Week 4: Testing/Regression

Brandon Stewart[1]

Princeton

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[1] Many of the slides are adapted from Matt Blackwell, Adam Glynn, Jens Hainmueller, and Erin Hartman

Where We've Been and Where We're Going...

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- Last Week
  - inference and estimator properties
  - point estimates, confidence intervals

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- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causal inference

We Secretly Already Covered This!

American Political Science Review Vol. 102, No. 1 February 2008
DOI: 10.1017/S000305540808009X

Social Pressure and Voter Turnout: Evidence from a Large-Scale Field Experiment

ALAN S. GERBER *Yale University*
DONALD P. GREEN *Yale University*
CHRISTOPHER W. LARIMER *University of Northern Iowa*

We Secretly Already Covered This!

Dear Registered Voter:

WHAT IF YOUR NEIGHBORS KNEW WHETHER YOU VOTED?

Why do so many people fail to vote? We've been talking about the problem for years, but it only seems to get worse. This year, we're taking a new approach. We're sending this mailing to you and your neighbors to publicize who does and does not vote.

The chart shows the names of some of your neighbors, showing which have voted in the past. After the August 8 election, we intend to mail an updated chart. You and your neighbors will all know who voted and who did not.

DO YOUR CIVIC DUTY — VOTE!

MAPLE DR	Aug 04	Nov 04	Aug 06
9995 JOSEPH JAMES SMITH	Voted	Voted	______
9995 JENNIFER KAY SMITH		Voted	______
9997 RICHARD B JACKSON		Voted	______
9999 KATHY MARIE JACKSON		Voted	______

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- Which we then approximate with $\hat{\theta} \sim \mathcal{N}(0, \widehat{\text{SE}}(\hat{\theta})^2)$

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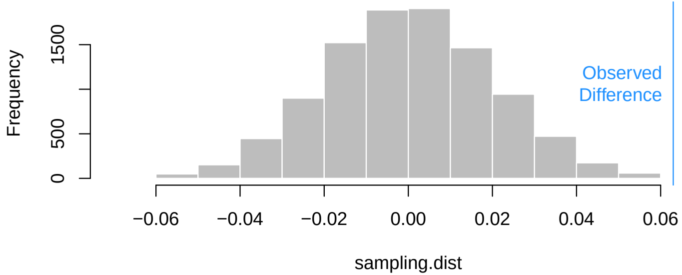
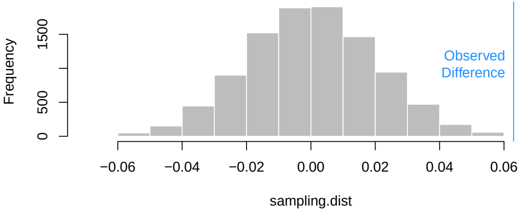
Example from Gerber, Green and Larimer (2008).



Overall Idea



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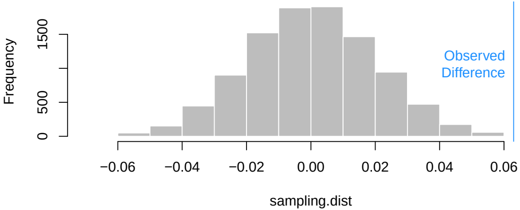

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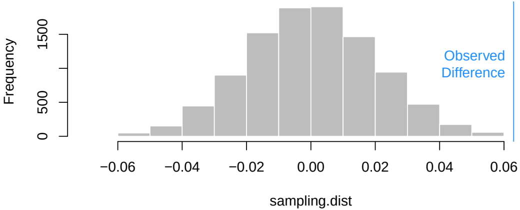

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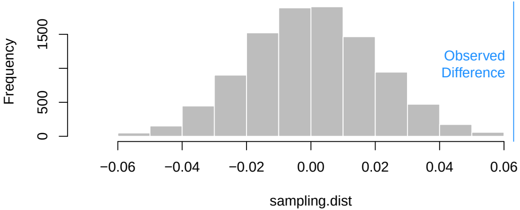

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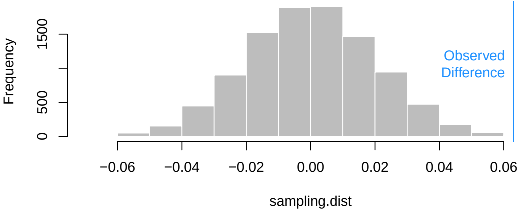

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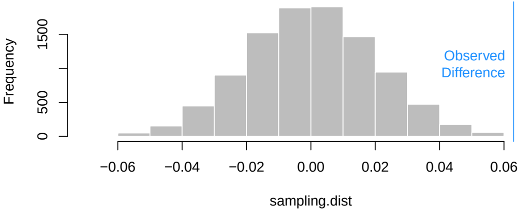

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Basically, we did a hypothesis test.

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- Hypothesis tests lead to discrete decisions.

Background on Drug Testing

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There are typically three phases of clinical trials before a drug is approved:

- Phase I: Toxicity (Will it kill you?)
- Phase II: Efficacy (Is there any evidence that it helps?)
- Phase III: Effectiveness (Is it better than existing treatments?)

Phase I trials are conducted on a small number of healthy volunteers, Phase II trial are either randomized experiments or within-patient comparisons, and Phase III trials are almost always randomized experiments with control groups.

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- The trial included 345 patients with initial systolic blood pressure between 140-159.
- Each subject was assigned to take the drug combination for 16 weeks.
- Systolic blood pressure was measured on each subject before and after the treatment period.

Example

Subject	SBP_{before}	SBP_{after}	Decrease
1			
2			
3			
4			
$\vdots$			
345			

Example

Subject	SBP_{before}	SBP_{after}	Decrease
1	147		
2	153		
3	142		
4	141		
$\vdots$	$\vdots$		
345	155		

Example

Subject	SBP_{before}	SBP_{after}	Decrease
1	147	135	
2	153	122	
3	142	119	
4	141	134	
⋮	⋮	⋮	
345	155	115	

Example

Subject	SBP_{before}	SBP_{after}	Decrease
1	147	135	12
2	153	122	31
3	142	119	23
4	141	134	7
$\vdots$	$\vdots$	$\vdots$	$\vdots$
345	155	115	40

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Question: Should the FDA allow the drug to proceed to the next stage of testing?

The FDA's Decision

We can think of the FDA's problem in terms of two dimensions:

- The true state of the world
- The decision made by the FDA

	Drug works	Drug doesn't work
FDA approves		
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Two kinds of bad decisions:

- False positive
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Elements of a Hypothesis Test

Important terms we are about to define:

- Null Hypothesis (assumed state of world for test)
- Alternative Hypothesis (all other states of the world)
- Type I and Type II Errors (two types of errors)
- Test Statistic (what we will observe from the sample)
- Test Level (the probability of a type I error)
- Rejection Region (the basis of our decision)

Hypotheses

- Null Hypothesis (H_0): The conservatively assumed state of the world we are accumulating evidence against (often "no effect").

  Example: There is not a difference in voting rates between those who received the social pressure mailer and those that received the civic duty mailer.

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- Alternative Hypothesis (H_a): The state of the world where the null hypothesis is not true and thus the claim to be indirectly tested.

  Example: There is a difference in voting rates between those who received the social pressure mailer and those that received the civic duty mailer.

Error Types

	(H_0 False)	(H_0 True)
Reject H_0		
Don't Reject H_0		

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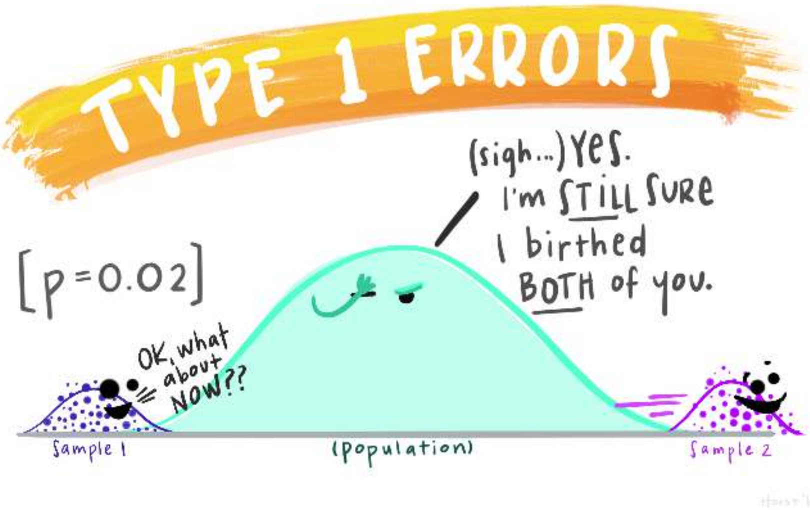
We generally make the normative judgment that we prefer an undetected finding (Type II error) to a false discovery (Type I error).

A Visual Reminder from Allison Horst



Artwork by @allison_horst

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Test Statistics and Null Distributions

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Test Statistic: which we denote T_n is a function of the sample, the estimator and the null hypothesis.

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Null Distribution: the sampling distribution of the statistic/test statistic <u>assuming that the null is true</u>.

Test Statistics and Null Distribution

Returning to the voting experiment. We know from the Central Limit Theorem that the standardized difference in means has a standard normal distribution asymptotically,

$$T_n = \frac{\hat{\theta} - (\mu_y - \mu_x)}{\widehat{\mathsf{SE}}[\hat{\theta}]} \xrightarrow{d} \mathcal{N}(0, 1)$$

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Under the null hypothesis of $\mu_y - \mu_x = 0$, we have

$$T_n = \frac{\hat{\theta}}{\widehat{\mathsf{SE}}[\hat{\theta}]} \xrightarrow{d} \mathcal{N}(0,1)$$

If T_n is very far from zero—in the sense that it has low probability under $\mathcal{N}(0,1)$—then we reject the null hypothesis as not plausible.

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- We call c the critical value.
- Much like the 95% confidence interval, we pick $\alpha = .05$ by convention.

The value for which P=0.05, or 1 in 20, is 1.96 or nearly 2; it is convenient to take this point as a limit in judging whether a deviation ought to be considered significant or not. Deviations exceeding twice the standard deviation are thus formally regarded as significant. Using this criterion we should be led to follow up a false indication only once in 22 trials, even if the statistics were the only guide available. Small effects will still escape notice if the data are insufficiently numerous to bring them out, but no lowering of the standard of significance would meet this difficulty.

- Ronald Fisher, *Design of Experiments* (1922)

Two-sided Rejection Region

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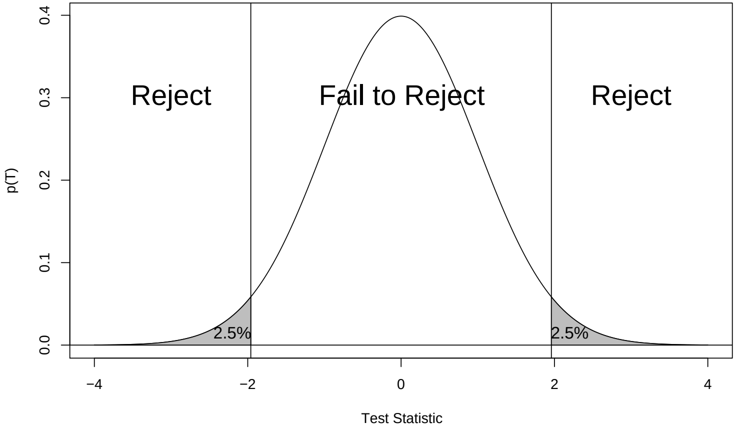
Rejection region with $\alpha = .05$, $H_0 : \theta = 0$, $H_A : \theta \neq 0$:



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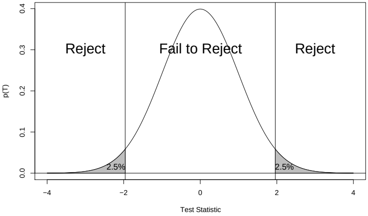

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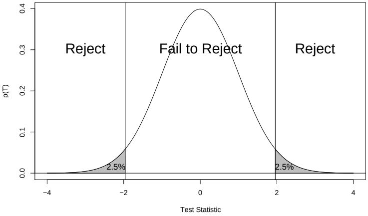

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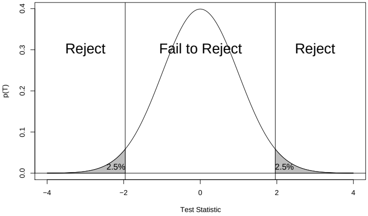

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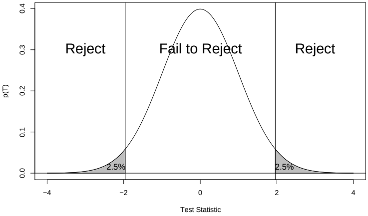

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- We want to find the point c such that $P_{H_0}(T_n < c) + P_{H_0}(T_n > c) = \alpha$ where we typically use equal probability on each side by convention.
- This is just the task of finding the quantile for $\alpha/2$. In the case of $\alpha = .05$, $\texttt{qnorm}(.05/2) = 1.96$

The Complete Recipe

The Complete Recipe

1. Define a null hypothesis and the alternative hypothesis.
2. Choose a test statistic T_n.
3. Determine the Type I error you will tolerate (α).
4. Determine your critical value and thus your rejection region.
5. Calculate your test statistic in your observed data.
6. If your observed data is sufficiently unlikely under your null hypothesis, reject your null.

The Gerber, Green and Larimer Example

```
diff <- mean(treated) - mean(control)
se_diff <- sqrt(var(treated)/length(treated) +
                var(control)/length(control))
test_statistic <- diff/se_diff
```

This yields 18.3 which is much larger in absolute value than our .05 critical value of 1.96.

We reject the null.

Back to the FDA

Let's go back to the FDA making a decision about drugs.

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Drug trials are expensive and *ex ante* we can specify that we only care about one direction in particular. Consider the Sowers et al (2006) case which claimed to decrease blood pressure.

Sowers et al. Example

We can define our hypotheses for a one-sided test.

$$H_0 : \mu_{decrease} \leq 0 \quad (1)$$
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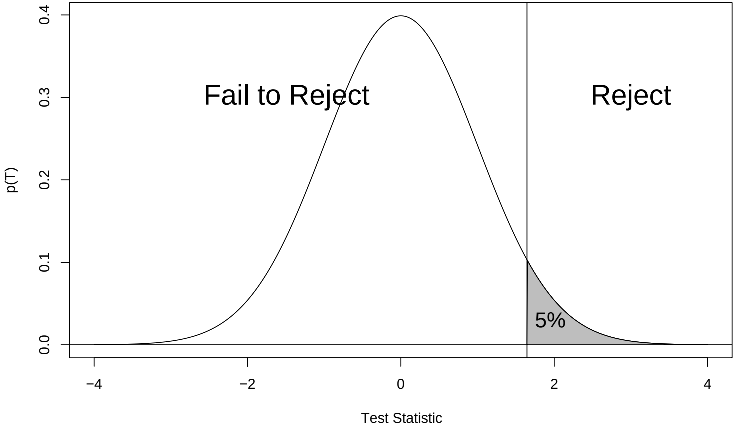
- $\hat{\sigma} = 14.3$
- $n = 345$

Therefore,

$$t_n = \frac{21.0 - 0}{\frac{14.3}{\sqrt{345}}} = 27.3$$

We construct our rejection region with $c = \texttt{qnorm(.95)} = 1.644$.

Rejection Region with $\alpha = .05$



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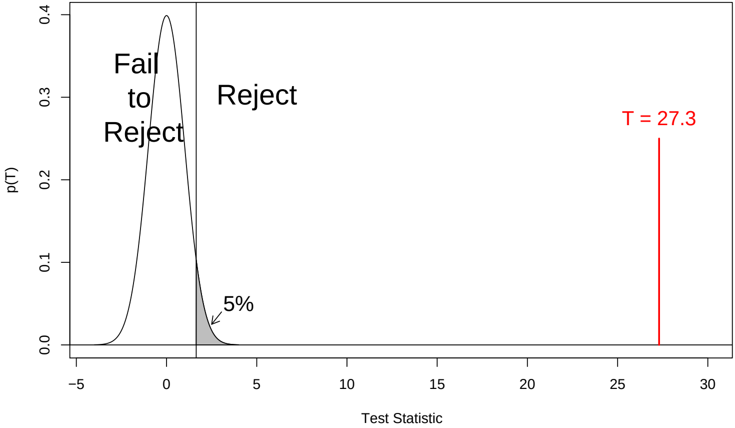

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- This will guarantee the nominal probability of Type I error as n gets large.

