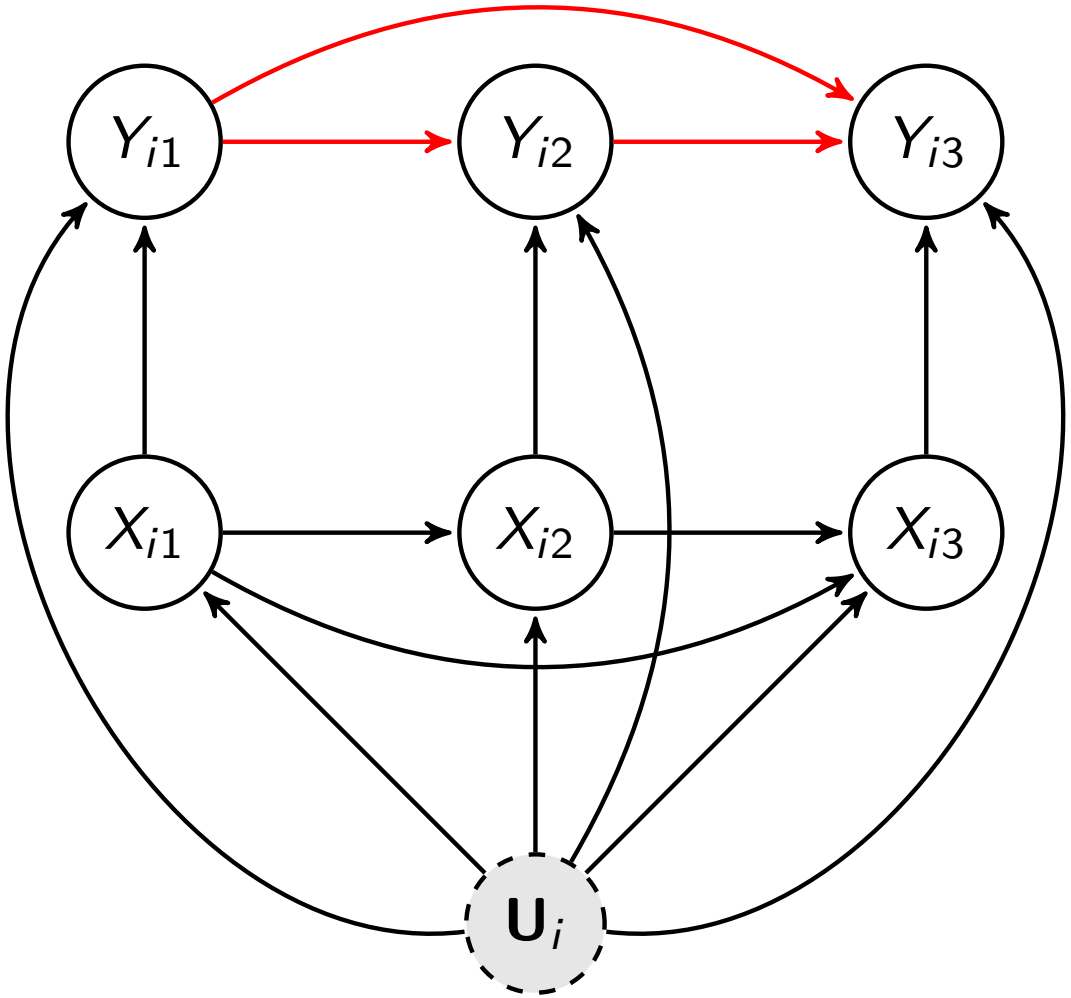

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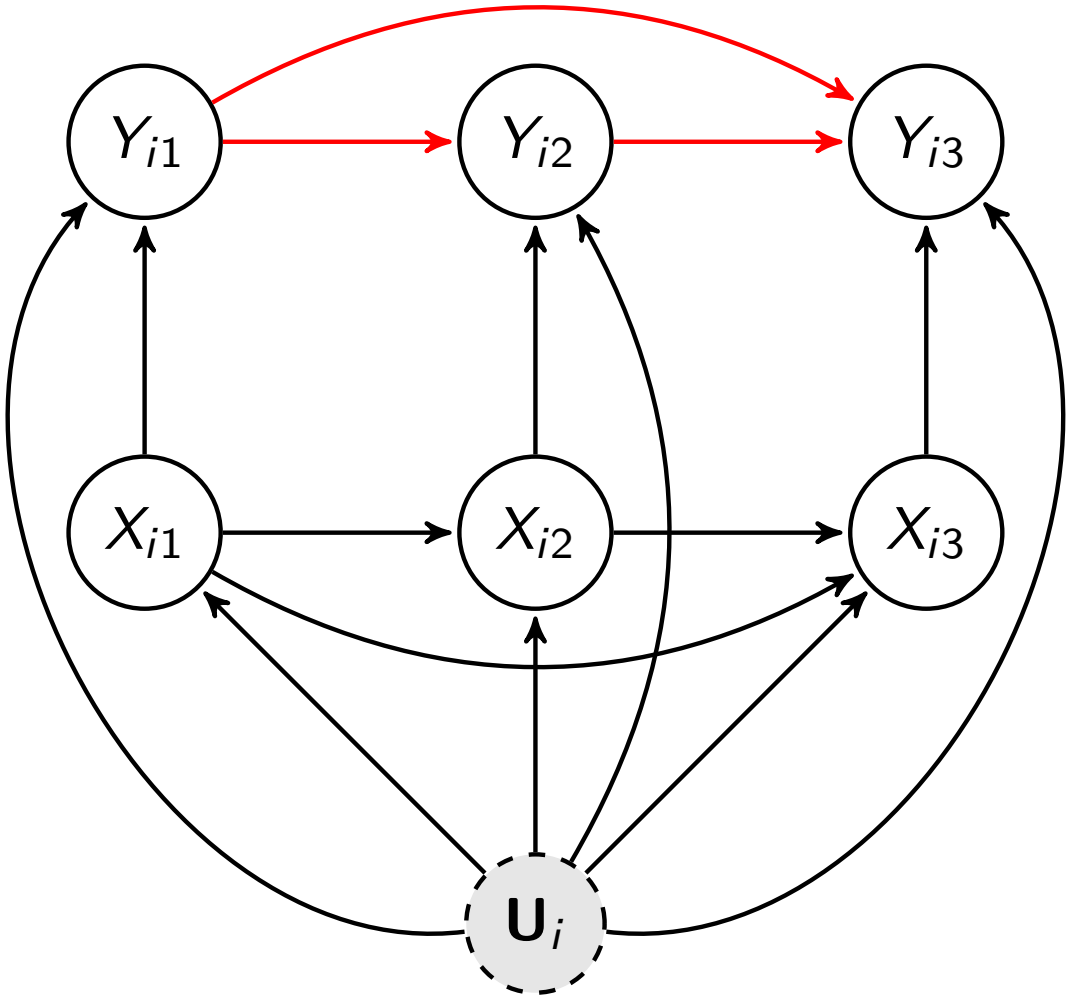

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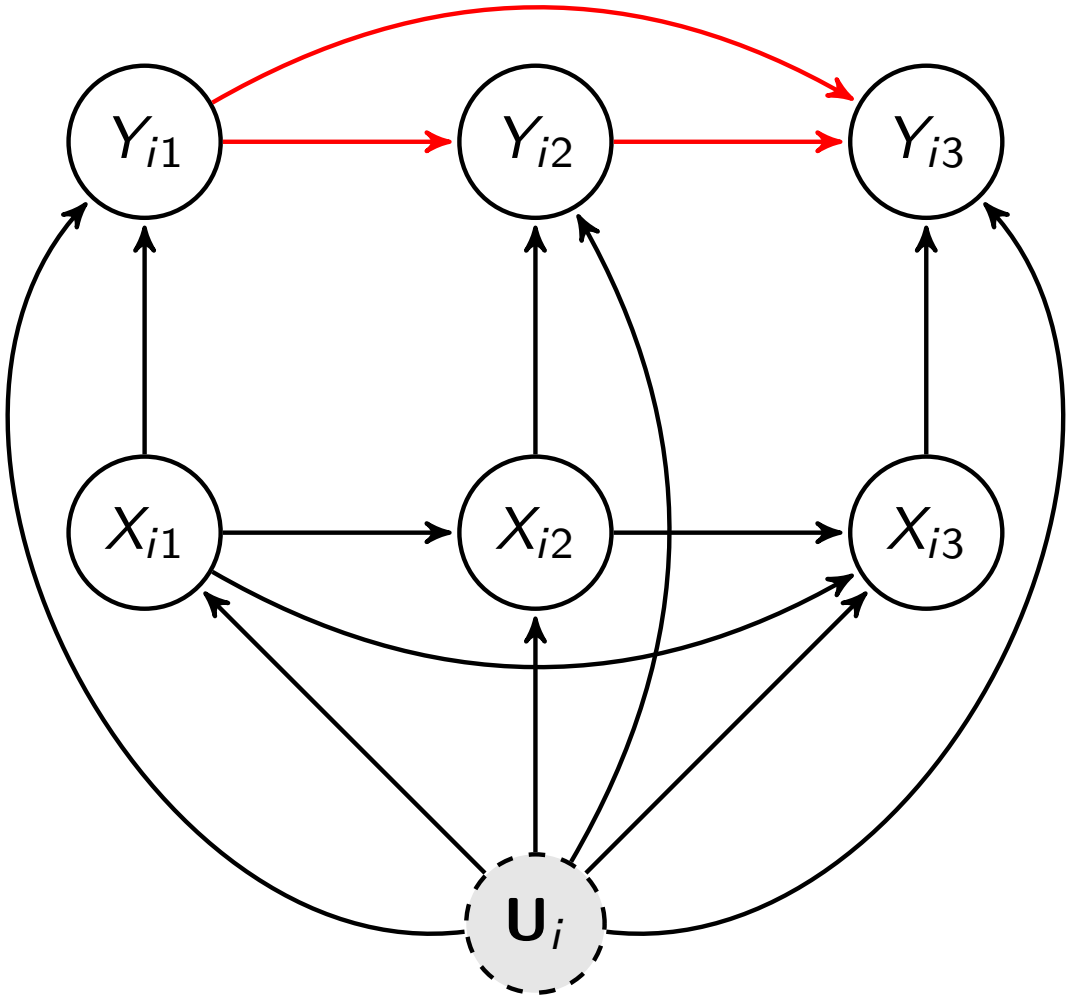

