

Week 9: Frameworks for Causal Inference

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¹These slides are heavily influenced by Matt Blackwell, Justin Grimmer, Jens Hainmueller, Erin Hartman and Kosuke Imai

Where We've Been and Where We're Going...

- Last Week
 - ▶ regression diagnostics
- This Week
 - ▶ core ideas in causal inference
 - ▶ potential outcomes
 - ▶ nonparametric structural equation models and DAGs
 - ▶ experiments
- Next Week
 - ▶ selection on observables
- Long Run
 - ▶ probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causal inference

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- Reason 2: When you **understand** how the methods work you can make choices based on **substantive knowledge** rather than what software spits out at you.

**DO ALL THE
MATH**



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Regression as a Tool: A Review

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- This is because they are about **different groups of units** not about **the same unit under intervention**.

1 Core Ideas in Causal Inference

2 Potential Outcomes

- Framework
- Estimands
- Three Big Assumptions
- What Gets to Be a Cause

3 Nonparametric Structural Equation Models and Causal Directed Acyclic Graphs

4 The Experimental Ideal

1 Core Ideas in Causal Inference

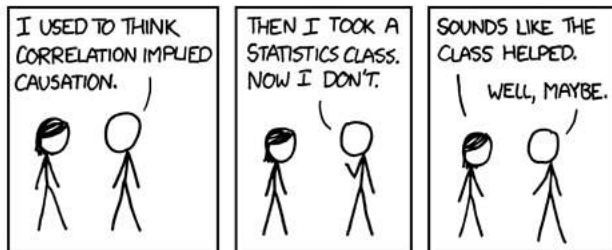
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Causation



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- By convention we often call the counterfactual levels we care about **treated** and **control** and we start with binary **treatment variables** (continuous things get much more complicated!).

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We will read Lundberg, Johnson, and Stewart 2021 where theoretical and empirical estimand terminology comes from.

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- Identification depends on **assumptions** not statistical models.
- As we will see this is **not** a conversation about estimation: in other words, if someone answers “regression” they have made a **category error**

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- Even when identification is possible, estimation may impose additional assumptions (i.e. that the linear approximation to the CEF is good enough)
- **Law of Decreasing Credibility (Manski)**: The credibility of inference decreases with the strength of the assumptions maintained

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- It is a way of thinking about counterfactuals and the assumptions required to make statements about them.
- We will first step through the **framework**, then discuss **estimands**, **three big assumptions** and finally what **counts as a cause**.

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T_i : Dichotomous Treatment assignment for unit i (multi-valued treatments are possible too—just more potential outcomes for each unit)

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Illustrated potential outcomes here and later courtesy of Erin Hartman

What is random in the potential outcomes framework?

Note that potential outcomes are thought of as **fixed**, and that they, and the difference between them, can vary by arbitrary amounts for each unit i . There is some true distribution of potential outcomes across the population.

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Treatment assignment is the source of randomness

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No methodology allows us to simultaneously observe both potential outcomes, $Y_i(1)$ and $Y_i(0)$, making τ_i unobservable—and unidentifiable without additional assumptions (**Fundamental Problem of Causal Inference** Holland (1986))

Causal Inference is a Missing Data Problem

Example: Aspirin's Impact on Headaches

Patient i		Pill T_i	Headache Status $Y_i(\text{red pill})$ $Y_i(\text{brown pill})$ Y_i	Age X_{1i}	Academic X_{2i}
1			 	25	Y
2			 	55	N
3			 	62	Y
4			 	80	N
5			 	32	Y
6			 	45	B
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Step 1: (Randomly) sample units

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Step 2: Randomly assign treatment

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Step 3: Measure revealed potential outcome

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- **Treatment effect heterogeneity:** Zero ATE doesn't mean zero effect for everyone

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Built in Assumptions

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- No simultaneity

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The notation implies three related assumptions:

- No simultaneity
- No interference
 - ▶ We are implicitly stating that the potential outcomes for that unit are unaffected by the treatment status of other units
 - ▶ If this is not true, the number of potential outcomes for unit i grows
 - ▶ Ex: in an experiment with 3 units, if the potential outcomes for unit i depend on the treatment assignment of units j and k , the potential outcomes for unit i are defined by $Y(i, j, k)$:

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Built in Assumptions

The notation implies three related assumptions:

- No simultaneity
- No interference
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We also need to assume **Positivity** $0 < P(T_i = 1) < 1 \forall i$ with probability 1.