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We can think of confidence intervals as a range of plausible values in the sense that we would not have rejected them had they been our null hypotheses.

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3. Null is false, the test is well-powered and we got incredibly unlucky.

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- Power analysis is a way of guiding the choice of sample size prior to an experiment to avoid this kind of mistake.

How Well Powered Are Social Science Studies?

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Quantitative Political Science Research is Greatly Underpowered[*]

Vincent Arel-Bundock[†] Ryan C. Briggs[‡]
Hristos Doucouliagos[§] Marco Mendoza Aviña[¶] T.D. Stanley[⊠]

July 05, 2022

Abstract

We analyze the statistical power of political science research by collating over 16,000 hypothesis tests from about 2,000 articles. Even with generous assumptions, the median analysis has about 10% power, and only about 1 in 10 tests have at least 80% power to detect the consensus effects reported in the literature. There is also substantial heterogeneity in tests across research areas, with some being characterized by high-power but most having very low power. To contextualize our findings, we survey political methodologists to assess their expectations about power levels. Most methodologists greatly overestimate the statistical power of political science research.

The median analysis has about 10% power ($\alpha = 0.05$), and only about 1 in 10 statistical tests have at least 80% power.

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5) Possibly repeat under different assumptions about the population.

Example: Power Analysis

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	Control	Civic Duty	Hawthorne	Self	Neighbors
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- For this example, let's ignore the household sampling (which is really important in practice, but we want to make this simple!).

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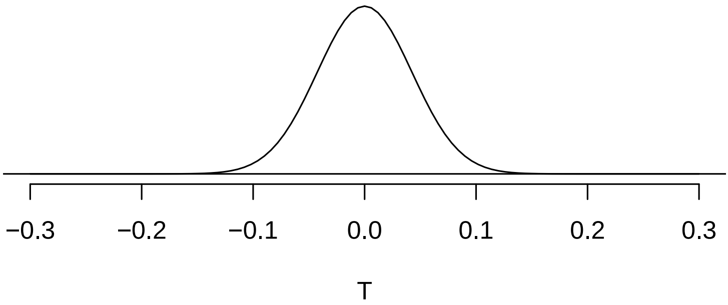
Yikes! That is not well powered.

Power Graph



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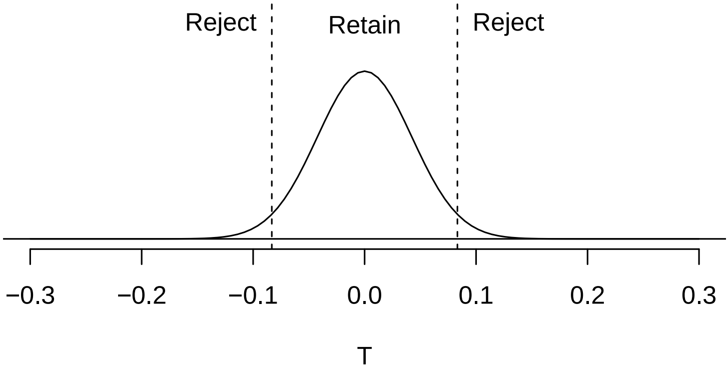

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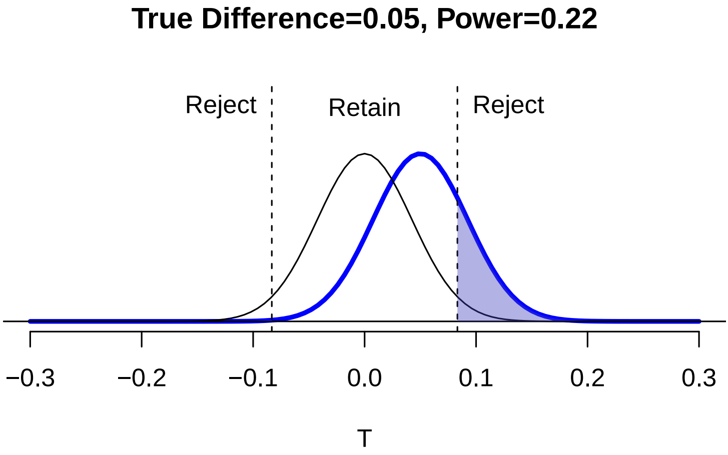
True Difference=0.05, Power=0.22
Reject
Retain
Reject
−0.3
−0.2
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0.0
0.1
0.2
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T

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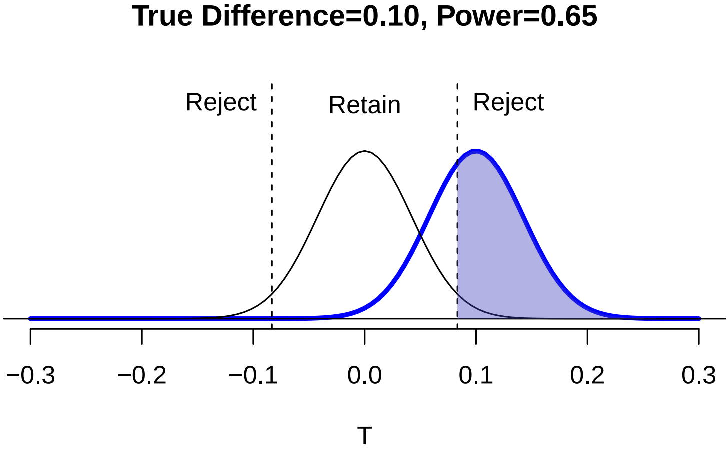
True Difference=0.10, Power=0.65
Reject
Retain
Reject
−0.3
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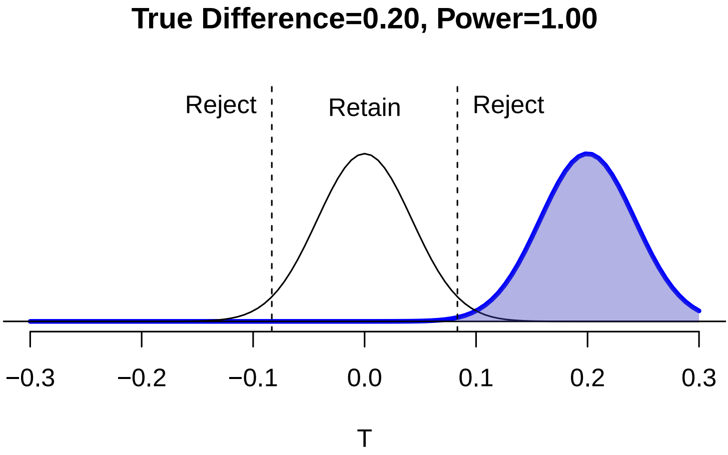
True Difference=0.20, Power=1.00
Reject
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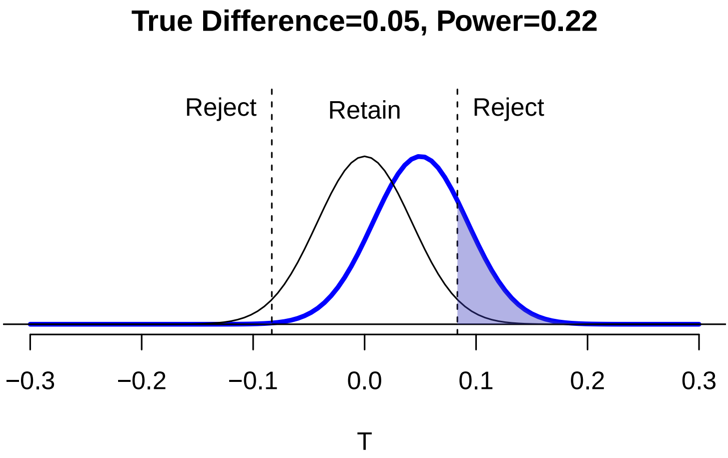

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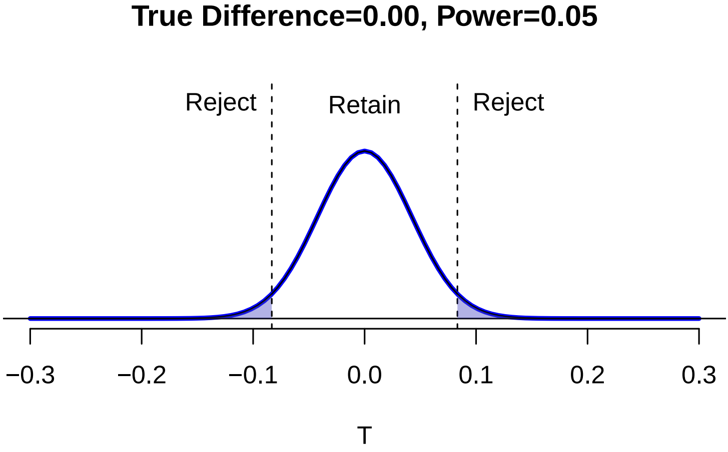
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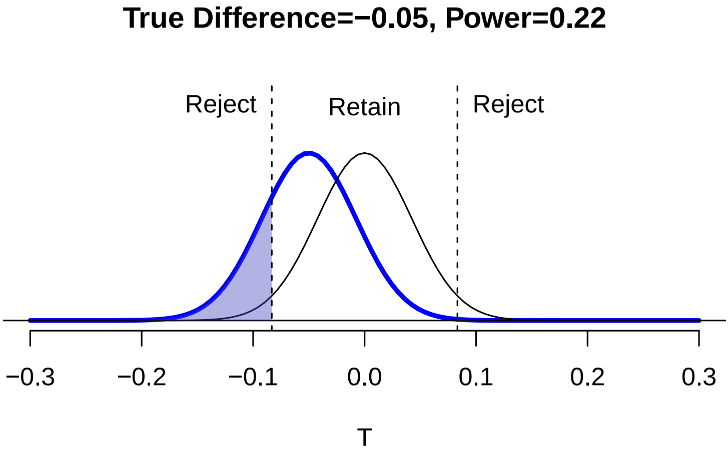

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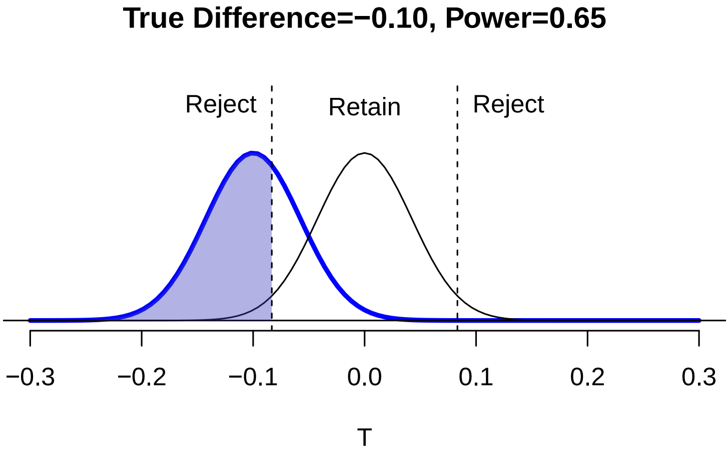

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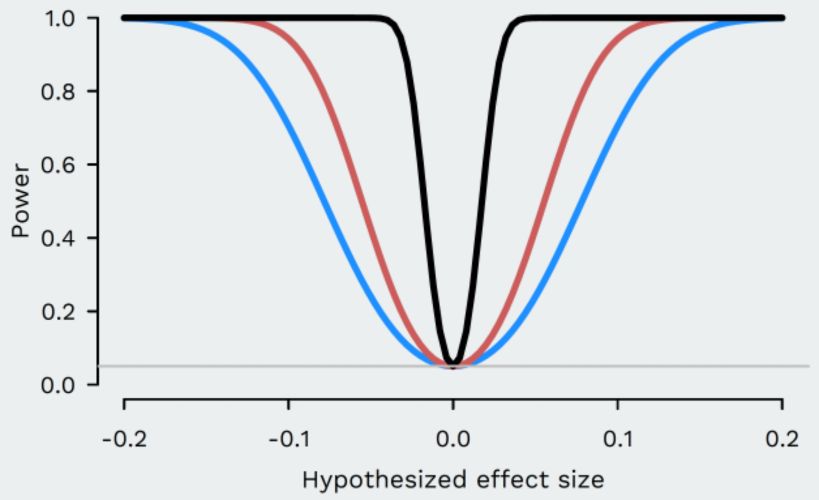
Yeah. . . kind of. In practice, this is why we calculate under many possible configurations.

Power Curve

You can graph power for various possible effect sizes (here for 500, 1000, and 10000 samples).



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- Power analysis is really important if you are planning experiments, but we will touch on it only cursorily in this class. The Gerber and Green Field Experiments book is an amazing resource for more on experiments in general.

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- We will cover more on subtleties of interpretation next.

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Under the null hypothesis, this corresponds to the probability of observing a test statistic as extreme or more extreme than the one in the observed data (where extreme is defined in terms of the alternative hypothesis).

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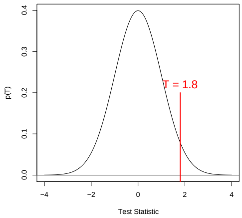

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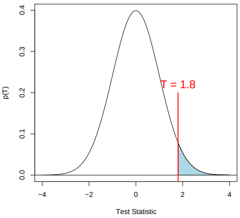

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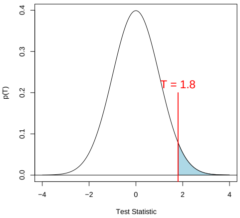

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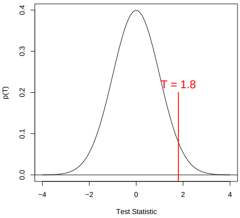

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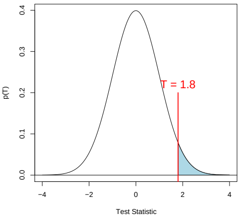

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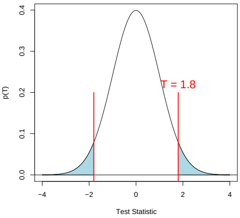

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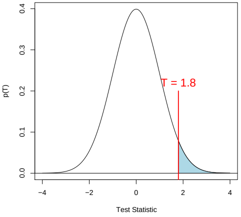

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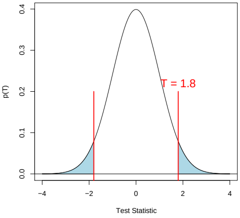

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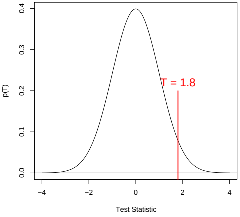

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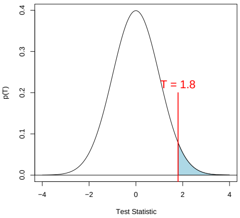

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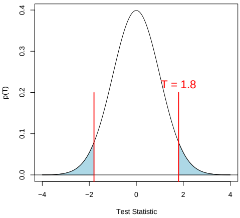

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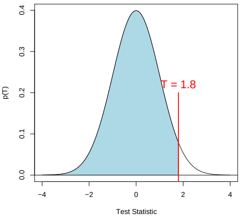

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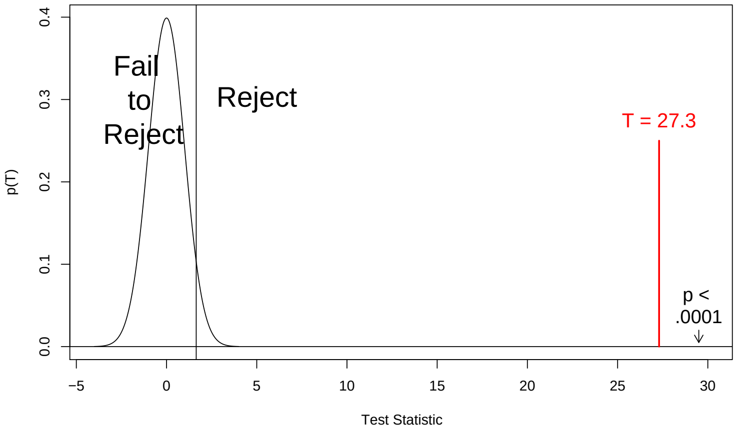
- That yields a p-value of 2.06×10^{-76}.
- By convention we would say it is statistically significant at level α for some α that the p-value is below.