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- Conclusion: The assumption can be relaxed

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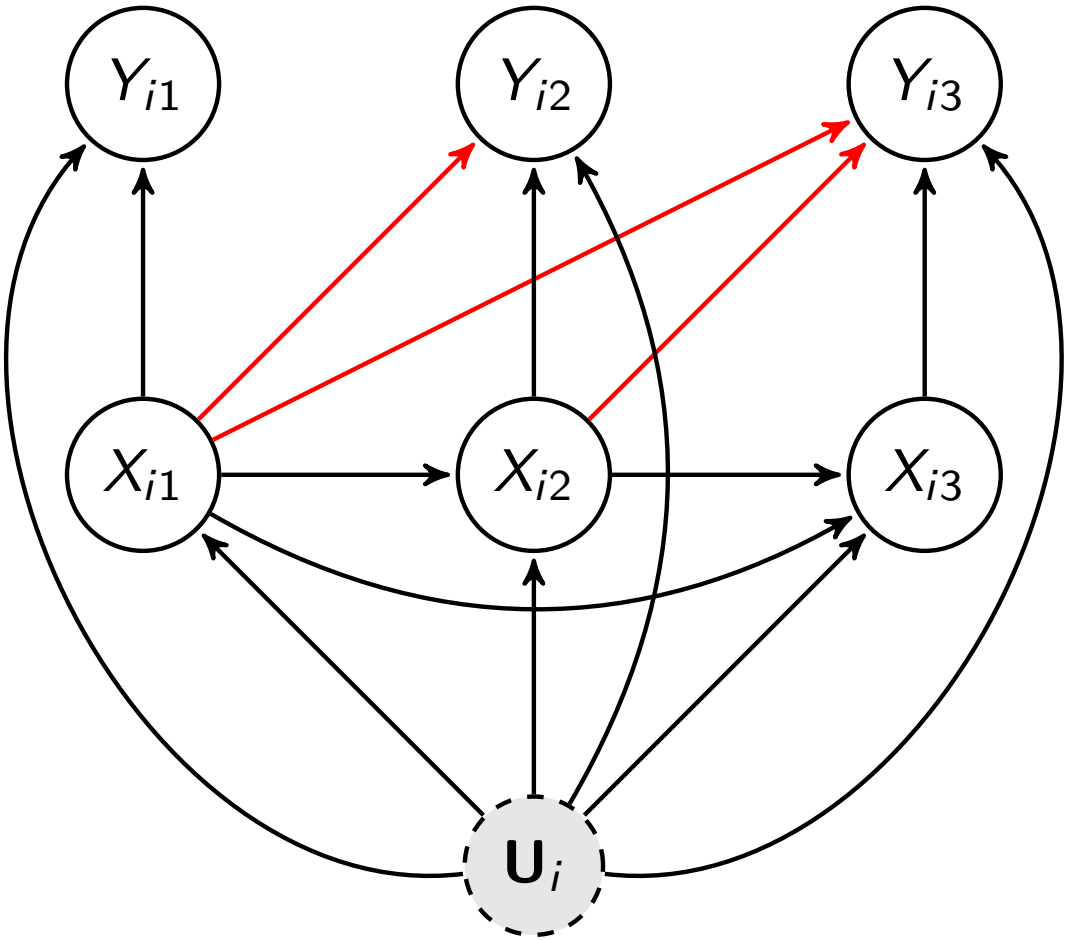

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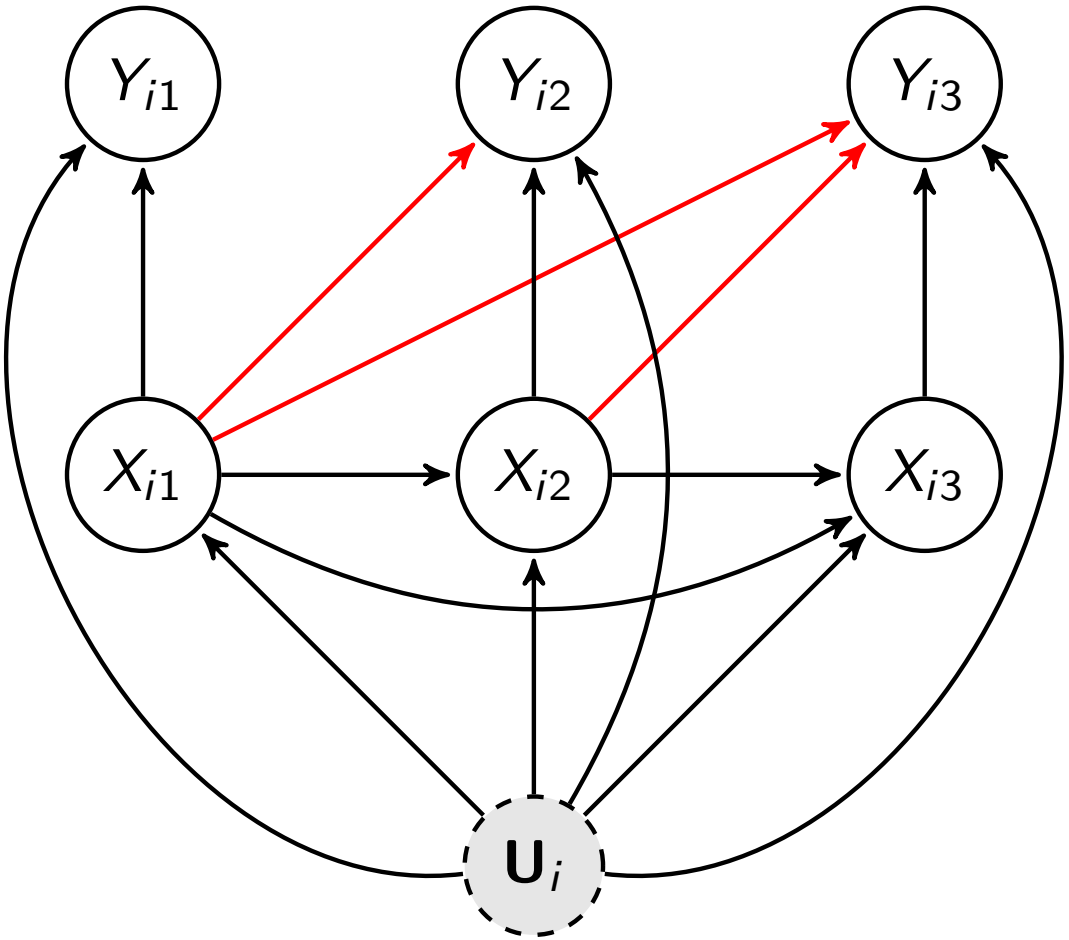