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- Another way of thinking of it: The distributions of the potential outcomes $(Y_i(1), Y_i(0))$ are the same for the treatment and control group.
- Yet another way of thinking of it: The treatment and control group are exchangeable, or balanced (on observables and unobservables) on average

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- Assume a completely randomized trial (fixed number of units assigned to treatment)
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How can we estimate the difference in means with regression?

$\mathbb{1}_m(y \sim T)$. Recall that, when our only regressor is binary, $\hat{\beta}$ estimates the difference-in-means.

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Design-based Inference for SATE

Variance of $\hat{\tau}$:

$$V[\hat{\tau}] = \frac{1}{n} \left(\frac{n_c}{n_t} S_1^2 + \frac{n_t}{n_c} S_0^2 + 2S_{0,1} \right)$$

where for $t = 0, 1$,

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Conservative estimator:

$$\hat{V}[\hat{\tau}] \leq \frac{\hat{S}_1^2}{n_t} + \frac{\hat{S}_0^2}{n_c}$$

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- This is **conditional ignorability**.

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- Naive estimator = Average Treatment Effect on Treated + Selection Bias
- Selection bias: how different the treated and control groups are in terms of their potential outcome under control.

Selection Makes Us Care About Assignment Mechanisms

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- **Unconfounded assignment:** Disallows dependence of the assignment mechanism on the potential outcomes

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Most statistical models of causal inference attain identification of treatment effects by restricting the assignment mechanism in some way.

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- Design Principle:
 - Pretend you could design any experiment
 - If that experiment does not exist, be concerned about interpretation

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- Causal inference is a missing data problem: we typically make assumptions about the assignment mechanism to go from descriptive inference to causal inference.

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2 Potential Outcomes

- Framework
- Estimands
- Three Big Assumptions
- What Gets to Be a Cause

3 Nonparametric Structural Equation Models and Causal Directed Acyclic Graphs

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- Identification involves using the do-calculus to write the **causal quantity** (which involves $\text{do}()$) in terms of an **empirical quantity** (which involves only observed data).

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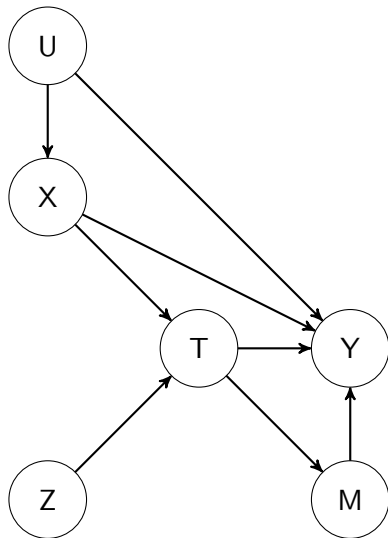
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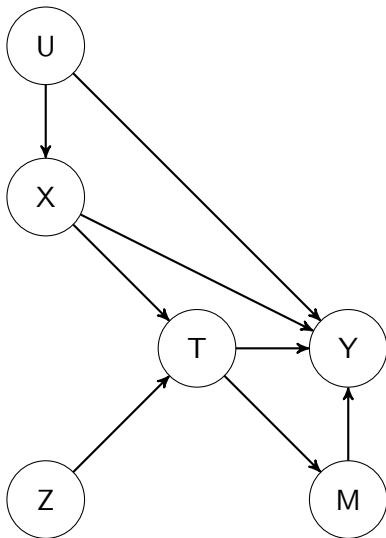
Code Idea: **causal reasoning before statistical reasoning**

Components of a DAG

- nodes represent variables
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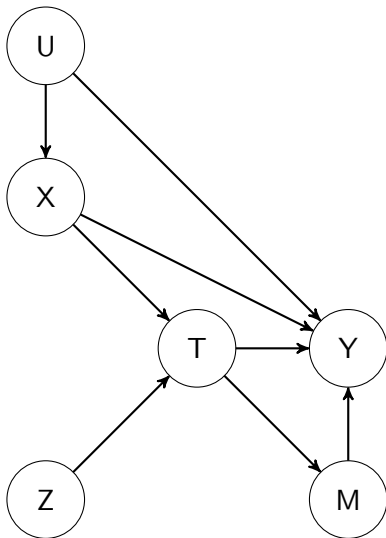