The Sowers et. al. Example



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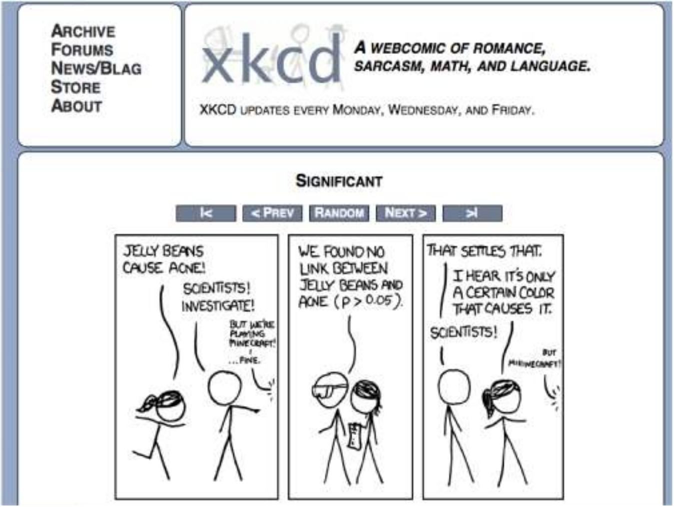
It only works for one test.

Star Chasing (aka there is an XKCD for everything)



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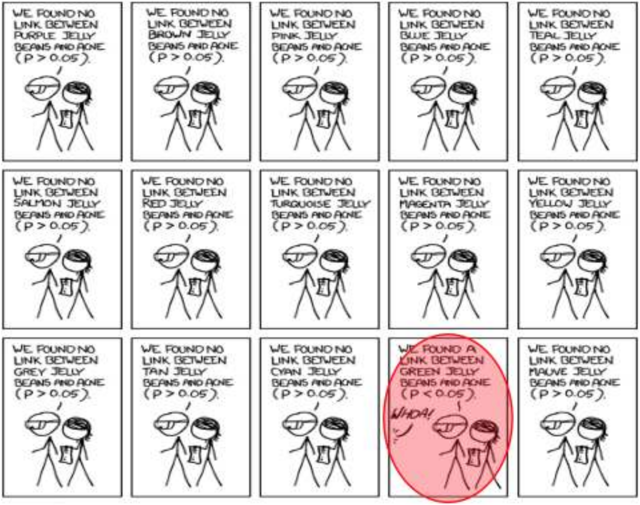

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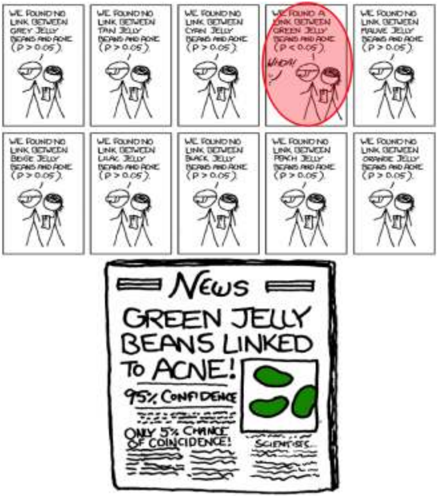

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- Illustration: randomly draw 21 variables, and run a regression of the first variable on the rest.
- By design, no effect of any variable on any other, but when we run the regression:

Multiple Test Example

```
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0280393  0.1138198  -0.246  0.80605
## X2          -0.1503904  0.1121808  -1.341  0.18389
## X3           0.0791578  0.0950278   0.833  0.40736
## X4          -0.0717419  0.1045788  -0.686  0.49472
## X5           0.1720783  0.1140017   1.509  0.13518
## X6           0.0808522  0.1083414   0.746  0.45772
## X7           0.1029129  0.1141562   0.902  0.37006
## X8          -0.3210531  0.1206727  -2.661  0.00945 **
## X9          -0.0531223  0.1079834  -0.492  0.62412
## X10          0.1801045  0.1264427   1.424  0.15827
## X11          0.1663864  0.1109471   1.500  0.13768
## X12          0.0080111  0.1037663   0.077  0.93866
## X13          0.0002117  0.1037845   0.002  0.99838
## X14         -0.0659690  0.1122145  -0.588  0.55829
## X15         -0.1296539  0.1115753  -1.162  0.24872
## X16         -0.0544456  0.1251395  -0.435  0.66469
## X17          0.0043351  0.1120122   0.039  0.96923
## X18         -0.0807963  0.1098525  -0.735  0.46421
## X19         -0.0858057  0.1185529  -0.724  0.47134
## X20         -0.1860057  0.1045602  -1.779  0.07910 .
## X21          0.0021111  0.1081179   0.020  0.98447
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9992 on 79 degrees of freedom
## Multiple R-squared:  0.2009, Adjusted R-squared:  -0.00142
## F-statistic: 0.993 on 20 and 79 DF,  p-value: 0.4797
```

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- The procedure by which tests/analyses are performed and shown to us matters a lot!

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  So for each coefficient you have a .90 confidence interval, but overall a .52 percent confidence interval.

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A Sketch of Two Styles of Solutions:
(1) statistical
and
(2) procedural.

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    - ★ Reject all H_k for $k \leq k^*$

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Pvalues: 0.011, 0.13, 0.06, 0.54, 0.008, 0.024, 0.001, 0.201, 0.78, 0.023

Step 1: Sort	Step 2: Find New Threshold	Step 3: Find k	Step 4: Reject
0.001			
0.008			
0.011			
0.023			
0.024			
0.06			
0.13			
0.201			
0.54			
0.78			

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0.001	0.05*1/10 = 0.005		
0.008	0.05*2/10 = 0.01		
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0.023	0.05*4/10 = 0.02		
0.024	0.05*5/10 = 0.025		
0.06	0.05*6/10 = 0.03		
0.13	0.05*7/10 = 0.035		
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  - Similar to preregistration in that it doesn't directly address multiple comparisons, but limits them.

Fun With Salmon

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Bennett, Baird, Miller and Wolford. (2009). “Neural correlates of interspecies perspective taking in the post-mortem Atlantic Salmon: an argument for multiple comparisons correction.”

Methods

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(a.k.a. the greatest methods section of all time)

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  "One mature Atlantic Salmon (Salmo salar) participated in the fMRI study. The salmon was approximately 18 inches long, weighed 3.8 lbs, and was not alive at the time of scanning."

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  "One mature Atlantic Salmon (Salmo salar) participated in the fMRI study. The salmon was approximately 18 inches long, weighed 3.8 lbs, and was not alive at the time of scanning."
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  "The task administered to the salmon involved completing an open-ended mentalizing task.

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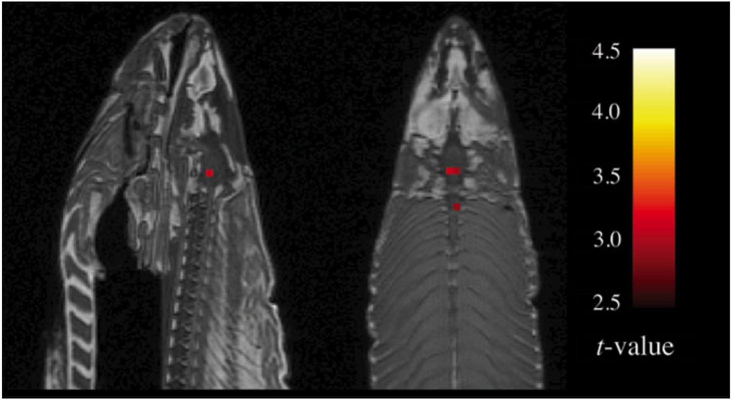
- Design
  "Stimuli were presented in a block design with each photo presented for 10 seconds followed by 12 seconds of rest.A total of 15 photos were displayed. Total scan time was 5.5 minutes."

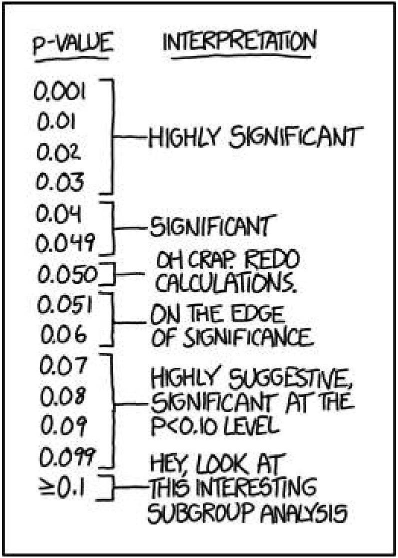
Results



"Several active voxels were discovered in a cluster located within the salmon's brain cavity. The size of this cluster was 81 mm^3 with a cluster-level significance of $p = .001$."

Okay, but what do they mean?

The Meaning of p-values (courtesy of XKCD)



The value of the p-value

Every experiment may be said to exist only in order to give the facts a chance of disproving the null hypothesis.

Ronald Fisher (1935)

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In social science (and I think in psychology as well), the null hypothesis is almost certainly false, false, false, and you don't need a p-value to tell you this. The p-value tells you the extent to which a certain aspect of your data are consistent with the null hypothesis. A lack of rejection doesn't tell you that the null hypothesis is likely true; rather, it tells you that you don't have enough data to reject the null hypothesis.

Andrew Gelman (2010)

Practical versus Statistical Significance

$$T_n = \frac{\hat{\theta} - \theta_0}{\widehat{\text{SE}}[\hat{\theta}]} = \frac{\bar{X} - \mu_0}{\sqrt{\hat{\sigma}^2 / n}}$$

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- In large samples even tiny effects will be significant, but the results may not be very important substantively. Always discuss both!

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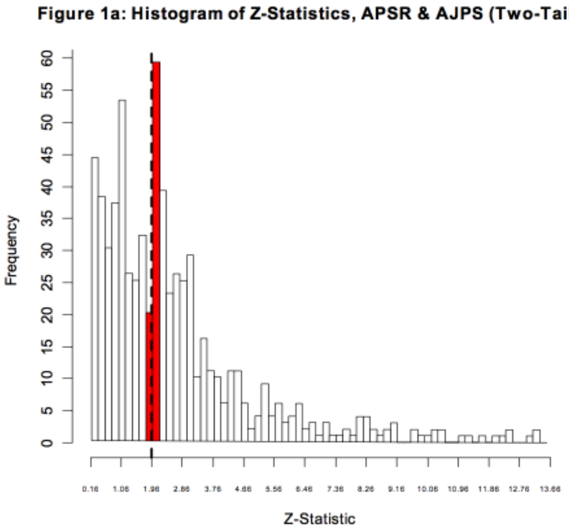
See also: The ASA's (American Statistical Association) Statement on p-Values: Context, Process, and Purpose (http://dx.doi.org/10.1080/00031305.2016.1154108)

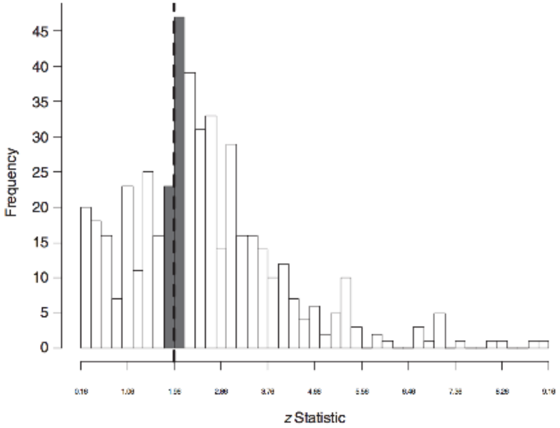
Arbitrary Publication Cutoffs



Gerber and Malhotra (2006) Top Political Science Journals

Arbitrary Publication Cutoffs

Figure 1
Histogram of *z* Statistics From the *American Sociological Review*, the *American Journal of Sociology*, and *The Sociological Quarterly* (Two-Tailed)



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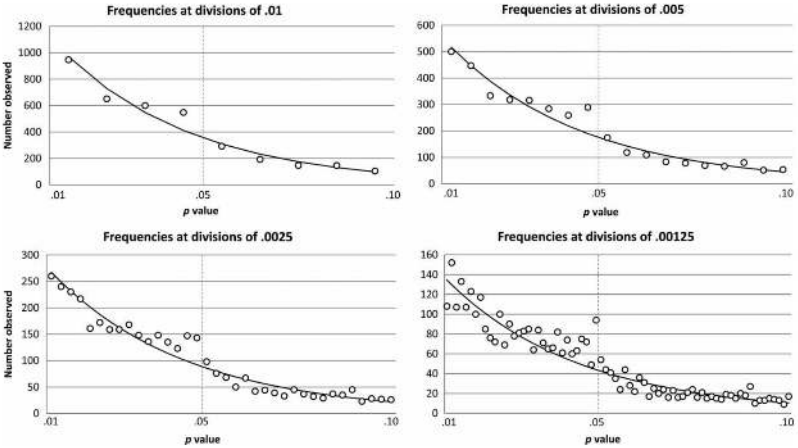

Figure 1.. *The graphs show the distribution of 3,627* p *values from three major psychology journals.*

Masicampo and Lalande (2012) Top Psychology Journals

Still Not Convinced?
The Real Harm of Misinterpreted p-values

ELSEVIER

Accident Analysis and Prevention 36 (2004) 495–500

ACCIDENT ANALYSIS & PREVENTION

www.elsevier.com/locate/aap

Viewpoint

The harm done by tests of significance

Ezra Hauer*

35 Merton Street, Apt. 1706, Toronto, Ont., Canada M4S 3G4

Abstract

Three historical episodes in which the application of null hypothesis significance testing (NHST) led to the mis-interpretation of data are described. It is argued that the pervasive use of this statistical ritual impedes the accumulation of knowledge and is unfit for use.

Keywords: Significance; Statistical hypothesis; Scientific method

Example from Hauer: Right-Turn-On-Red

Table 1
The Virginia RTOR study

	Before RTOR signing	After RTOR signing
Fatal crashes	0	0
Personal injury crashes	43	60
Persons injured	69	72
Property damage crashes	265	277
Property damage (US$)	161243	170807
Total crashes	308	337

The Point in Hauer

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- Two other interesting examples in Hauer (2004)

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The Point in Hauer

- Two other interesting examples in Hauer (2004)
- Core issue is that lack of significance is not an indication of a zero effect, it could also be a lack of power (i.e. a small sample size relative to the difficulty of detecting the effect)
- On the opposite end, large tech companies rarely use significance testing because they have huge samples which essentially always find some non-zero effect. But that doesn't make the finding significant in a colloquial sense of important.

What if I need to show evidence of a zero effect?

AMERICAN JOURNAL of POLITICAL SCIENCE

An Equivalence Approach to Balance and Placebo Tests

Erin Hartman University of California Los Angeles
F. Daniel Hidalgo Massachusetts Institute of Technology

Abstract: *Recent emphasis on credible causal designs has led to the expectation that scholars justify their research designs by testing the plausibility of their causal identification assumptions, often through balance and placebo tests. Yet current practice is to use statistical tests with an inappropriate null hypothesis of no difference, which can result in equating nonsignificant differences with significant homogeneity. Instead, we argue that researchers should begin with the initial hypothesis that the data are inconsistent with a valid research design, and provide sufficient statistical evidence in favor of a valid design. When tests are correctly specified so that difference is the null and equivalence is the alternative, the problems afflicting traditional tests are alleviated. We argue that equivalence tests are better able to incorporate substantive considerations about what constitutes good balance on covariates and placebo outcomes than traditional tests. We demonstrate these advantages with applications to natural experiments.*

Replication Materials: The data, code, and any additional materials required to replicate all analyses in this article are available on the *American Journal of Political Science* Dataverse within the Harvard Dataverse Network, at: https://doi.org/10.7910/DVN/RYNSDG.