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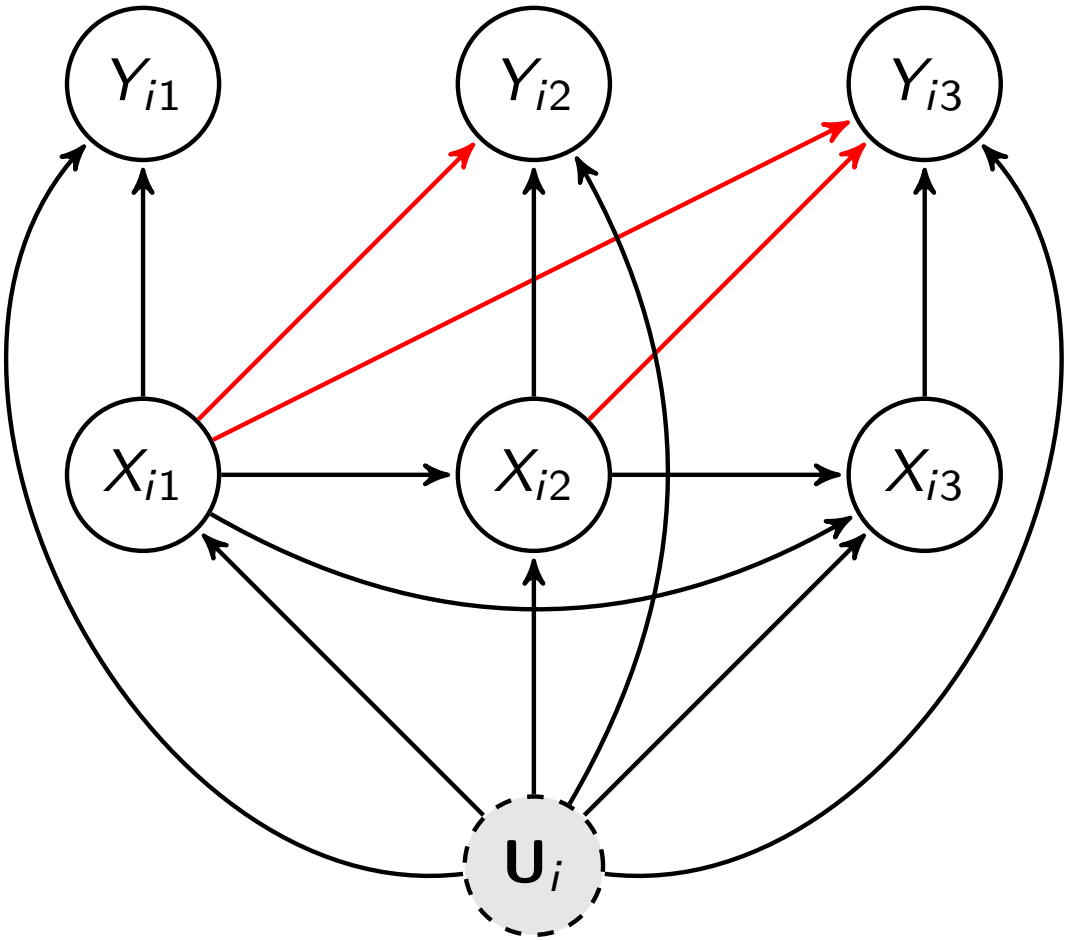

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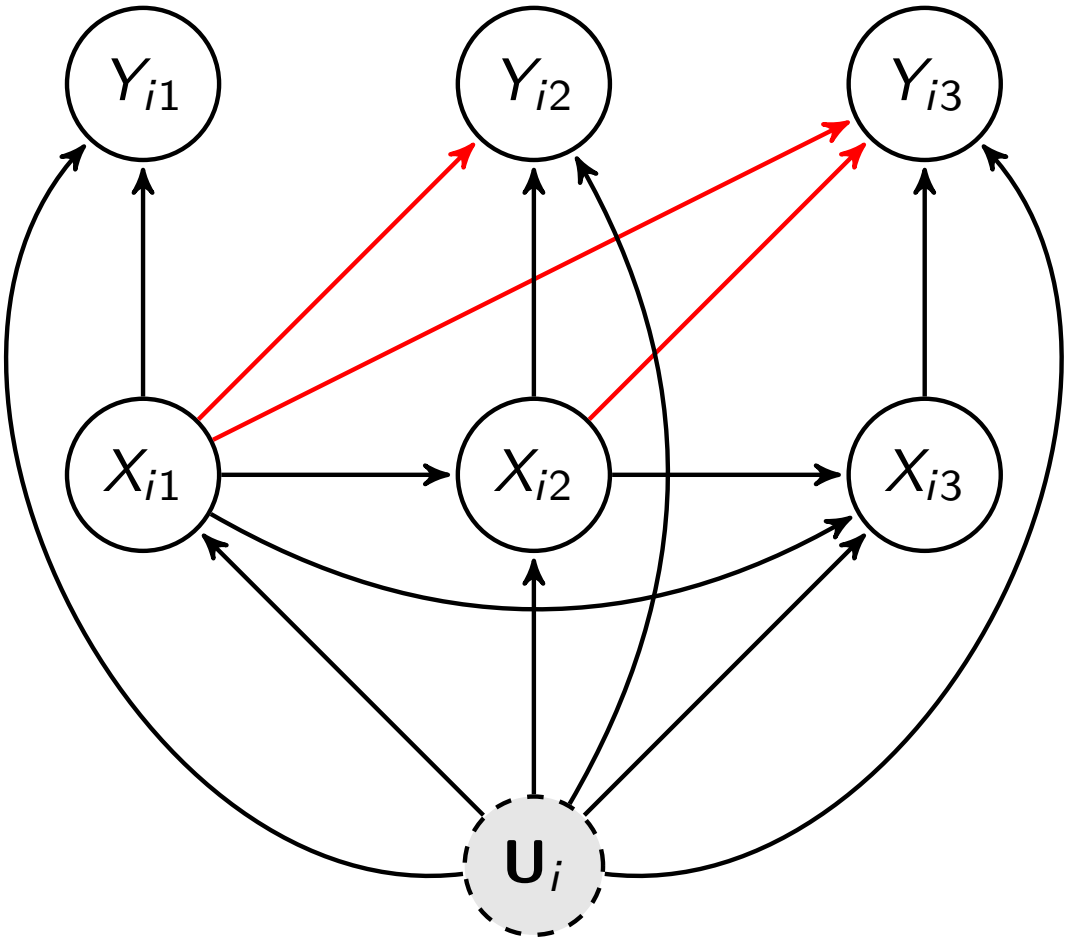

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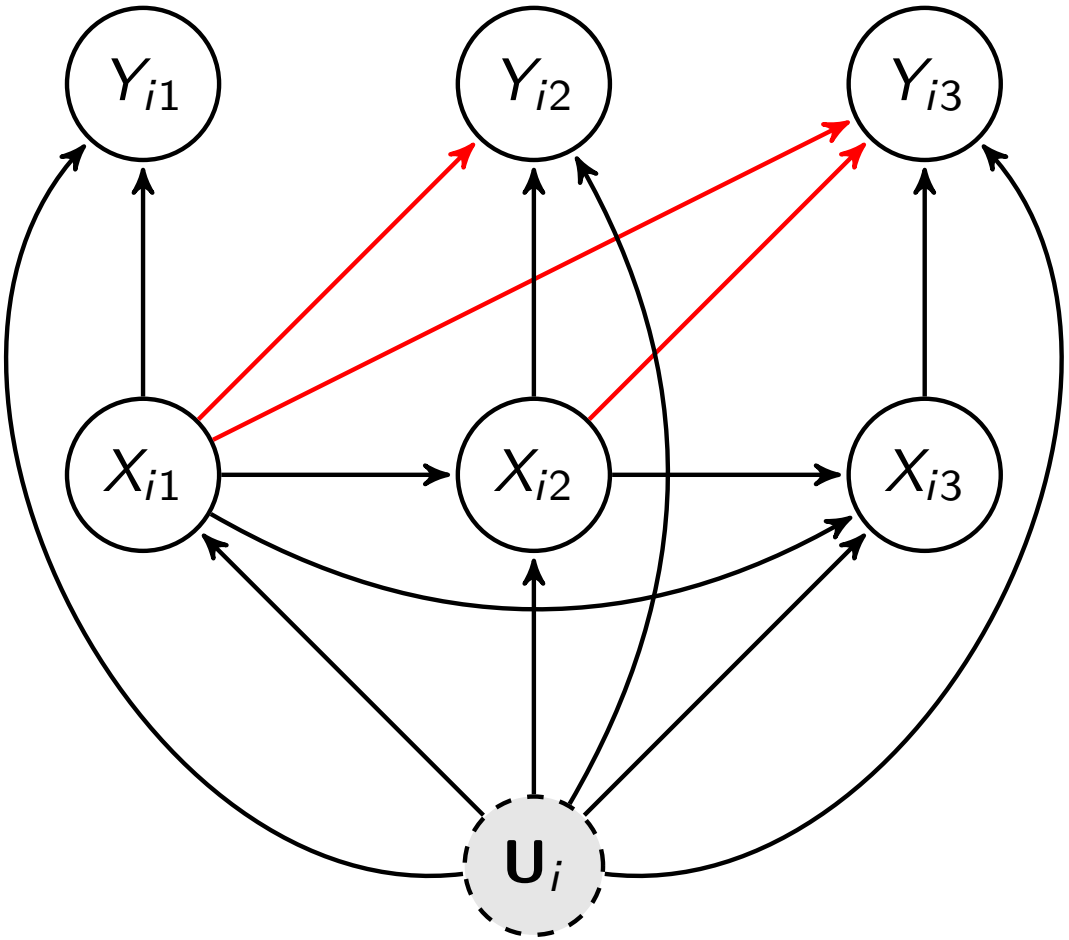