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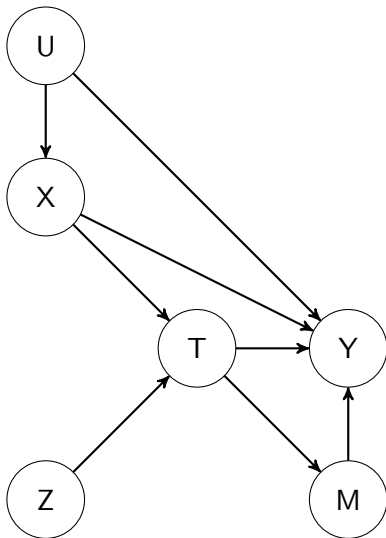
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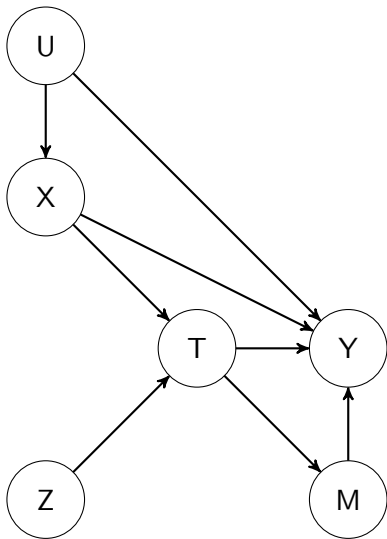
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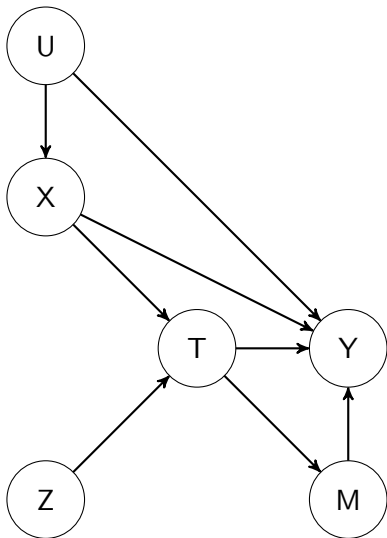
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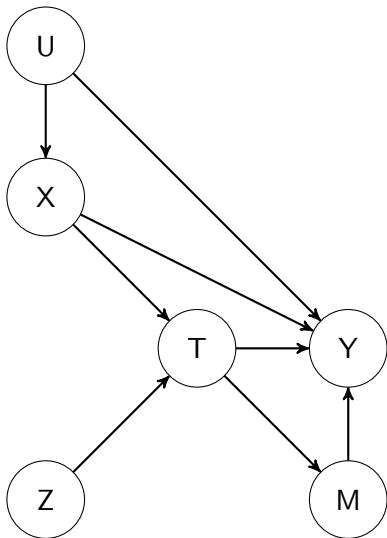
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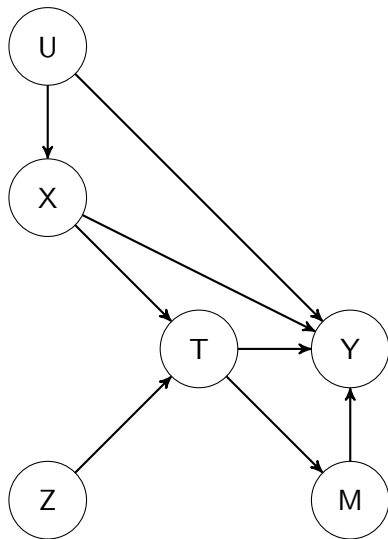
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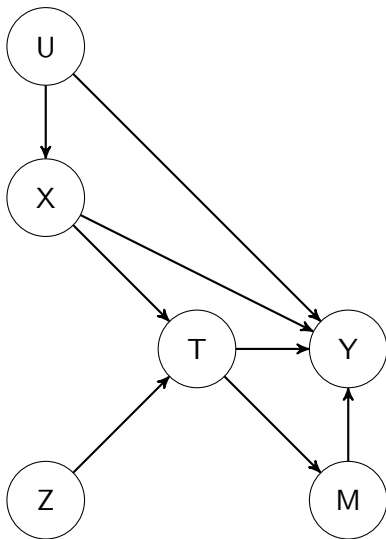
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- all relationships are **non-parametric**