One Step Deeper: Equivalence Tests

Solution: Flip the hypotheses:

$$H_0 : \theta_T - \theta_C \leq \epsilon_L \text{ or } \theta_T - \theta_C \geq \epsilon_U$$
$$\text{versus}$$
$$H_A : \epsilon_L < \theta_T - \theta_C < \epsilon_U$$

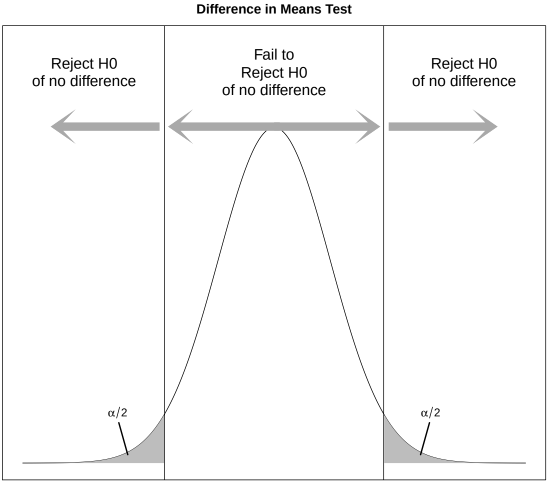
Tests of Difference vs. Equivalence Tests

$$H_0 : \frac{\mu_T - \mu_C}{\sigma} = 0 \qquad \text{versus} \qquad H_A : \frac{\mu_T - \mu_C}{\sigma} \neq 0$$

Type I Error

Test has α probability of declaring the two means different when they are, in fact, the same.

Problem: Controlling for the incorrect type of error if we're trying to provide evidence in favor of equivalence.



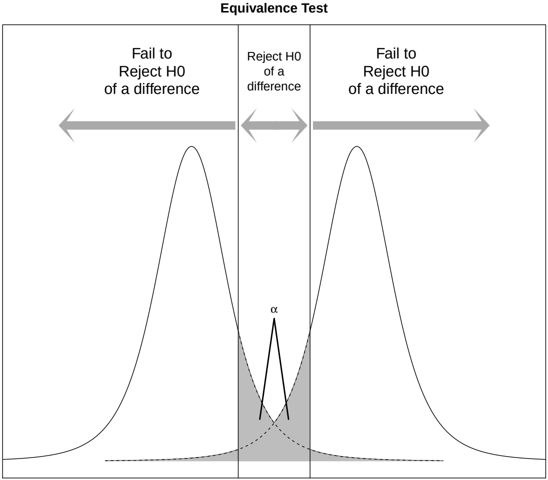
Tests of Difference vs. **Equivalence Tests**

$H_0 : \frac{\mu_T - \mu_C}{\sigma} \geq \epsilon_U$ or $\frac{\mu_T - \mu_C}{\sigma} \leq \epsilon_L$ versus $H_A : \epsilon_L < \frac{\mu_T - \mu_C}{\sigma} < \epsilon_U$

Type I Error

Test has α probability of declaring the two means equivalent when they are, in fact, the different.

Solution: Now control for the correct type of false positive.



p-values

p-values

General Message:

Don't misinterpret, or rely too heavily, on your p-values. They are evidence against your null, not evidence in favor of your alternative.

We Covered

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- hypothesis testing

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- hypothesis testing
- power

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- p-values

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- hypothesis testing
- power
- *p*-values
- multiple testing

We Covered

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- power
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Next Time: What is Regression?

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Next Time: What is Regression?

Bonus reading for those interested to learn more:

- Hauer. 2004 "The harm done by tests of significance." *Accident Analysis & Prevention*.
- Gigerenzer. 2004. "Mindless statistics." *Journal of Socio-Economics*.
- Nuzzo. 2014. "Statistical Errors." *Nature*
- Ward et al. 2010. "The perils of policy by p-value: Predicting civil conflicts." *Journal of Peace Research*
- Cohen. 1994. "The Earth is Round ($p < 0.05$)." *American Psychologist*
- Schwab. 2011. "Researchers should make thoughtful assessments instead of null-hypothesis significance tests." *Organizational Science*.

Where We've Been and Where We're Going...

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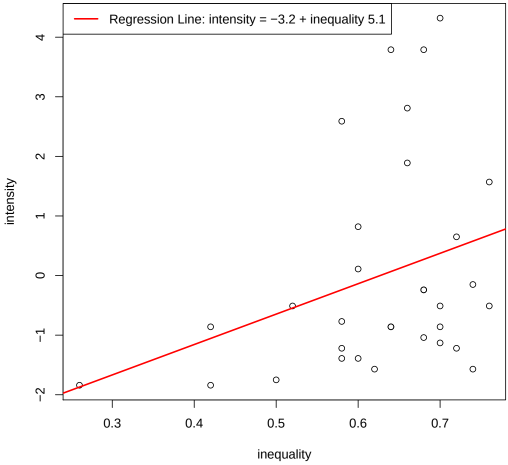
- Last Week
  - inference and estimator properties
  - point estimates, confidence intervals
- This Week
  - hypothesis testing
  - what is regression?
  - nonparametric and linear regression
- Next Week
  - inference for simple regression
  - properties of ordinary least squares
- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causal inference

1. Hypothesis Testing
   - Terminology and Procedure
   - One-Sided Tests
   - Connections
   - Power
2. p-values
   - Mechanics
   - Multiple Testing
   - Fun With Salmon
   - The Significance of Significance
3. What is Regression?
   - Conditional Expectation Functions
   - Nonparametric Regression
   - Best Linear Predictor
   - Ordinary Least Squares
4. Interpreting Regression
   - Fun With Linearity

What You've Probably Seen This



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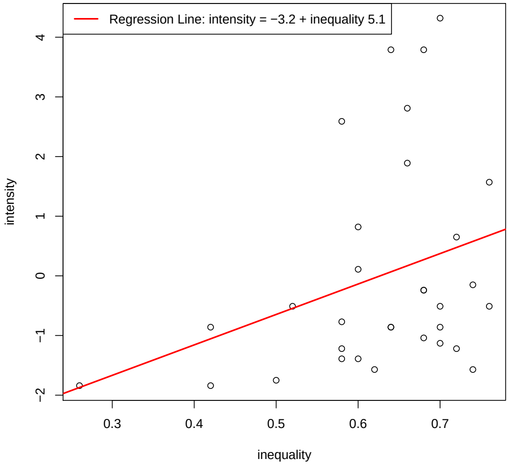

We are going to go about this a slightly different way.

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- X - the independent variable or explanatory variable or regressor or right-hand-side variable or treatment or predictor
  - Social pressure mailer versus Civic Duty Mailer
  - Applicant race
  - Incarcerated parent

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  - Prediction: for a random sample of the population and given a value of x, $\mu(x)$ is the best predictor in terms of squared error.
  - Causal Inference: with additional assumptions (later in the semester) we can talk about how intervening to change the value of X will change Y.

Reminder of Definitions

Definition (Conditional Expectation (Discrete))

Let Y and X be discrete random variables. The conditional expectation of Y given $X = x$ is defined as:

$$E[Y|X = x] = \sum_y y\, P(Y = y|X = x) = \sum_y y\, p_{Y|X}(y|x)$$

Definition (Conditional Expectation (Continuous))

Let Y and X be continuous random variables. The conditional expectation of Y given $X = x$ is given by:

$$E[Y|X = x] = \int_{-\infty}^{\infty} y\, f_{Y|X}(y|x) dy$$

Review of CEF Concepts

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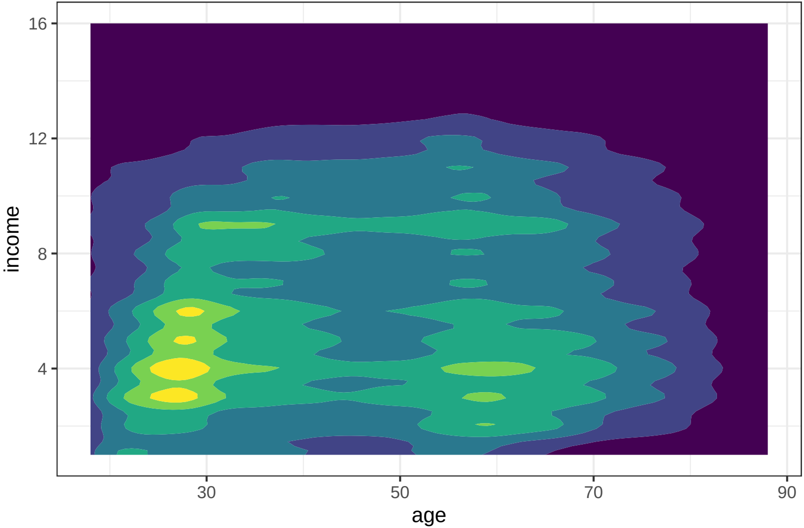

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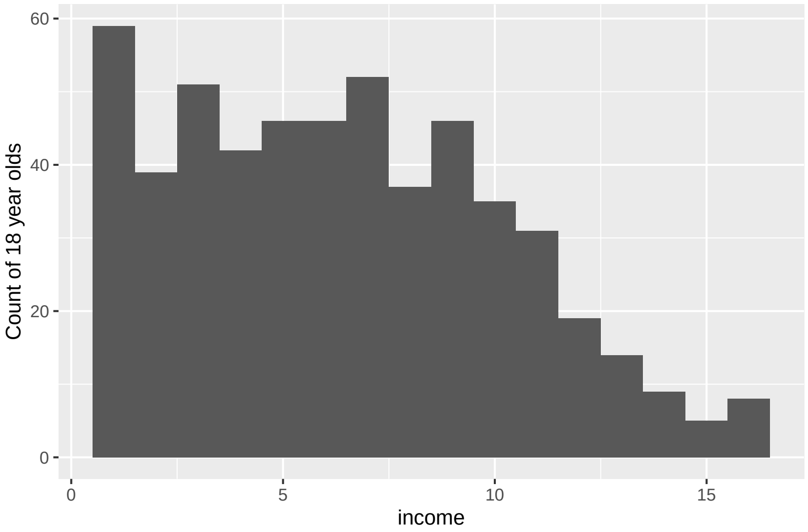

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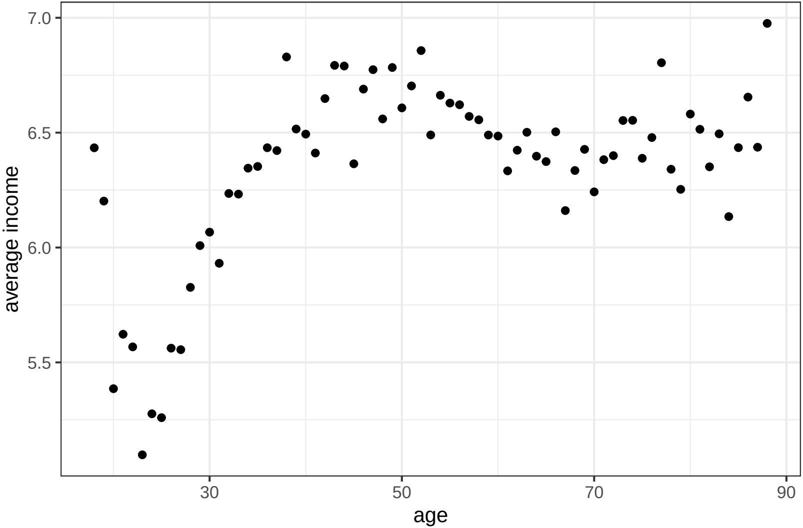

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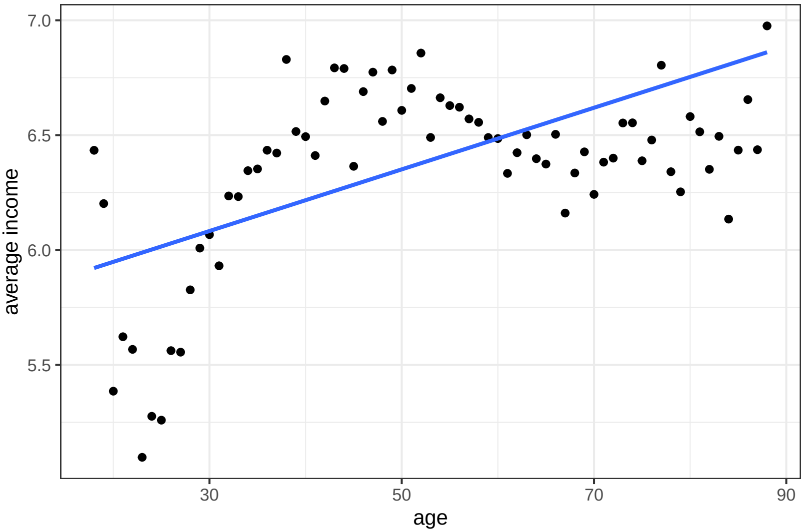

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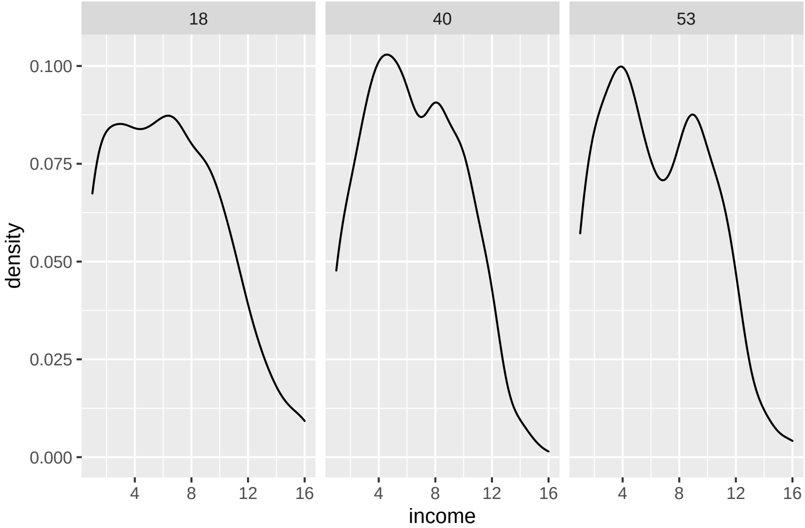

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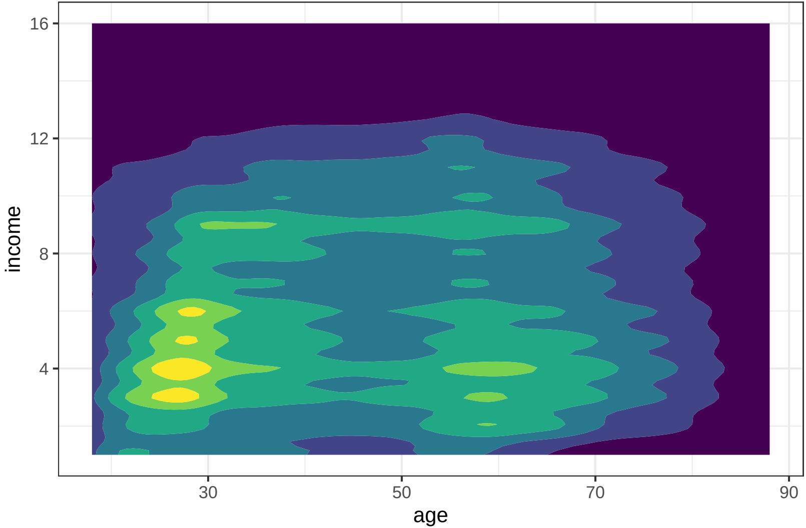

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- and the CEF is the lowest mean squared error predictor of Y_i given X_i

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- Notice here that since X can only take on two values, 0 and 1, then these two conditional means completely summarize the CEF.
- Estimation just involves taking the means within the groups.

Non-binary CEFs

- If X is discrete with not too many categories, we can estimate the conditional expectation using the means within groups:
  - $\hat{\mu}(1) = \hat{E}[Y|X = 1] = \frac{1}{n_{x=1}} \sum_{i:X_i=1} Y_i$
  - $\hat{\mu}(2) = \hat{E}[Y|X = 2] = \frac{1}{n_{x=2}} \sum_{i:X_i=2} Y_i$
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- When X can take on many possible values (think income) or we have few observation for a given value of X, we have to write out a more general function.
- These functional forms are unknown which makes life hard.