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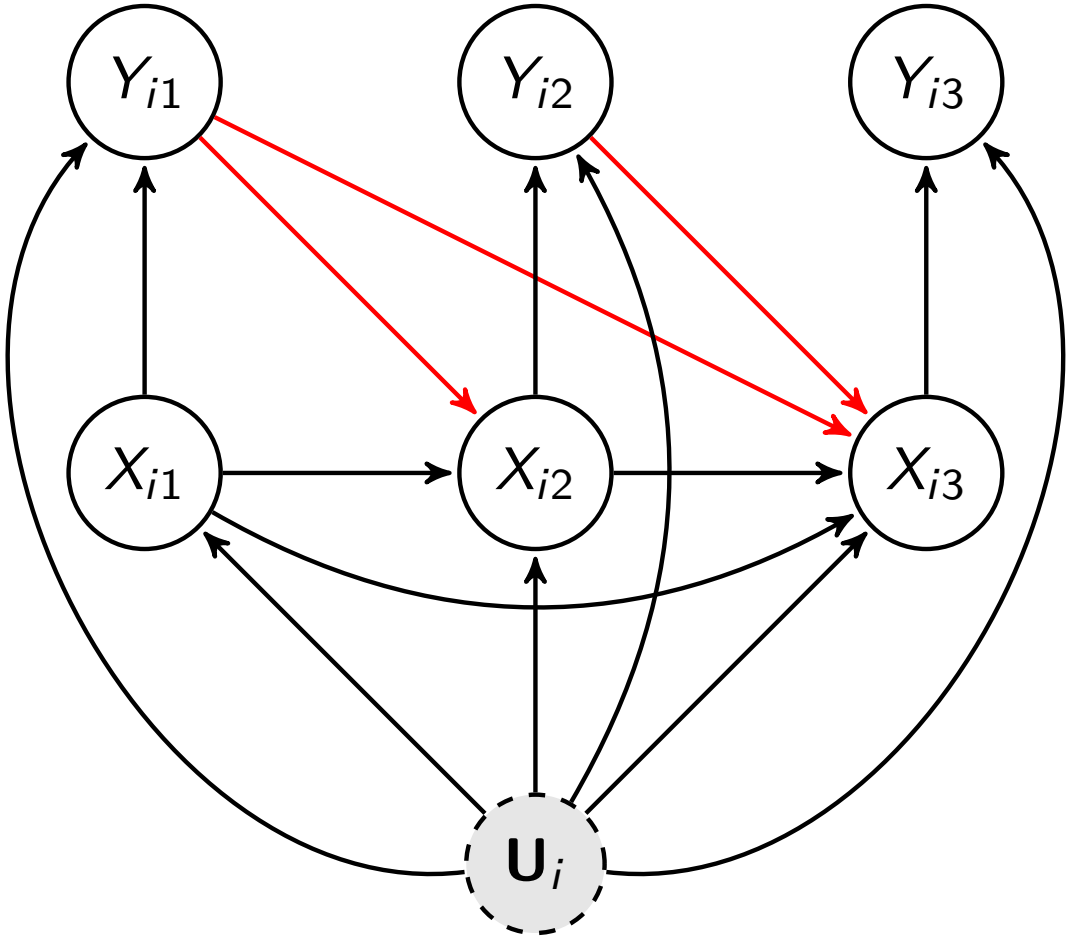

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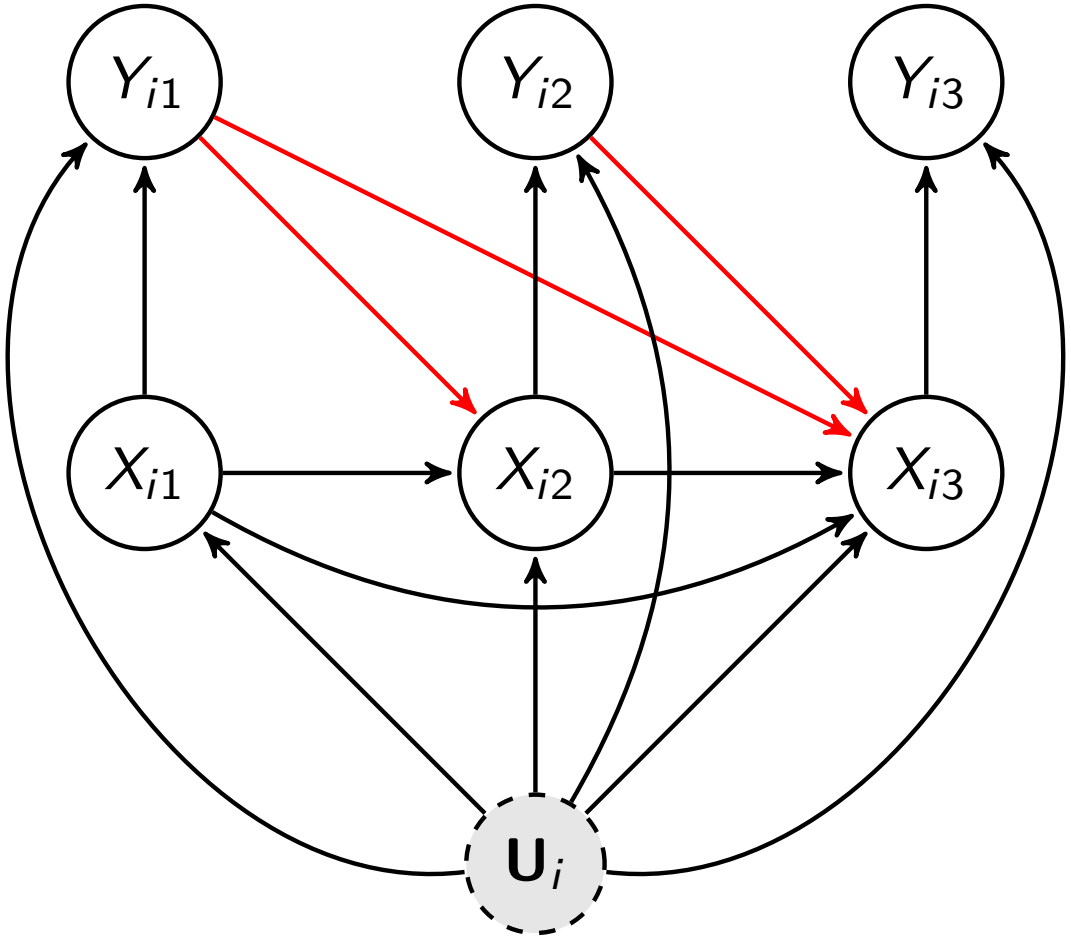

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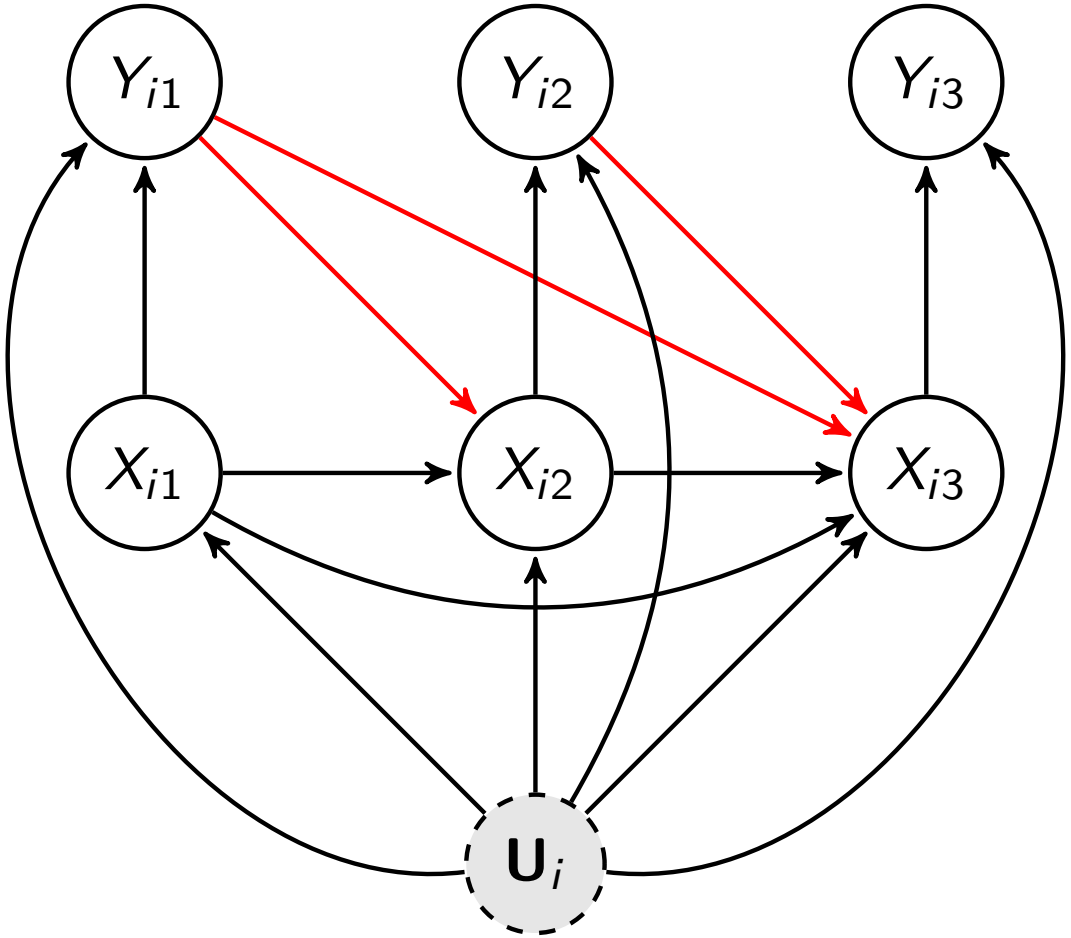