Relationships in a DAG



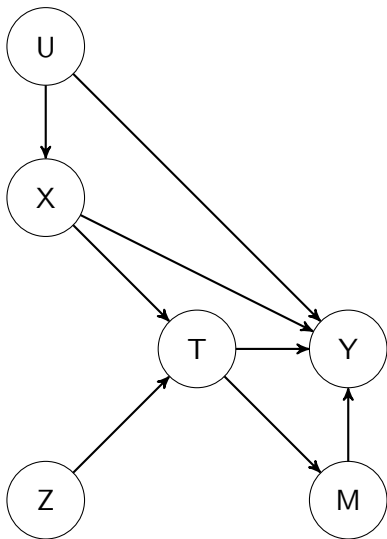
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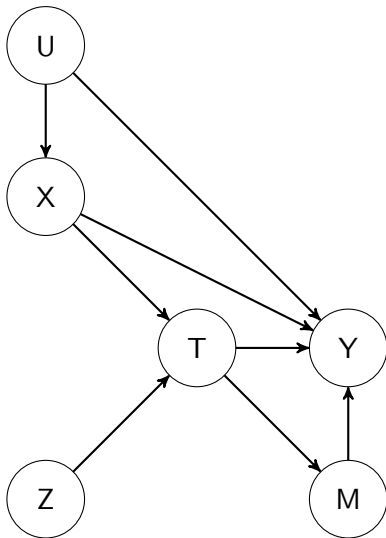
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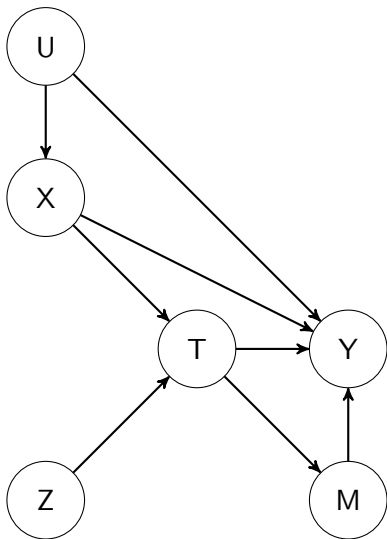
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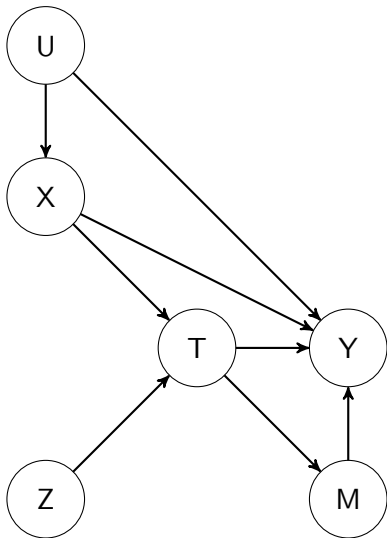
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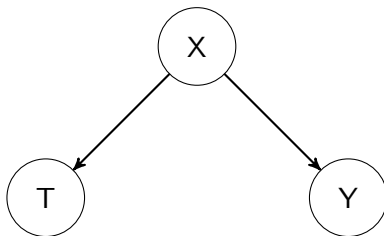
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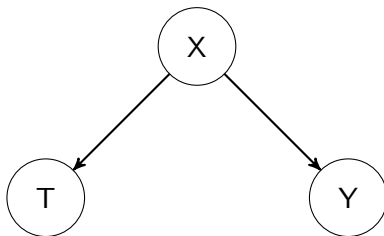
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- We will talk in depth about two types of relationships: **confounders** and **colliders**