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- We use two variables:
  - Y: income
  - X: educational attainment
- Goal is to characterize the conditional expectation $E[Y|X = x]$, i.e. how average income varies with education level

Nonparametric Regression with Discrete X

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`educ`: Respondent's education:

- 1. 8 grades or less and no diploma or
- 2. 9-11 grades
- 3. High school diploma or equivalency test
- 4. More than 12 years of schooling, no higher degree
- 5. Junior or community college level degree (AA degrees)
- 6. BA level degrees; 17+ years, no postgraduate degree
- 7. Advanced degree

Nonparametric Regression with Discrete X

`income`: Respondent's family income:

- 1. None or less than \$2,999
- 2. \$3,000-\$4,999
- 3. \$5,000-\$6,999
- 4. \$7,000-\$8,999
- 5. \$9,000-\$9,999
- 6. \$10,000-\$10,999

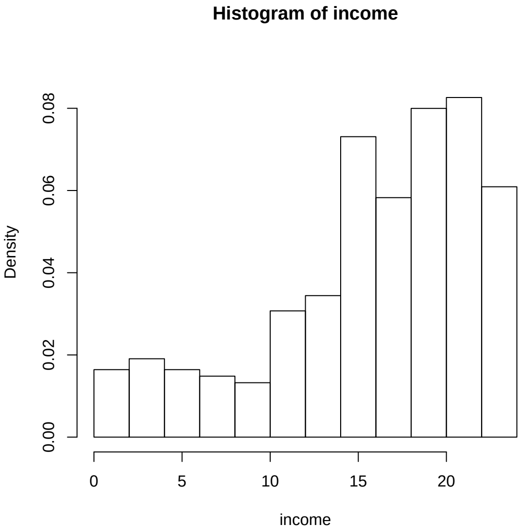
$\vdots$

- 17. \$35,000-\$39,999
- 18. \$40,000-\$44,999

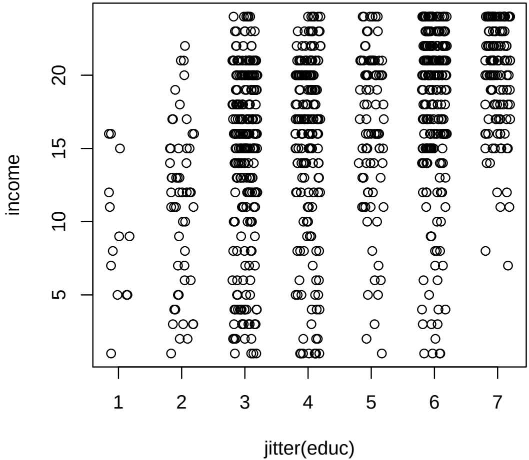
$\vdots$

- 23. \$90,000-\$104,999
- 24. \$105,000 and over

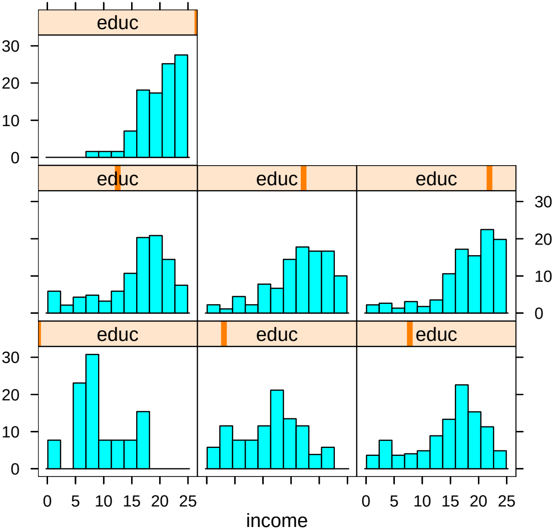
Marginal Distribution of Y (income)



Joint Distribution of X and Y (Income and Education)



Distribution of income given education $p(y|x)$



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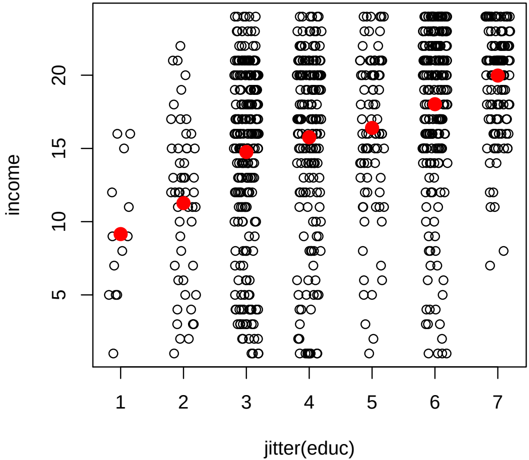

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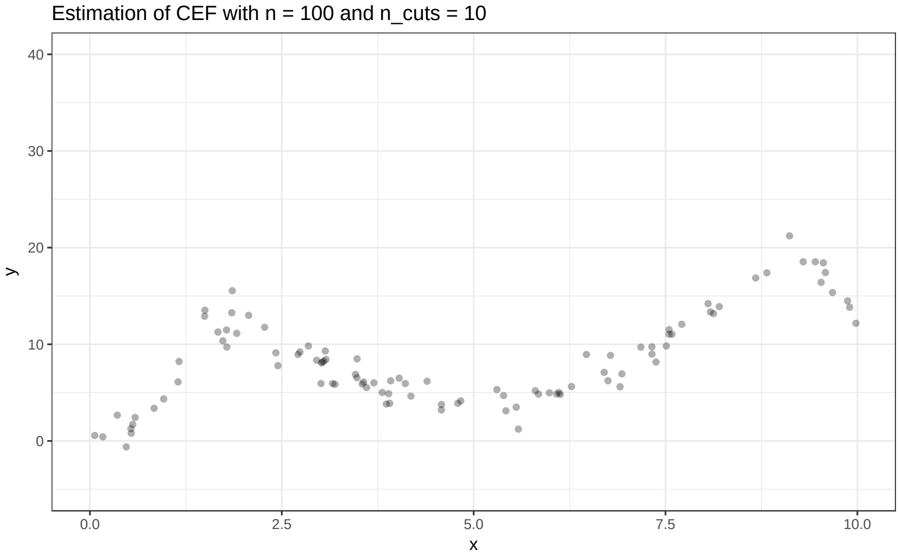
- This approach works well as long as
  - X is discrete
  - there are a small number values of X
  - a small number of X variables
  - a lot of observations at each X value
- But what do we do when X is continuous and has many values?
- Let's talk through a few options.

A Binning Approach to CEF for continuous random variables



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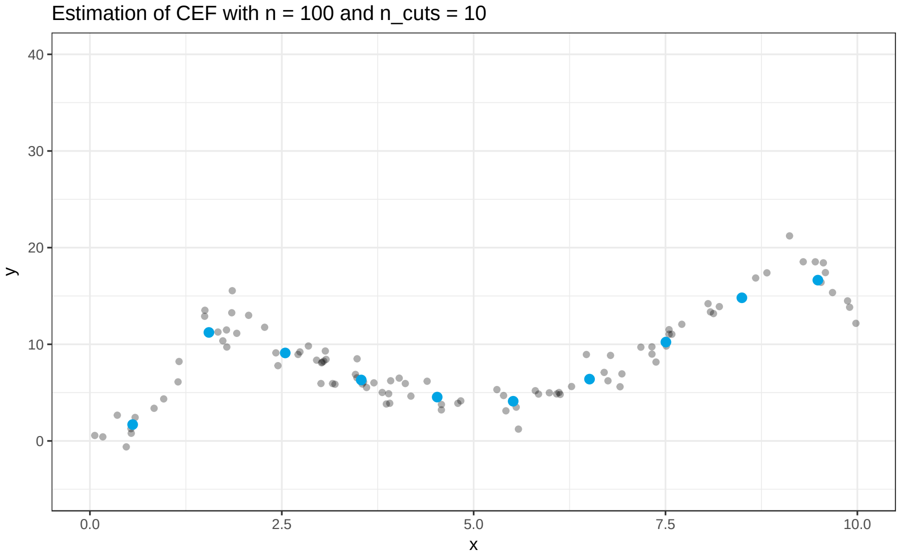

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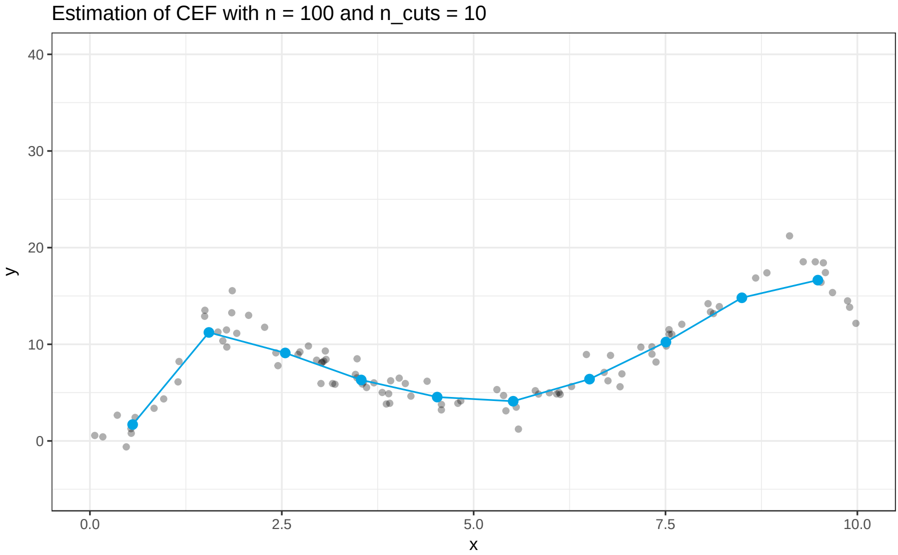

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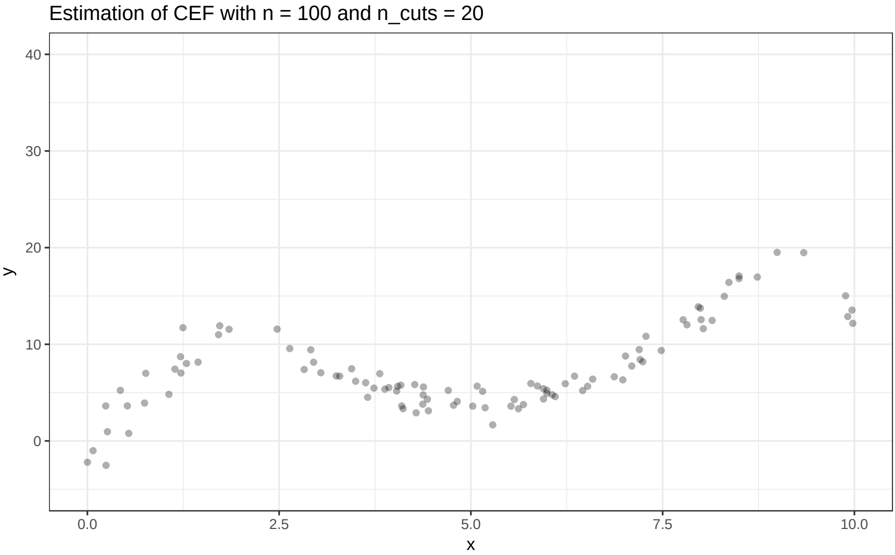

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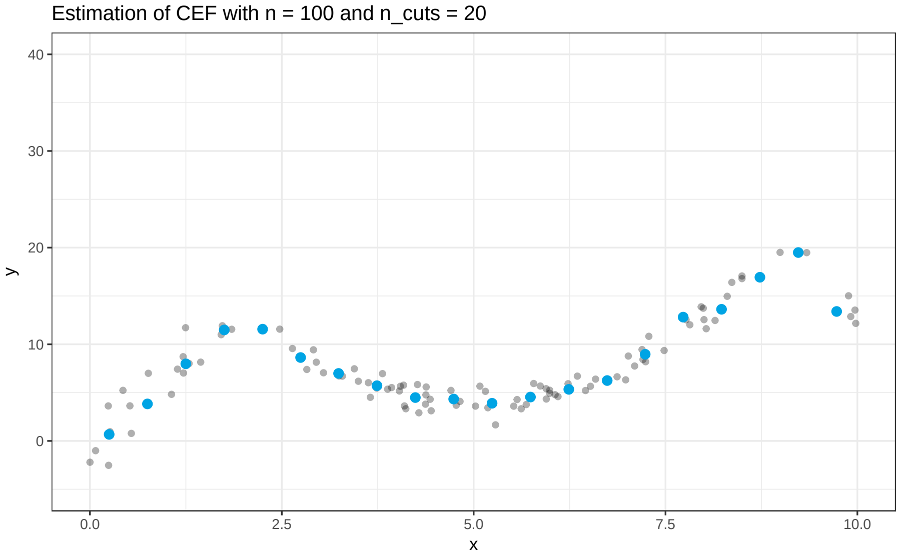

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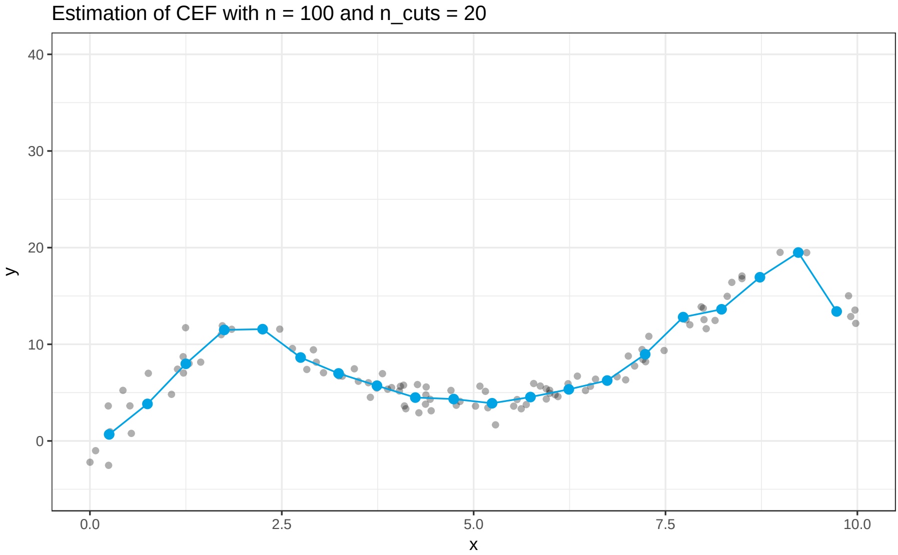

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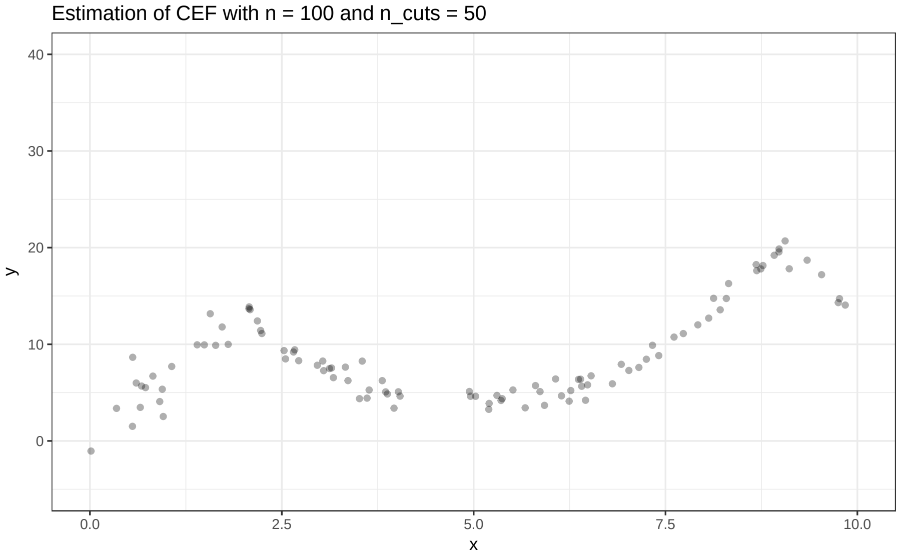

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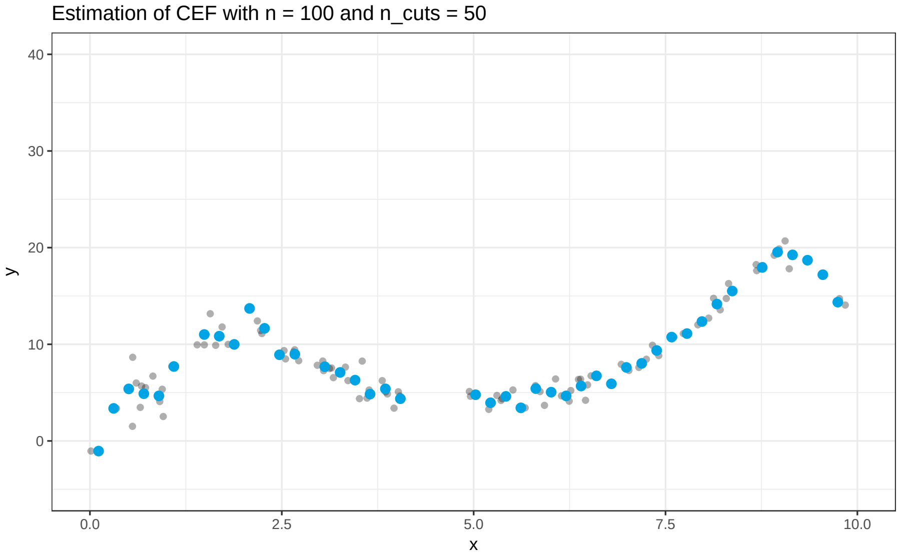