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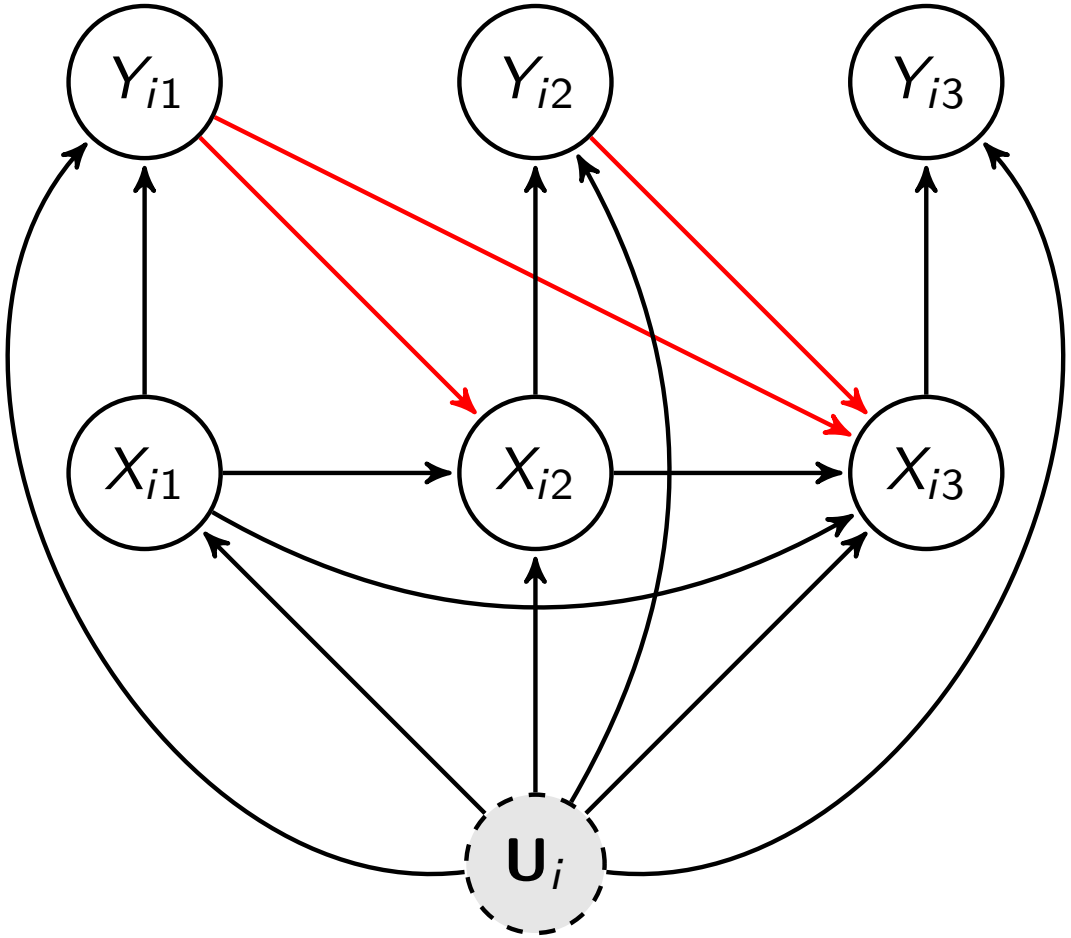

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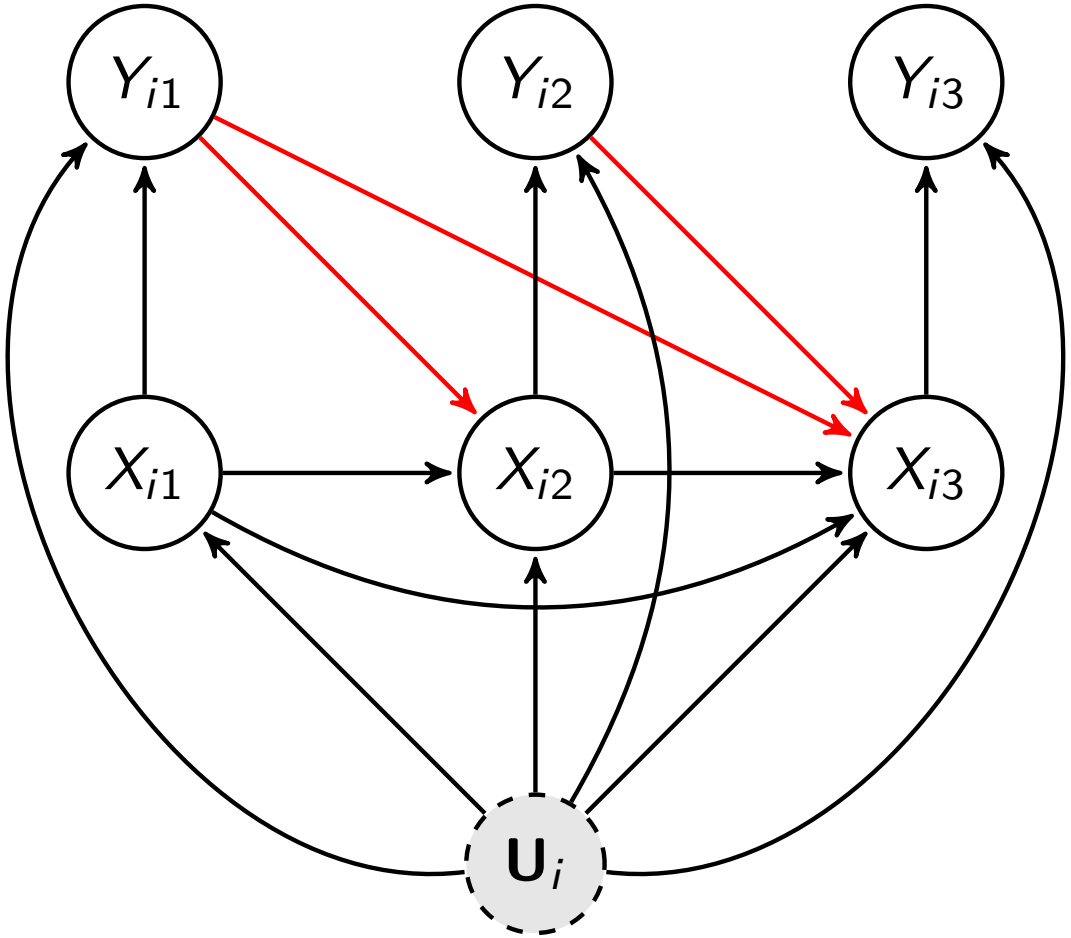

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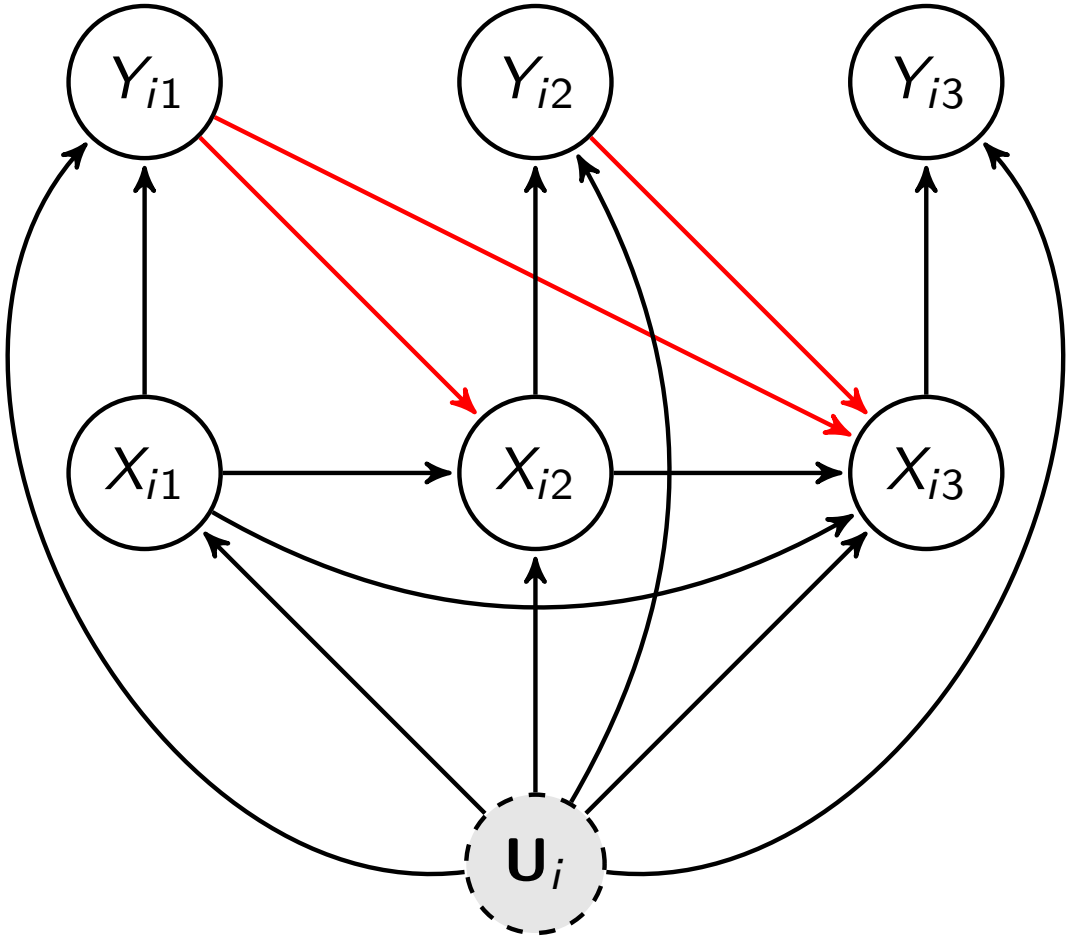