Confounders



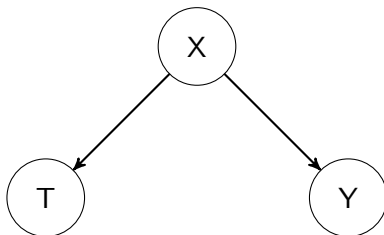
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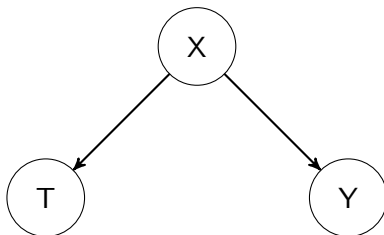
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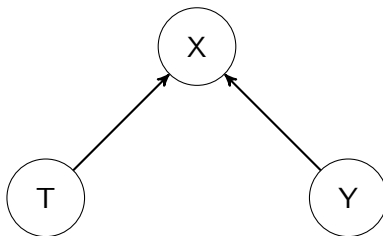
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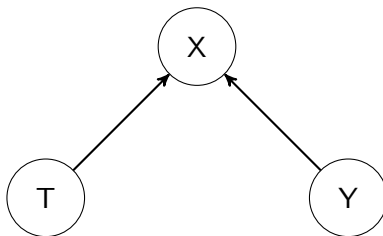
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- We can think of conditioning on a confounder as blocking the flow of association.

Colliders



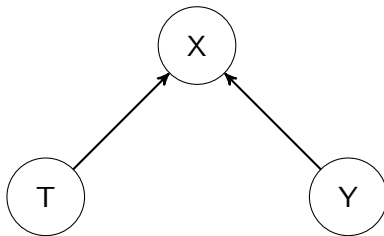
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- In this scenario T and Y are **not marginally associated**
- If we control for X they become associated and create a connection between T and Y

Colliders are scary because you can induce dependence



Endogenous Selection Bias: The Problem of Conditioning on a Collider Variable

Felix Elwert¹ and Christopher Winship²

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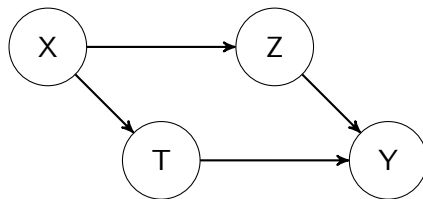
Keywords

causality, directed acyclic graphs, identification, confounding,
selection

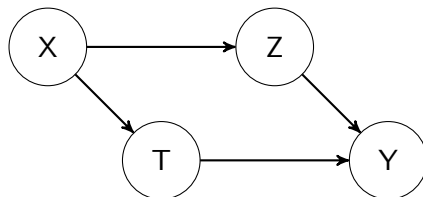
Abstract

Endogenous selection bias is a central problem for causal inference. Recognizing the problem, however, can be difficult in practice. This article introduces a purely graphical way of characterizing endogenous selection bias and of understanding its consequences (Elwert et al. 2009). We use causal graphs (direct acyclic graphs, or DAGs) to highlight that endogenous selection bias stems from conditioning (e.g., controlling, stratifying, or selecting) on a so-called collider variable, i.e., a variable that is itself caused by two other variables, one that is (or is associated with) the treatment and another that is (or is associated with) the outcome. Endogenous selection bias can result from direct conditioning on the outcome variable, a post-outcome variable, a post-treatment variable, and even a pre-treatment variable. We highlight the difference between endogenous selection bias, common-cause confounding, and overcontrol bias and discuss numerous examples from social stratification, cultural sociology, social network analysis, political sociology, social demography, and the sociology of education.