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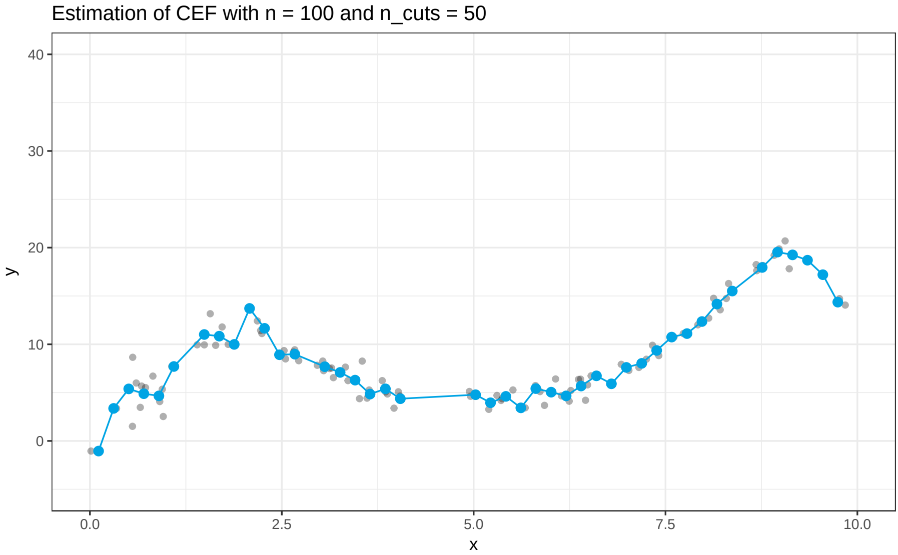

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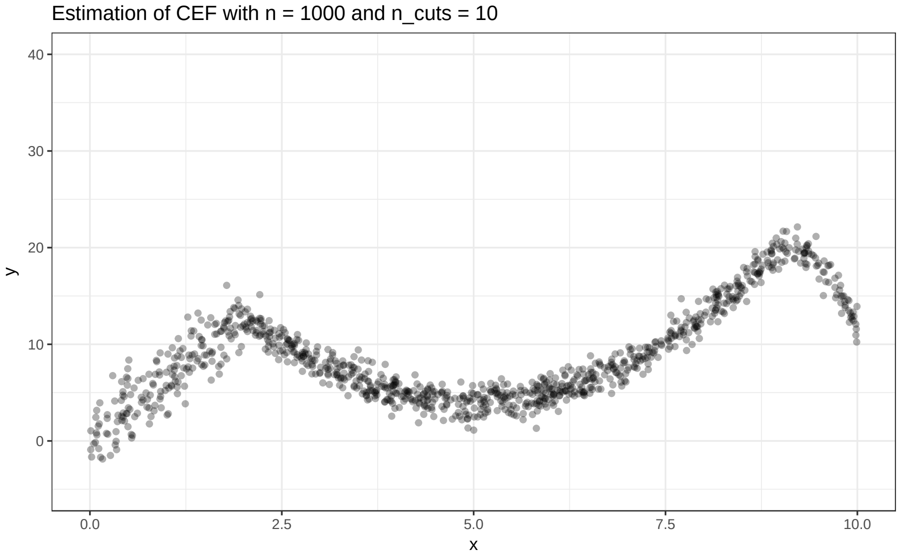

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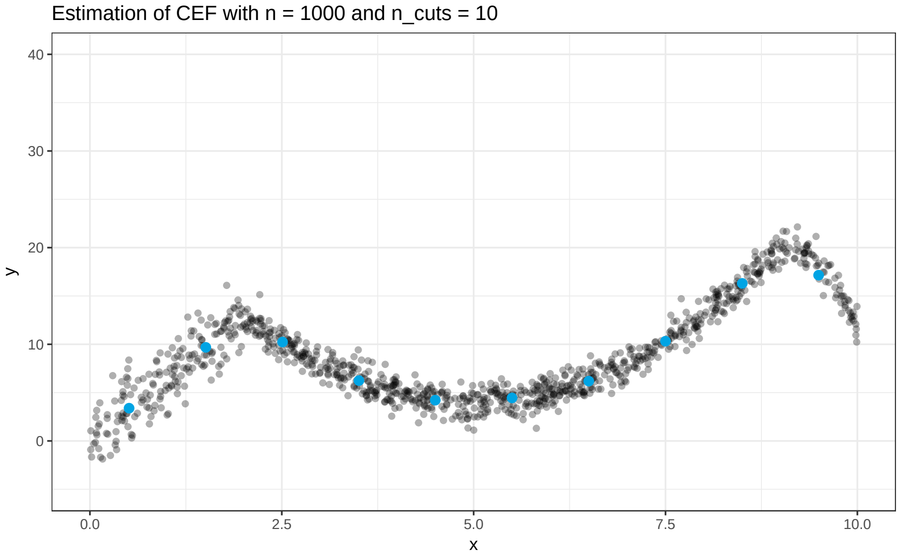

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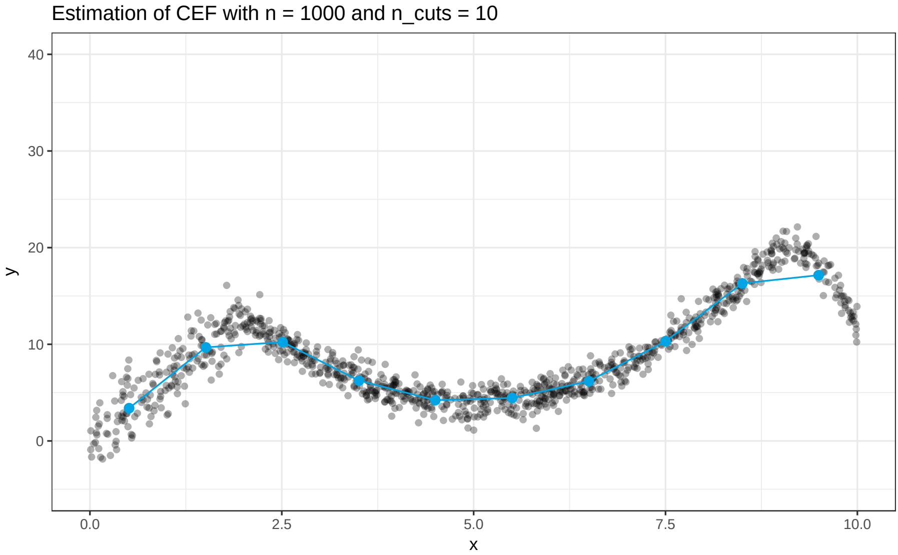

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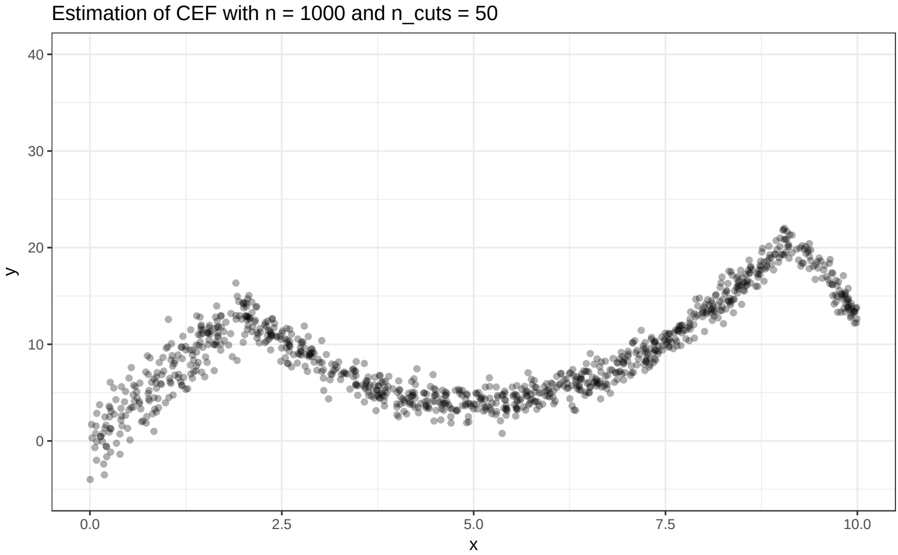

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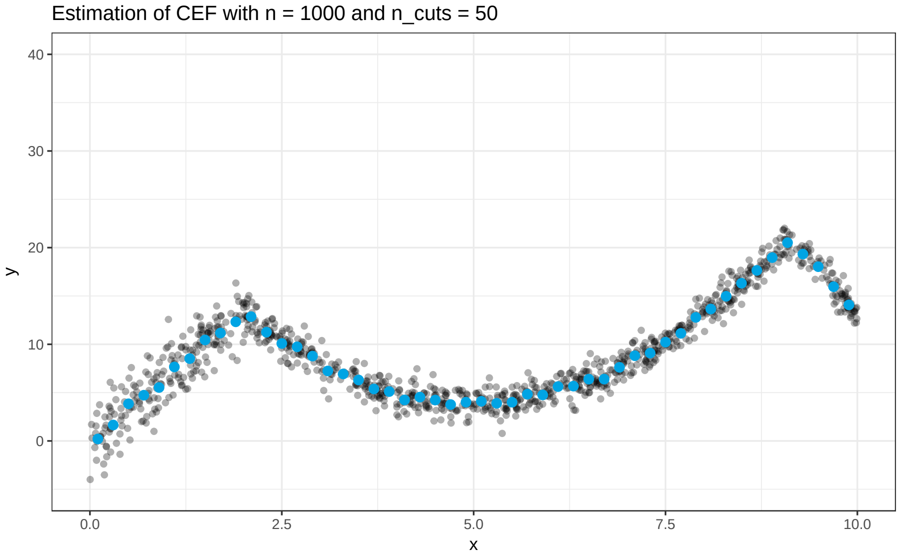

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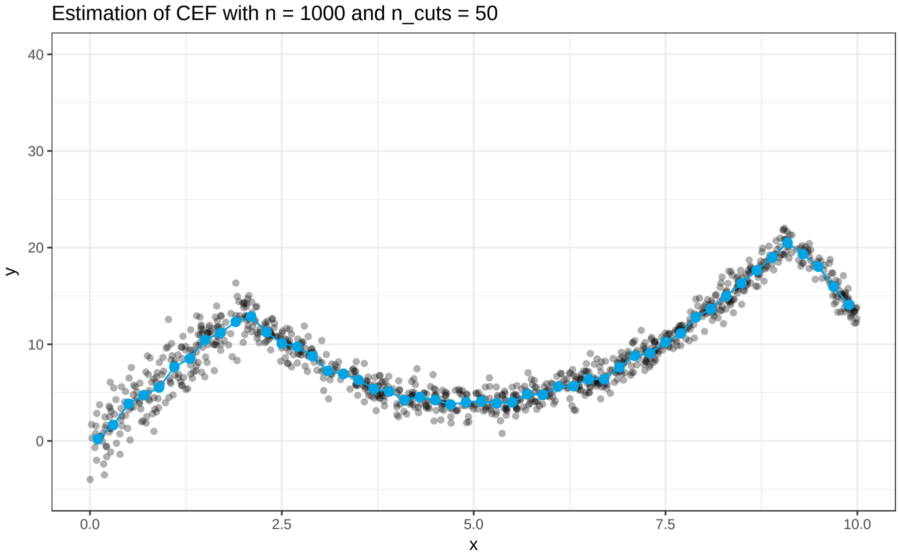

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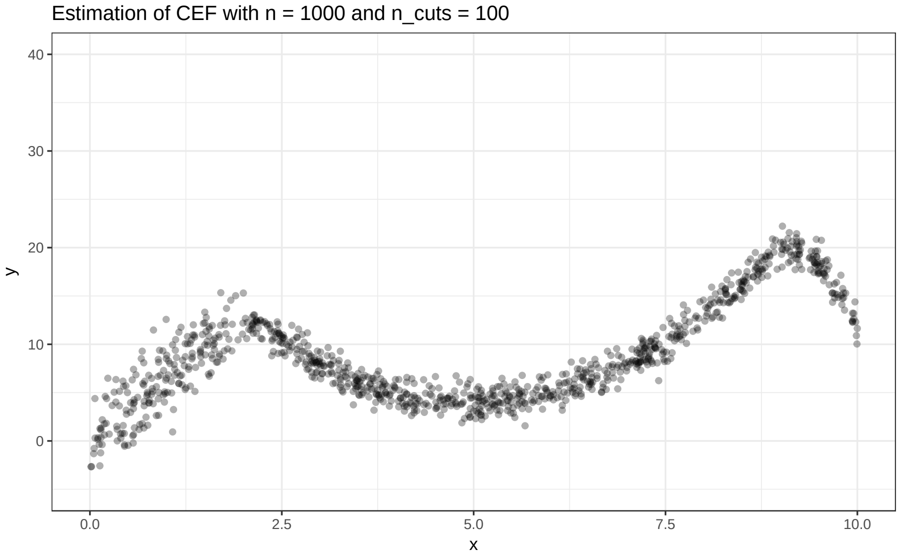

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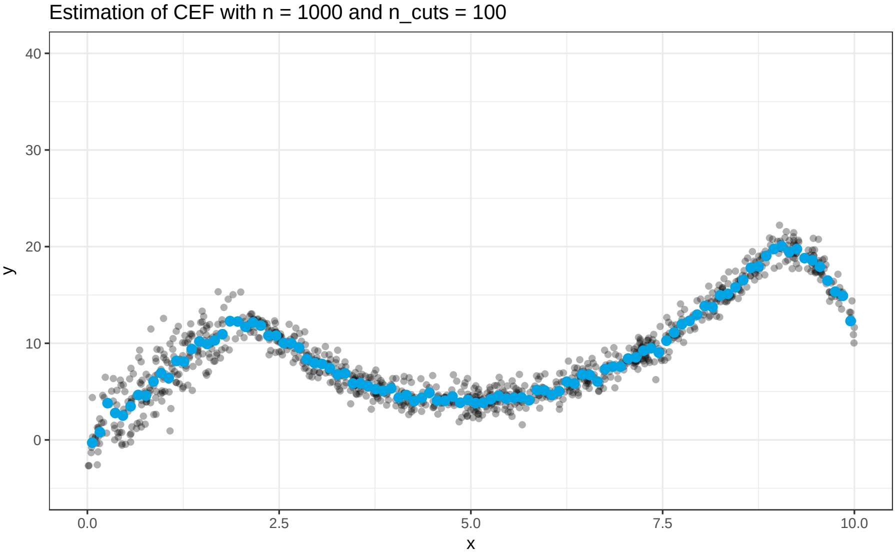

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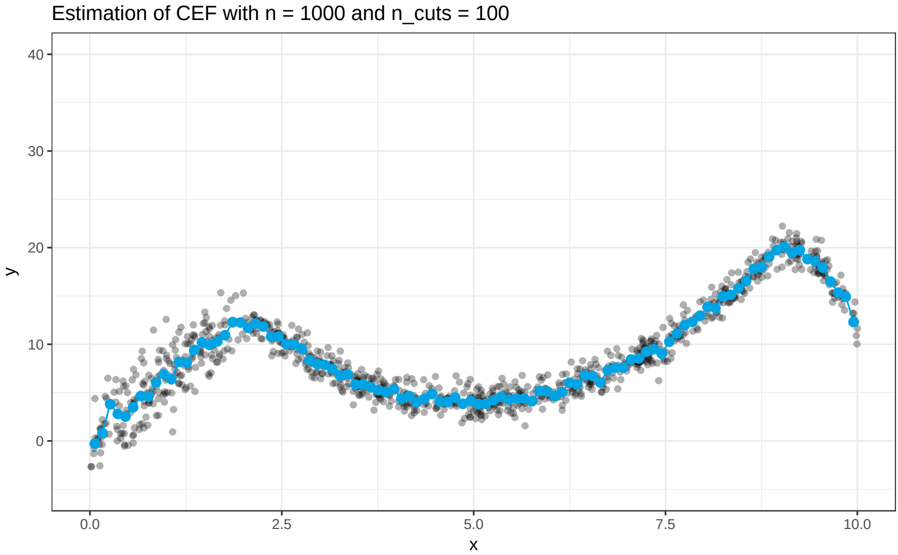