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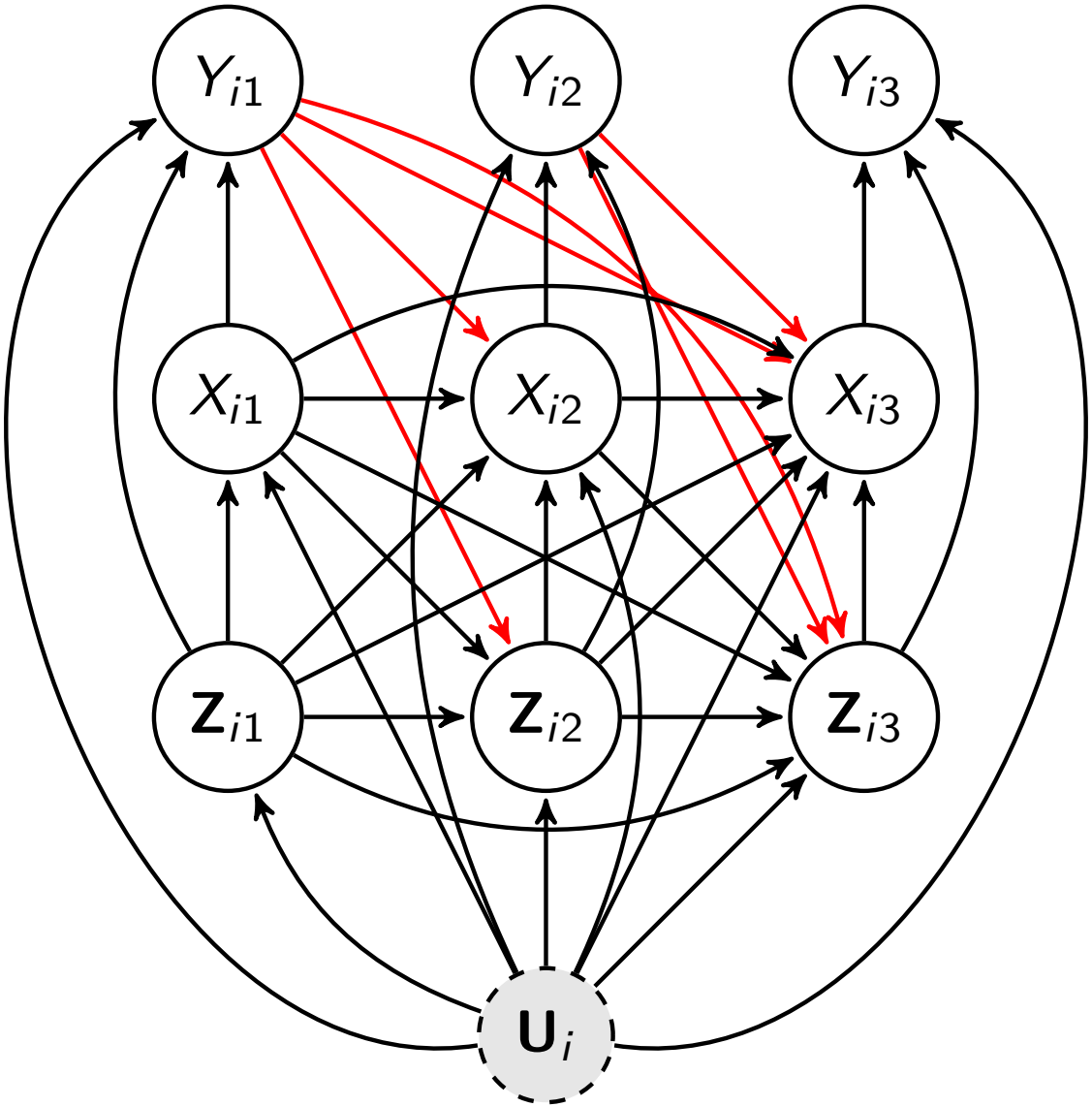

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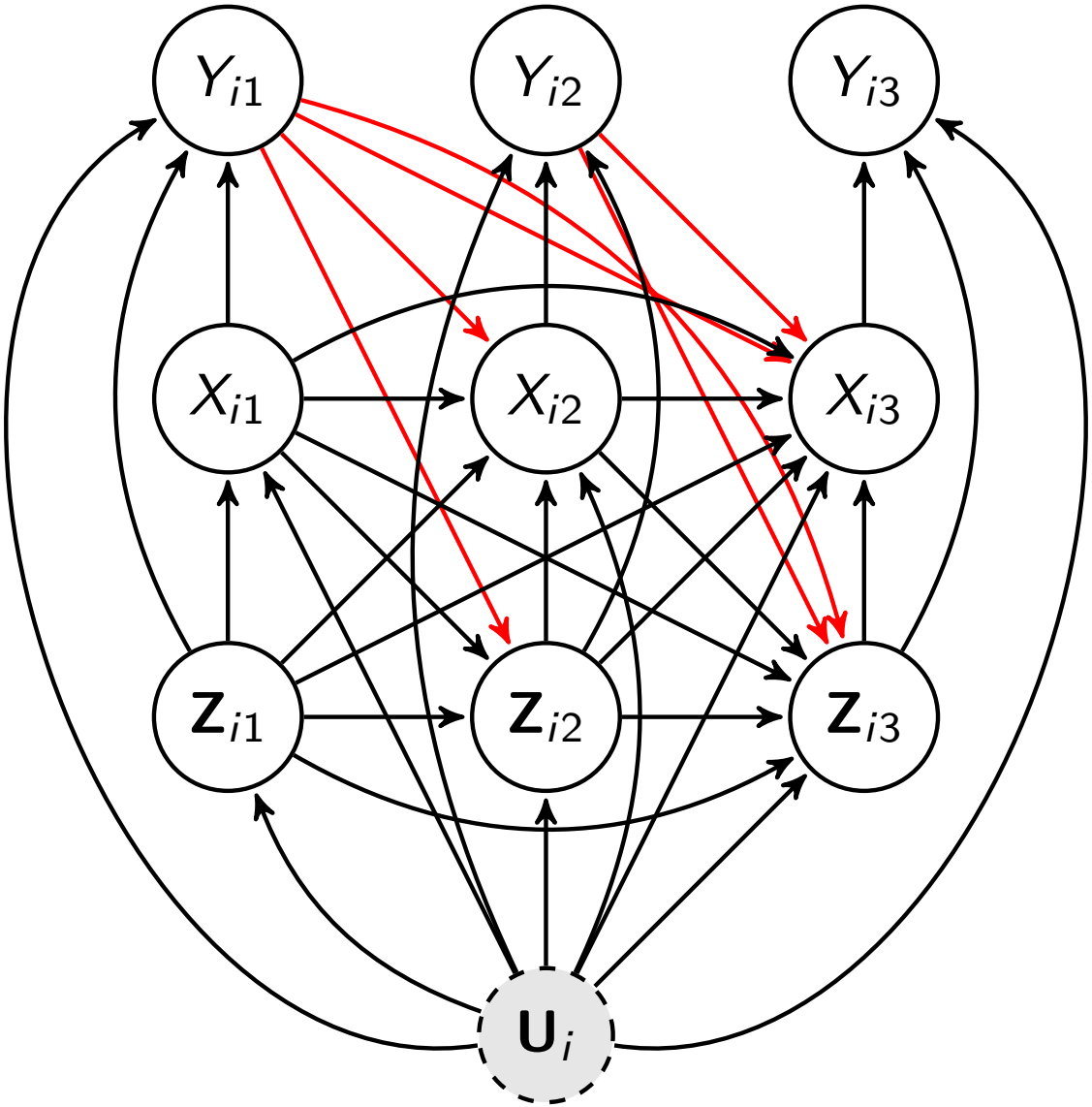