From Confounders to Back-Door Paths

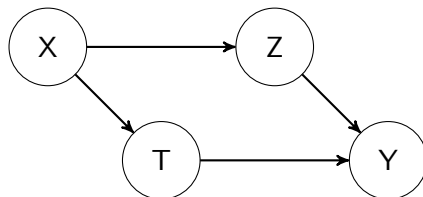


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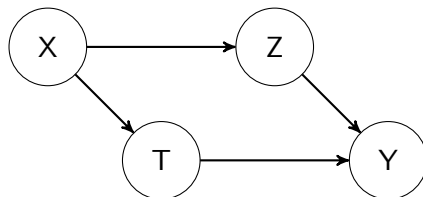
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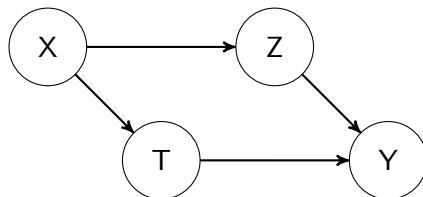
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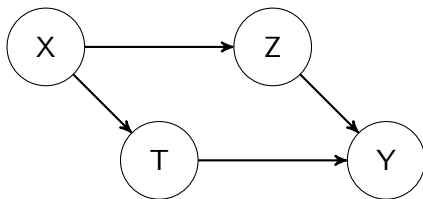
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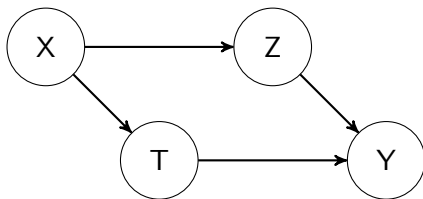
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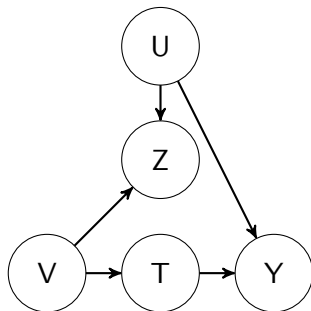
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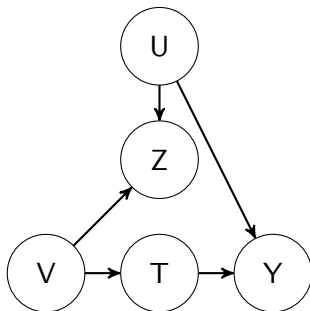
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- We want to **block** the back-door path to leave only the causal effect

Colliders and Back-Door Paths

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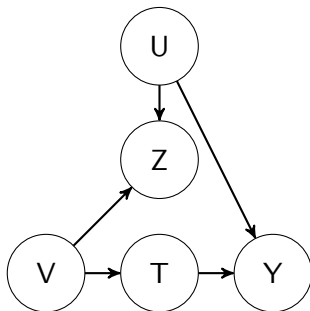


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- So how do we know which back-door paths to block?

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- If A is not D -separated from B by C we say that A is **D-connected** to B by C

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 - ① No node in X is a descendent of T
(i.e. don't condition on post-treatment variables!)

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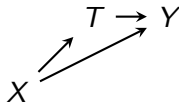
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- We will see some other approaches late in the semester.

Blocking backdoor paths: College and earnings

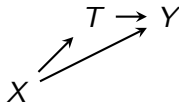
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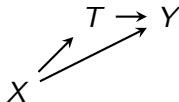
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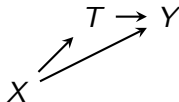
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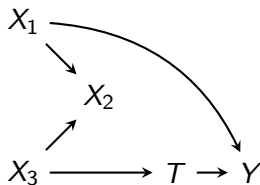
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Ability, parents' income, parents' education, extended family who pay for college and help you find a job, neighborhood characteristics that affect high school quality and also the availability of local jobs, ... **lots of things!**

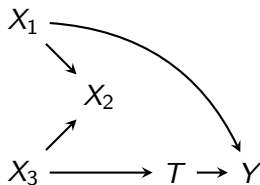
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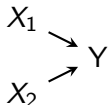
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No need to condition! X_2 already blocks this path. it is a **collider**.

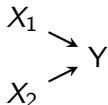
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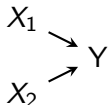


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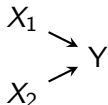


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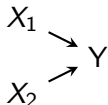


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Example extended from Elwert & Winship 2014

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Should we worry about this design? It depends on our **theory** about how these variables are related. We can argue about identification with a DAG.