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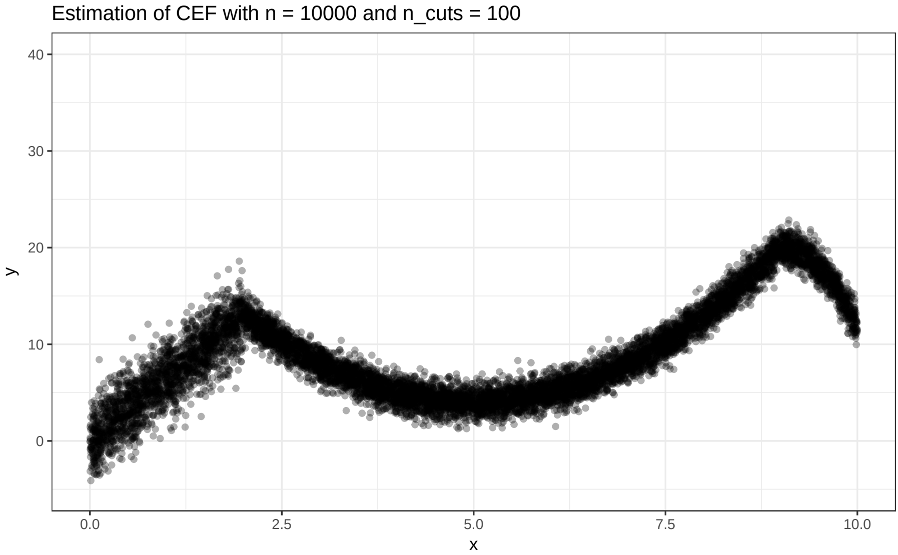

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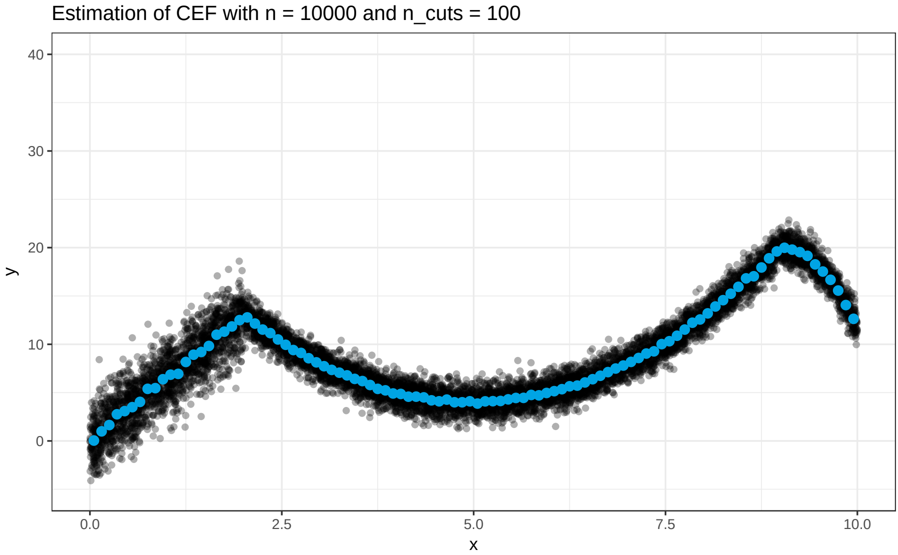

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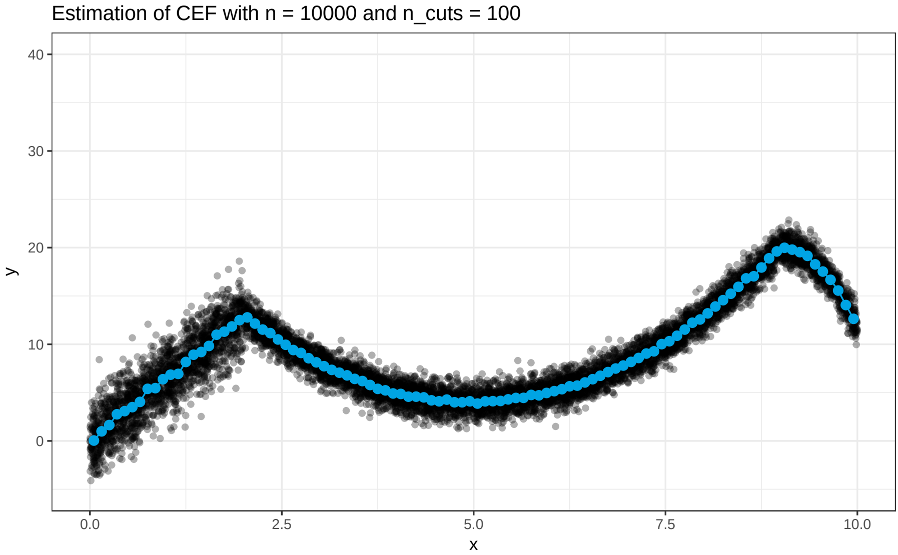

Uniform Kernel Regression: Simple Local Averages

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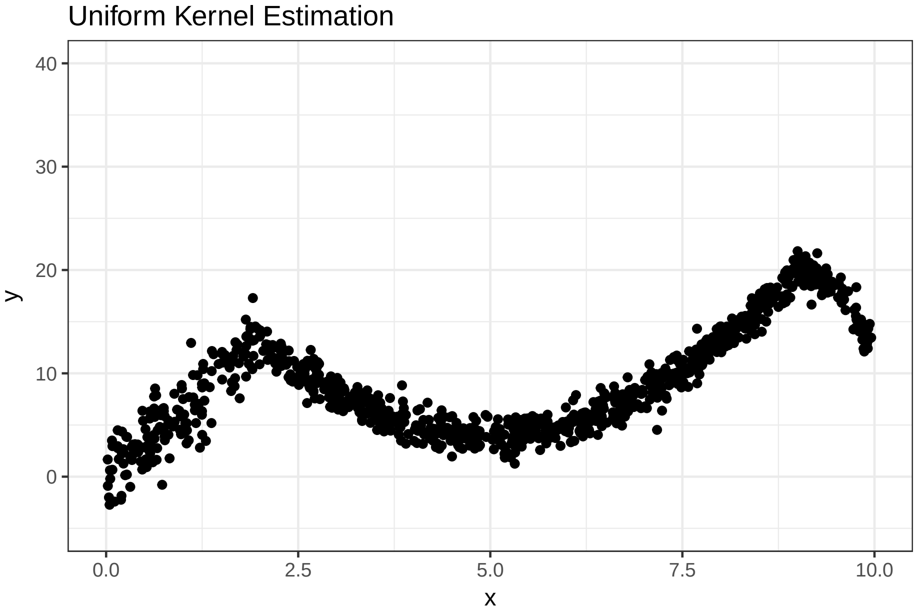
- We will call this approach uniform kernel regression for a reason that will become clear shortly.

Example of Kernel CEF Estimation



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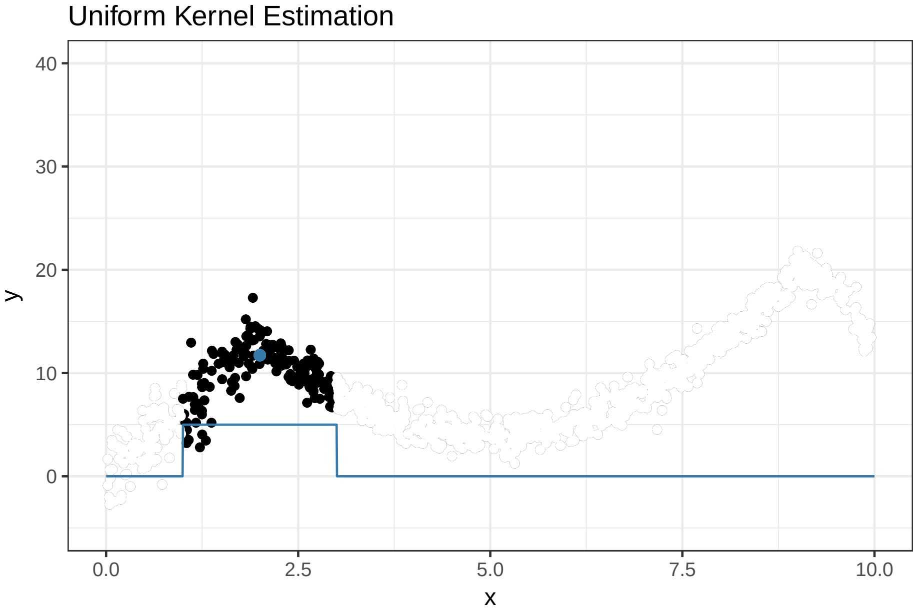

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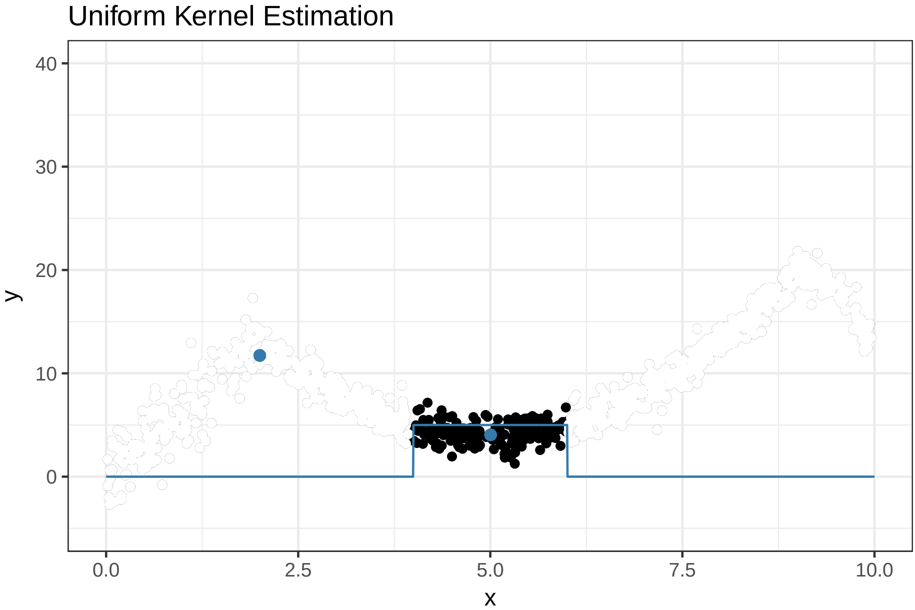

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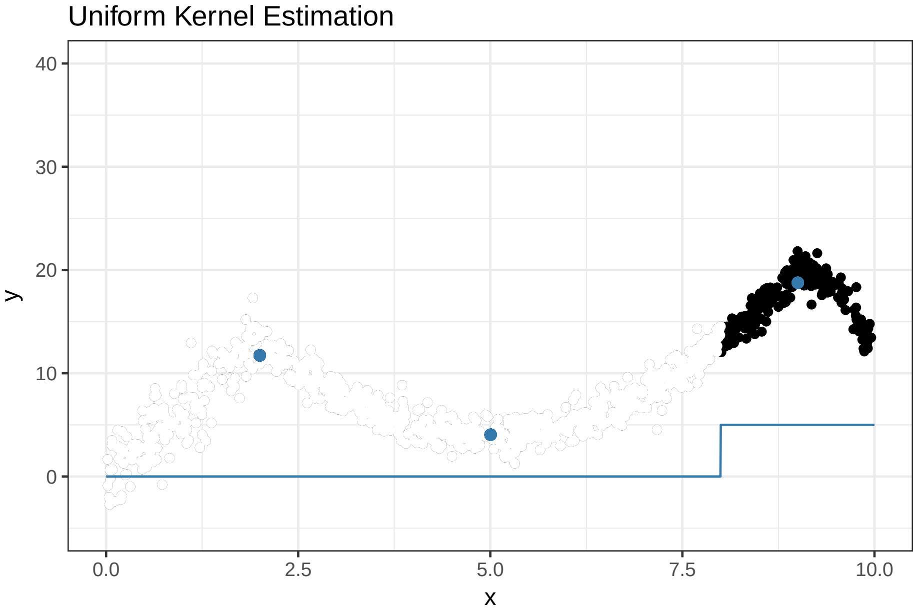

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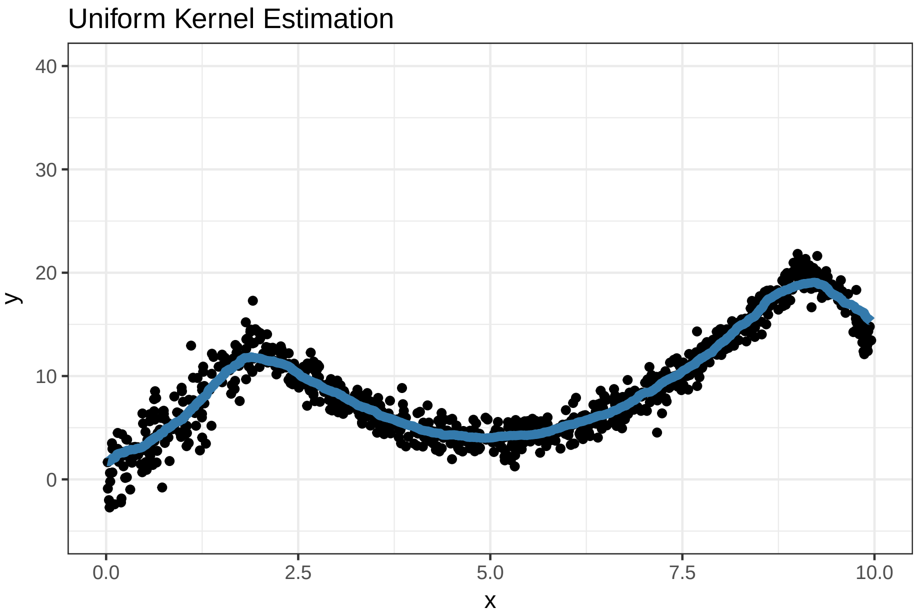

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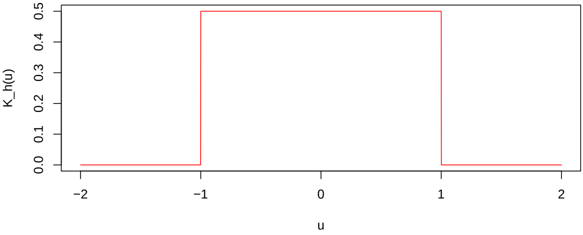

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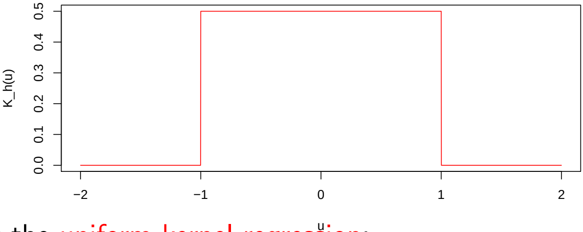
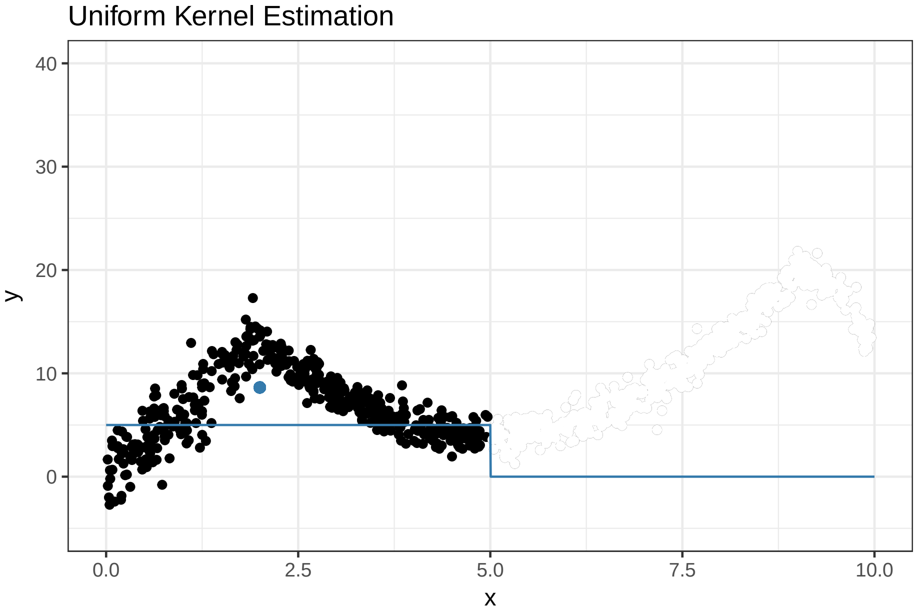