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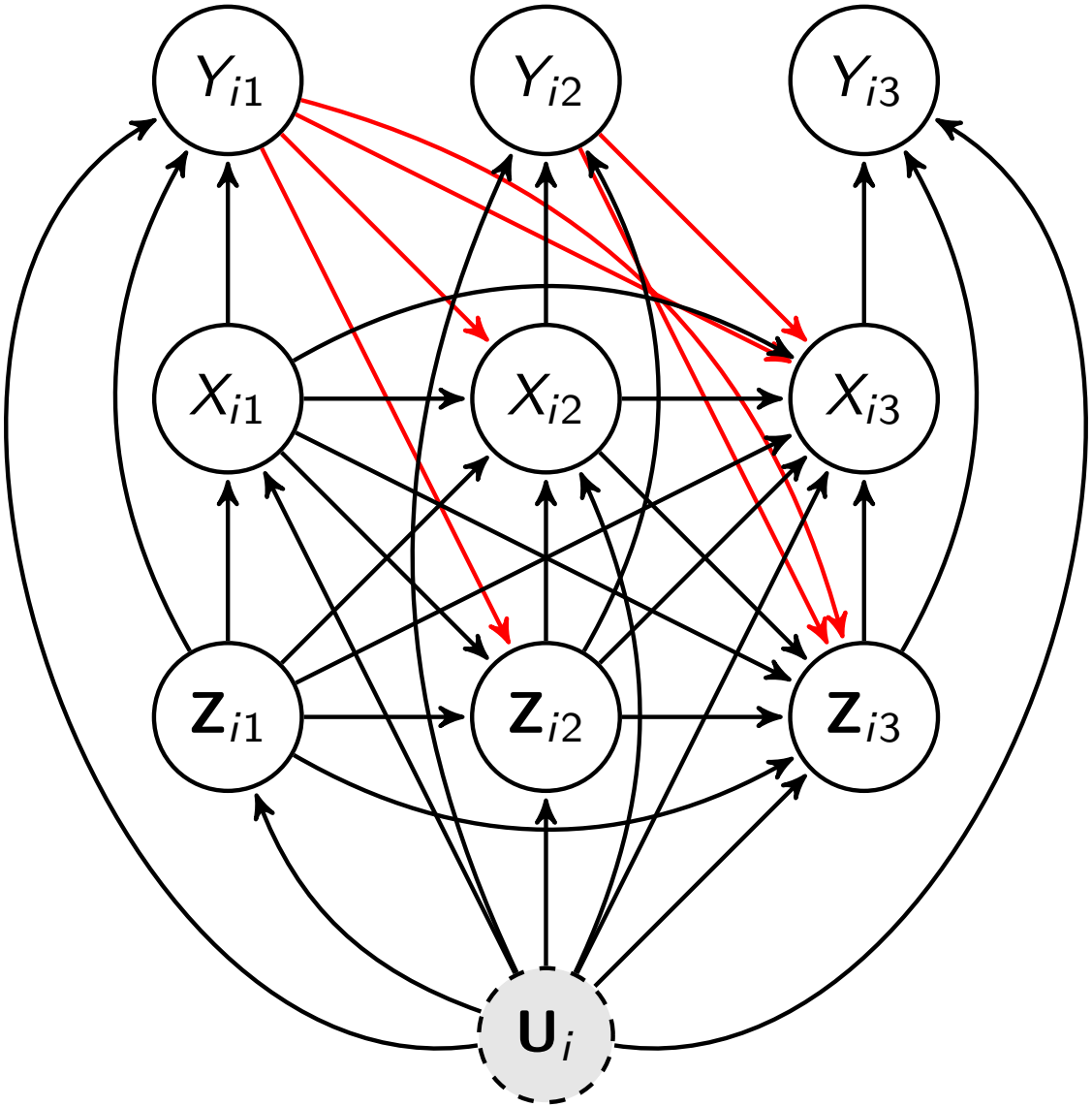

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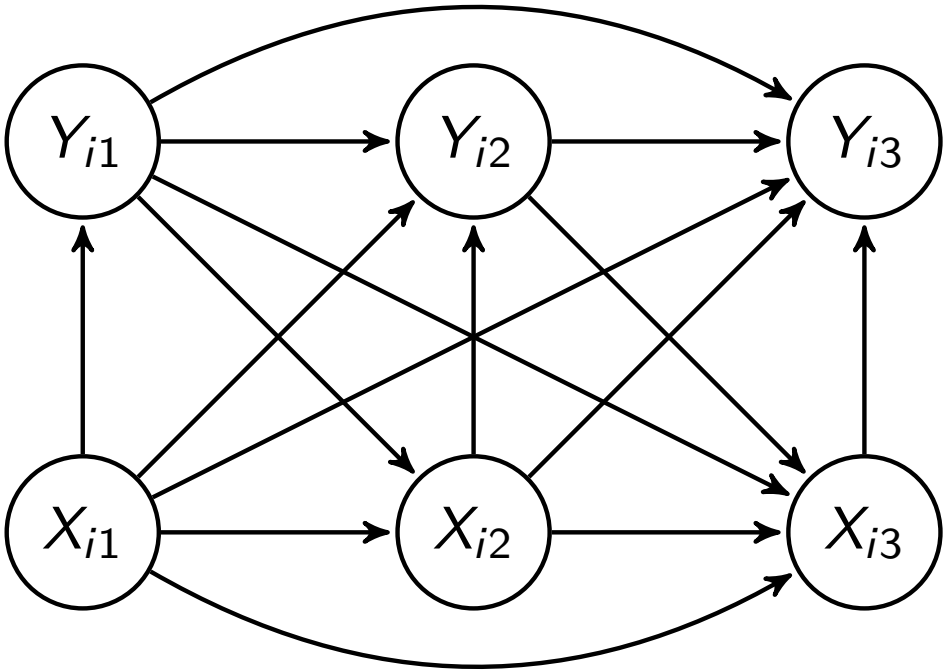

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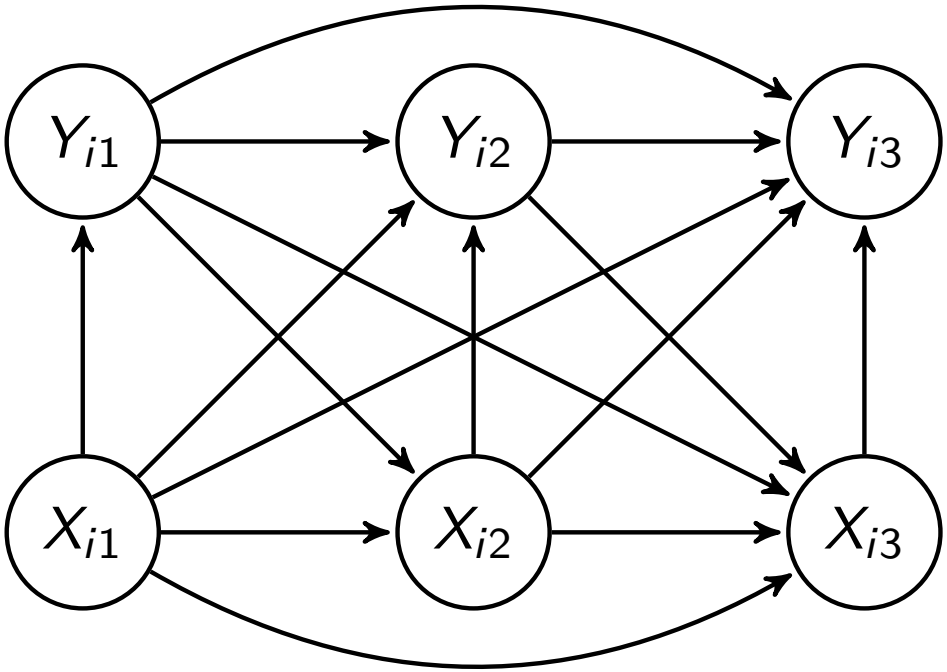

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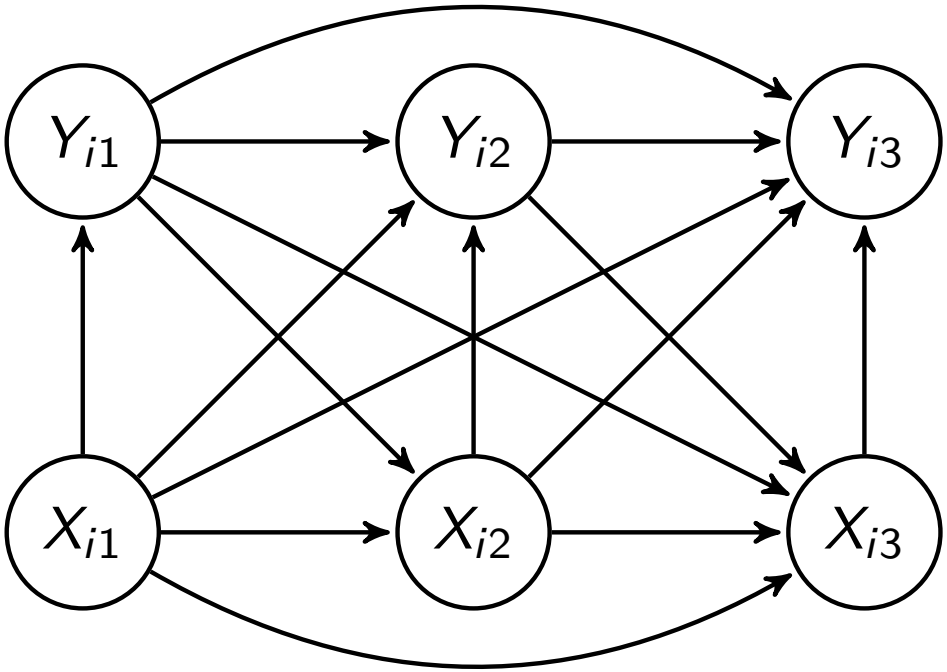