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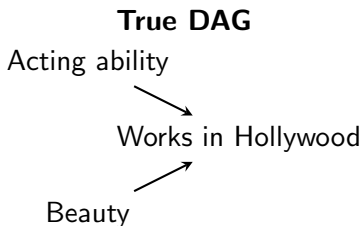
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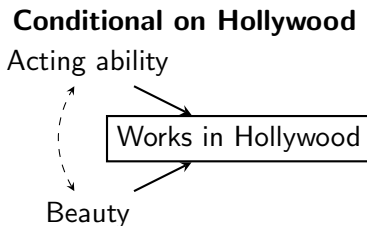
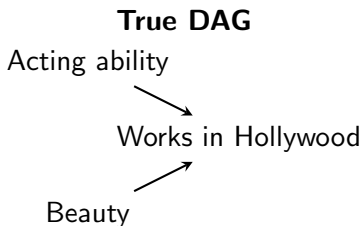


Conditional on Hollywood

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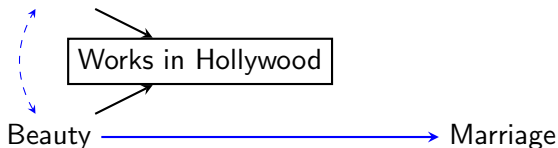


Colliders: When drawing a DAG helps

Example extended from Elwert & Winship 2014

This is an example of conditioning on a collider! We induce a negative association between acting ability and beauty.

Acting ability



Under the assumptions above, our results are driven by collider conditioning!

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- Estimands
- Three Big Assumptions
- What Gets to Be a Cause

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Lancet 2001: negative correlation between coronary heart disease mortality and level of vitamin C in bloodstream (controlling for age, gender, blood pressure, diabetes, and smoking)

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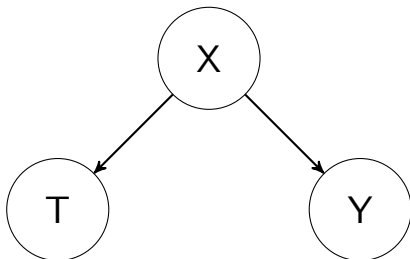


Lancet 2003: comparing among individuals with the same age, gender, blood pressure, diabetes, and smoking, those with higher vitamin C levels have lower levels of obesity, lower levels of alcohol consumption, are less likely to grow up in working class, etc.

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Confounders



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 - ▶ “The planner of an observational study should always ask himself [sic]: How would the study be conducted if it were possible to do it by controlled experimentation” (Cochran 1965)

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 - ★ Sometimes written as $T_i \perp\!\!\!\perp (\mathbf{Y}(1), \mathbf{Y}(0))$

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Remember selection bias?

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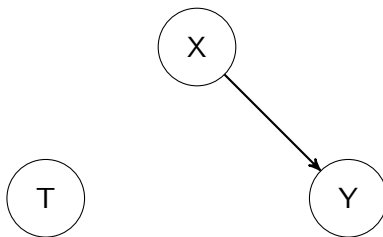
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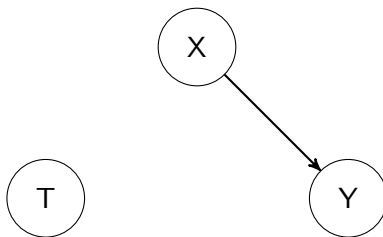
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When all goes well, an experiment eliminates selection bias.

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This ensures any observed or unobserved pretreatment covariates have the same distribution in the treatment and the control group. That is, they are **balanced**.

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- Conditional randomization assumptions:
 - ▶ Positivity: $0 < P(T_i = 1 | X_i = x) < 1$ for all i and x .
 - ▶ Unconfoundedness: $P(T_i = 1 | \mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0)) = P(T_i = 1 | X_i)$

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- ATE is just the weighted average of the within-strata differences in means.
- Identified because the last line is a function of observables.
- The averaging is over the distribution of the strata $\rightsquigarrow$ size of the blocks.

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- We cover a bit because experiments are a dominant **metaphor**. Much of the work on observational studies is trying to find a circumstance where we can pretend we have a **stratified experiment**.
- Of course, in real experiments sometimes people don't always do what we say (compliance problems).

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- estimation of causal effects does not require identical treatment and control groups
- you need a **clear counterfactual** to have a well-defined causal effect. For example of 'the recession was caused by Wall Street' may make intuitive sense but is it well-defined?

<http://egap.org/methods-guides/10-things-you-need-know-about-causal-inference>

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Next Time: Causality with Measured Confounding