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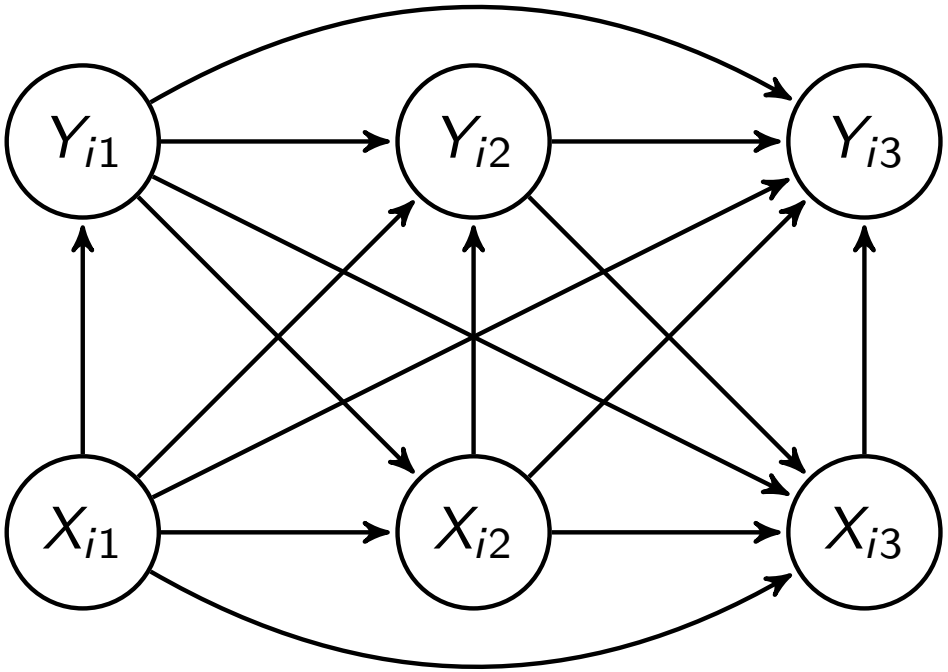

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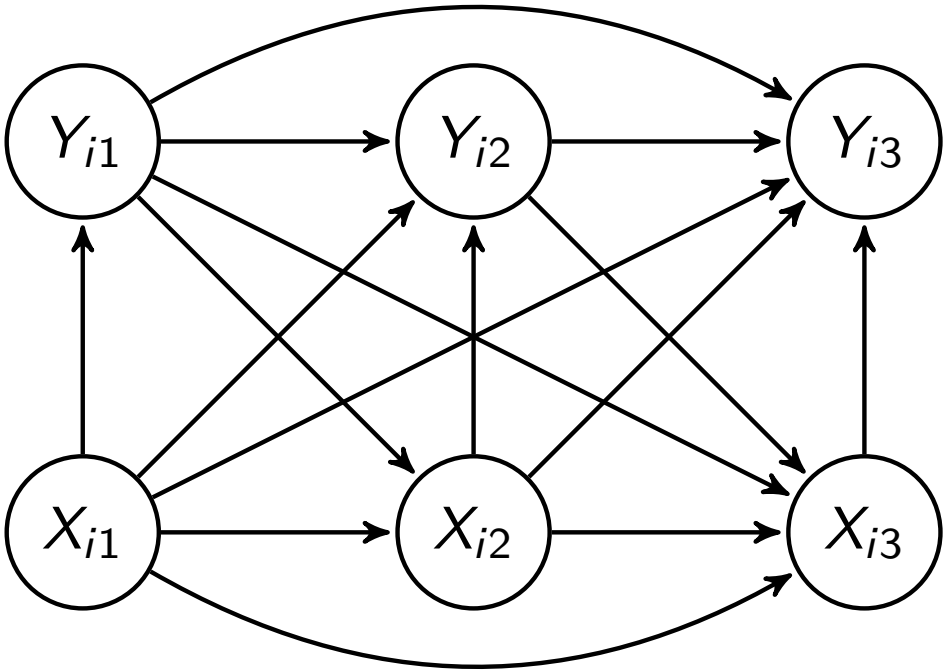

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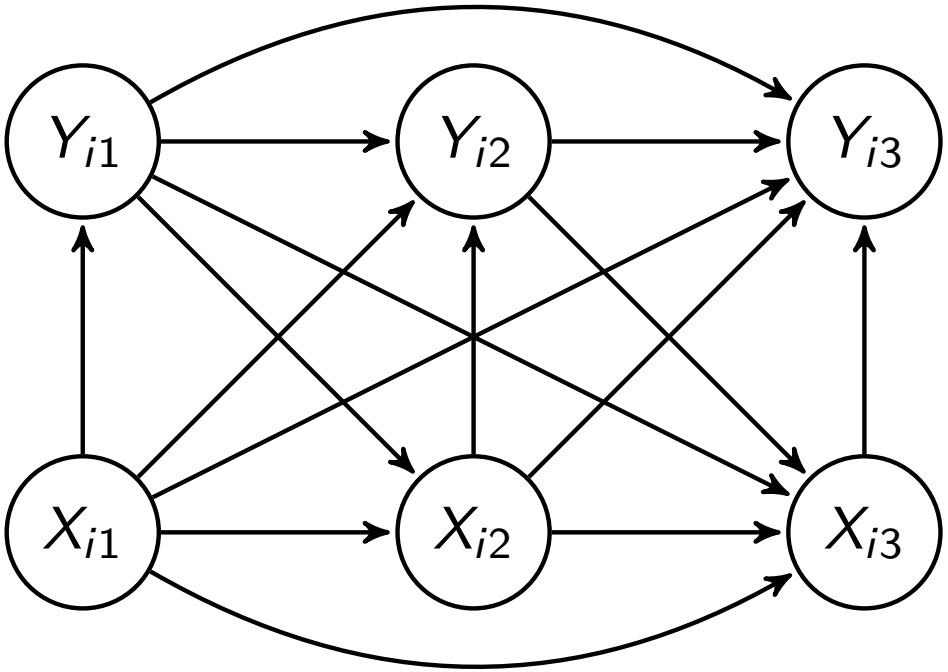

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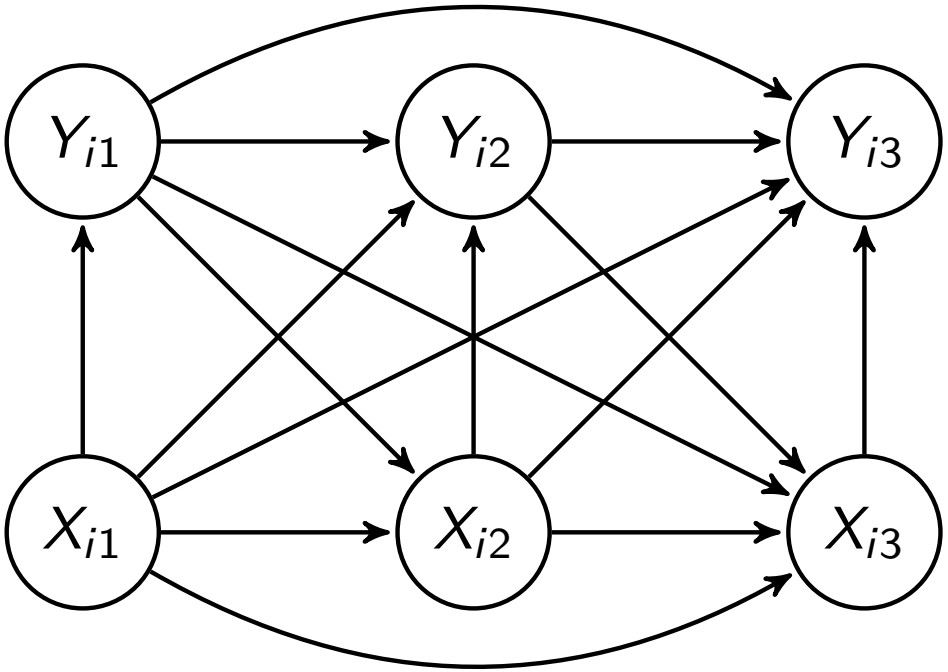

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- Trade-off $\rightsquigarrow$ no free lunch

Conclusions and Nonparametric Estimation

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Summary Table (Imai and Kim 2019)

TABLE 1 Identification Assumptions of Various Estimators

	Linearity	Time-Invariant Unobservables	Past Outcomes Affect Current Treatment	Past Treatments Affect Current Outcome
$Y_{it} = \alpha_i + \beta X_{it} + \epsilon_{it}$	Yes	Allowed	Not allowed	Not allowed
$Y_{it} = \alpha_i + \beta X_{it} + \rho Y_{i,t-1} + \epsilon_{it}$	Yes	Allowed	Allowed	Not allowed
$Y_{it} = \alpha_i + \beta_1 X_{it} + \beta_2 X_{i,t-1} + \epsilon_{it}$	Yes	Allowed	Not allowed	Allowed
$Y_{it} = \alpha_i + \beta_1 X_{it} + \beta_2 X_{i,t-1} + \rho Y_{i,t-1} + \epsilon_{it}$	Yes	Allowed	Partially allowed	Partially allowed
Marginal structural models	No	Not allowed	Allowed	Allowed

What to read next?

- Morgan and Winship Chapter 11 Repeated Observations and the Estimation of Causal Effects
- Imai and Kim (2019) "When Should We Use Unit Fixed Effects Regression Models for Causal Inference with Longitudinal Data?" *American Journal of Political Science*, `http://dx.doi.org/10.1111/ajps.12417`
- Liu, Wang and Xu "A Practical Guide to Counterfactual Estimators for Causal Inference with Time-Series Cross-Sectional Data" *American Journal of Political Science*.

We Covered

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- Regression based estimation of causal effects under measured confounding.
- Imputation estimators which generalize to broader class of machine learning estimators for the conditional expectation function.
- Matching as nonparametric preprocessing
- Fixed Effects
- Sensitivity Analysis

Next Time: What to Do About Unmeasured Confounding (Week 11)