- This gives the uniform kernel regression:

$$\widehat{E}[Y|X = x_0] = \frac{\sum_{i=1}^{N} K_h((X_i - x_0)/h)Y_i}{\sum_{i=1}^{N} K_h((X_i - x_0)/h)} \text{ where } K_h(u) = \frac{1}{2}\,\mathbf{1}_{\{|u|\leq 1\}}$$

Changing the Bandwidth



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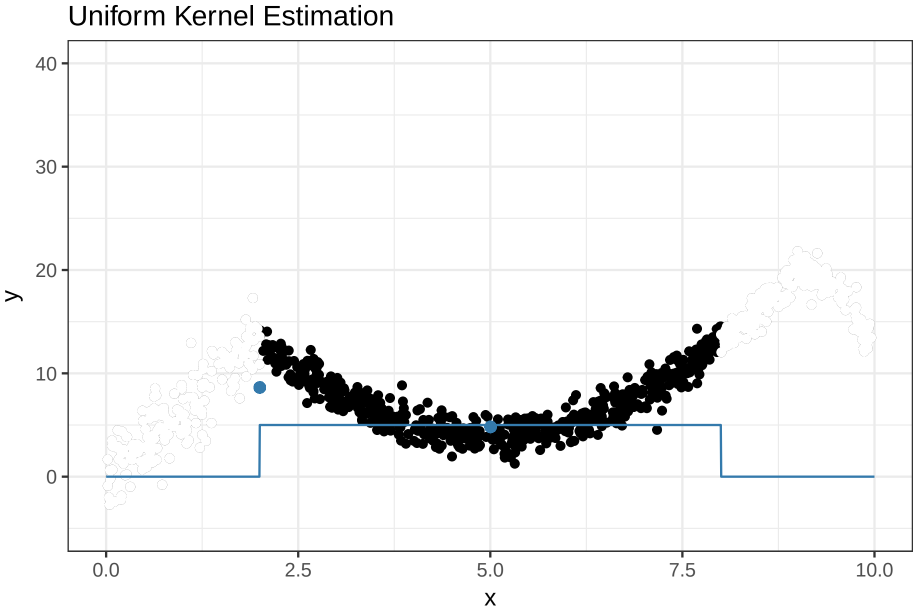

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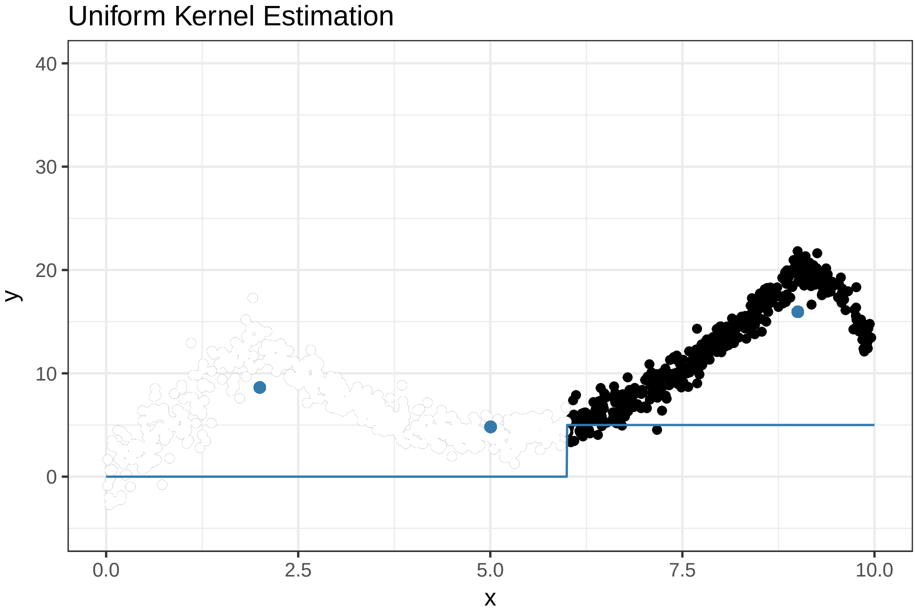

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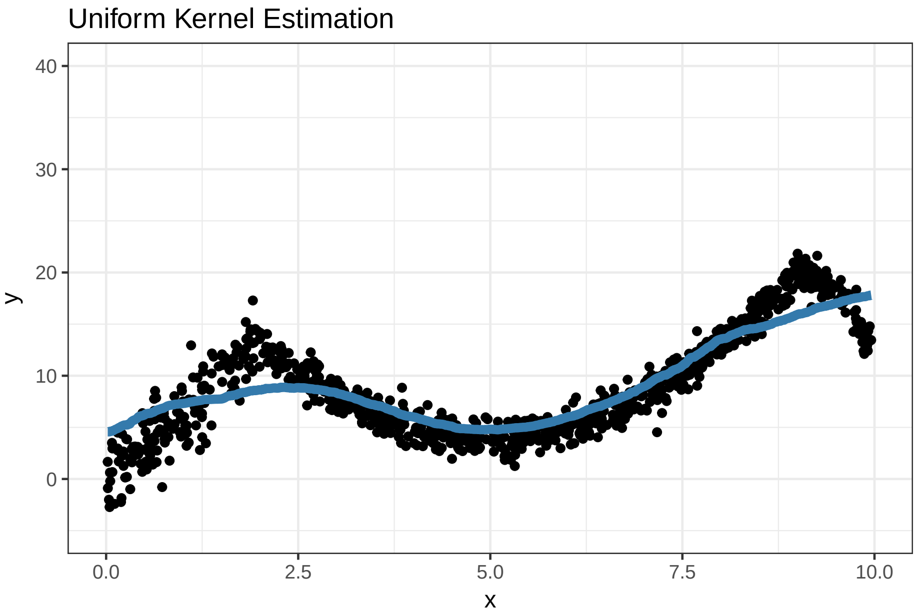

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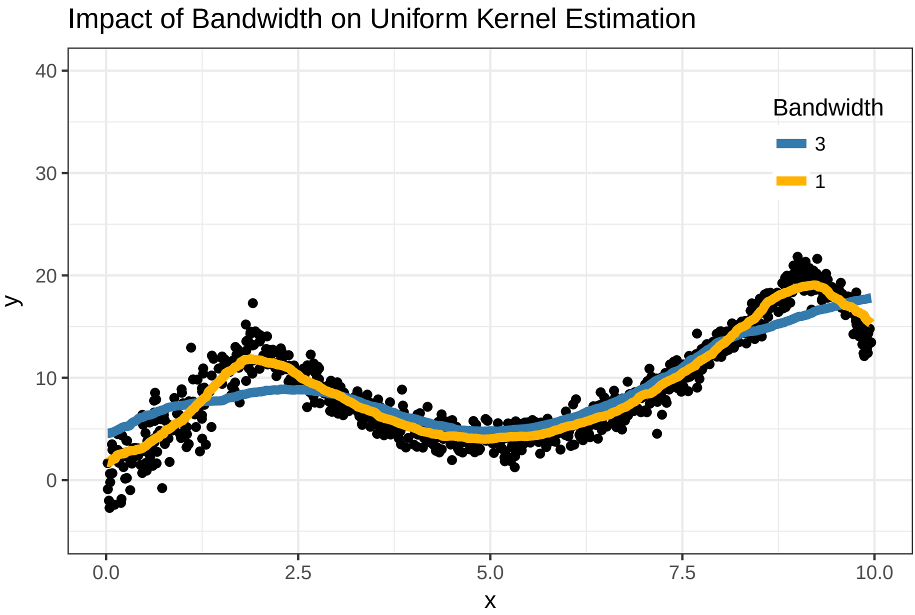

Uniform Kernel Regression: Properties

Theorem (Consistency of the Uniform Kernel Density Estimator)

For iid continuous random variables $X_1, X_2, \ldots X_n$, $\forall x \in \mathbb{R}$,

- *if the kernel is uniform, and*
- *if $h \to 0$ and*
- *$nh \to \infty$ as*
- *$n \to \infty$, then*

$$\hat{f}_K(x) \xrightarrow{p} f(x).$$

Aronow and Miller Theorem 3.3.8. Proof by weak law of large numbers and the plug-in principle.

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Let $K : \mathbb{R} \to \mathbb{R}$ be a function which is symmetric about the y-axis satisfying $\int_{-\infty}^{\infty} K(x)dx = 1$, and let $K_h(x) = \frac{1}{h}K(\frac{x}{h}) \forall x \in \mathbb{R}$ and $h > 0$.

The More General Form of the Estimator

Definition (Kernel Density Estimator)

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Then a kernel density estimator of $f(x)$ is

$$\hat{f}_K(x) = \frac{1}{n}\sum_{i=1}^{n} K_h(x - X_i), \forall x \in \mathbb{R}$$

The function K is called the kernel and the scaling parameter h is called the bandwidth.
(Aronow and Miller Definition 3.3.7)

Kernel Estimation

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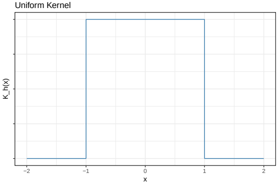
- Uniform Kernel
- Calculate the average of the observed y points that have x values in the interval $[x_0 - h, x_0 + h]$
- Each observation within the interval is given equal weight, each observation outside the interval is given 0 weight



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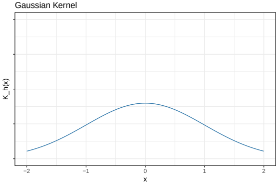
- Gaussian Kernel
- Distance weighted by how far from x_0 following the normal density

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$



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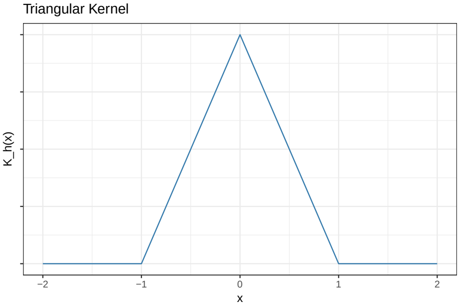
- Distance weighted by how far from x_0 using linear distance



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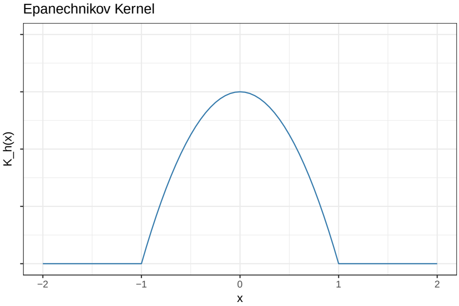
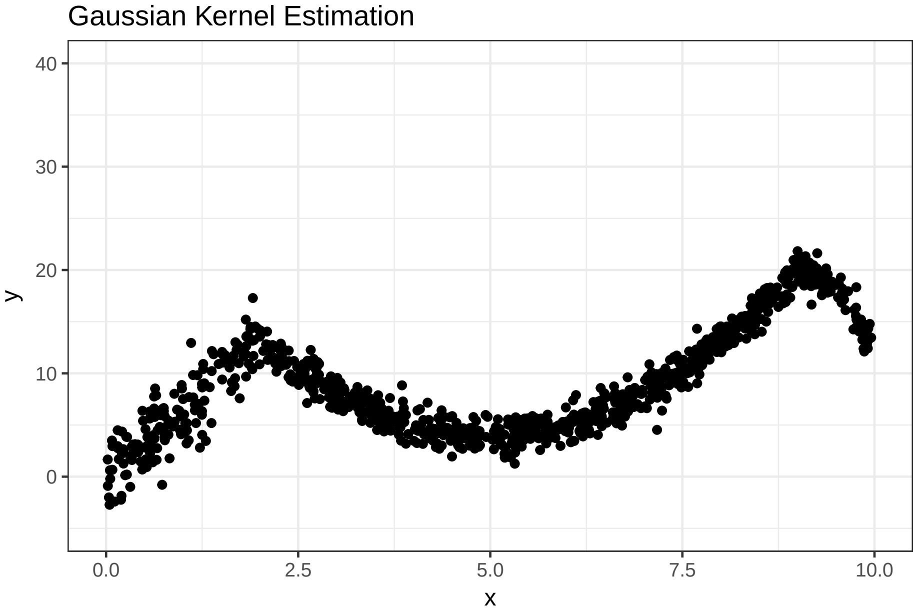


Example of Gaussian Kernel



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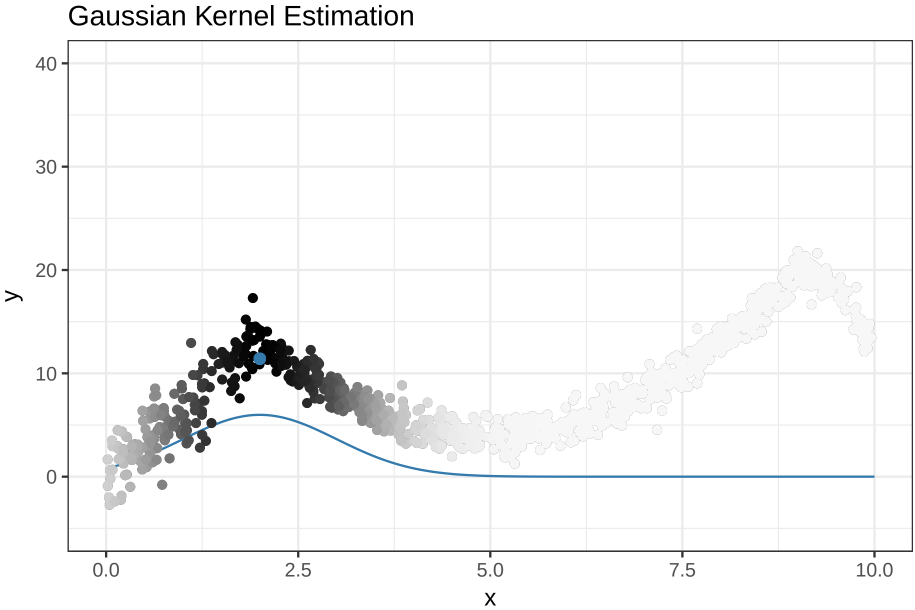

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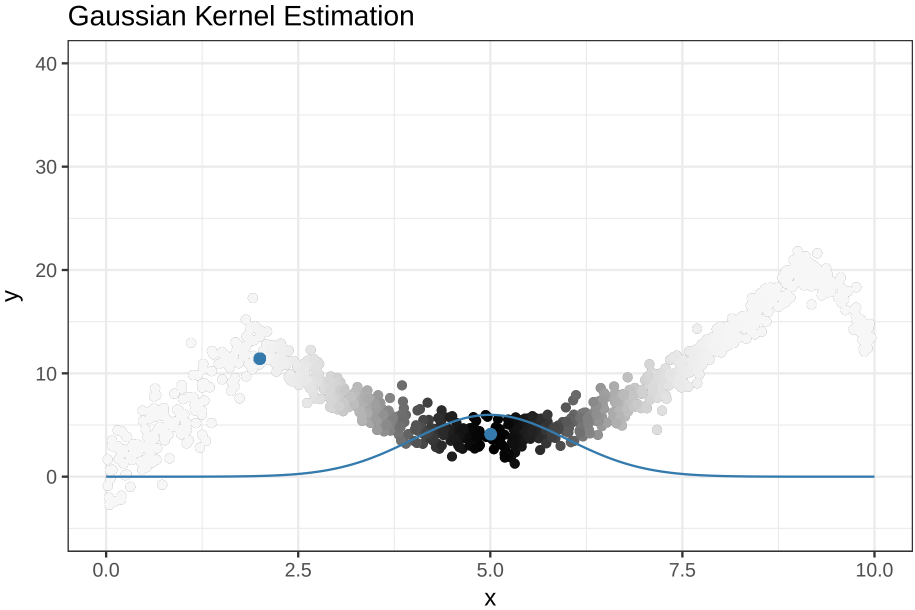

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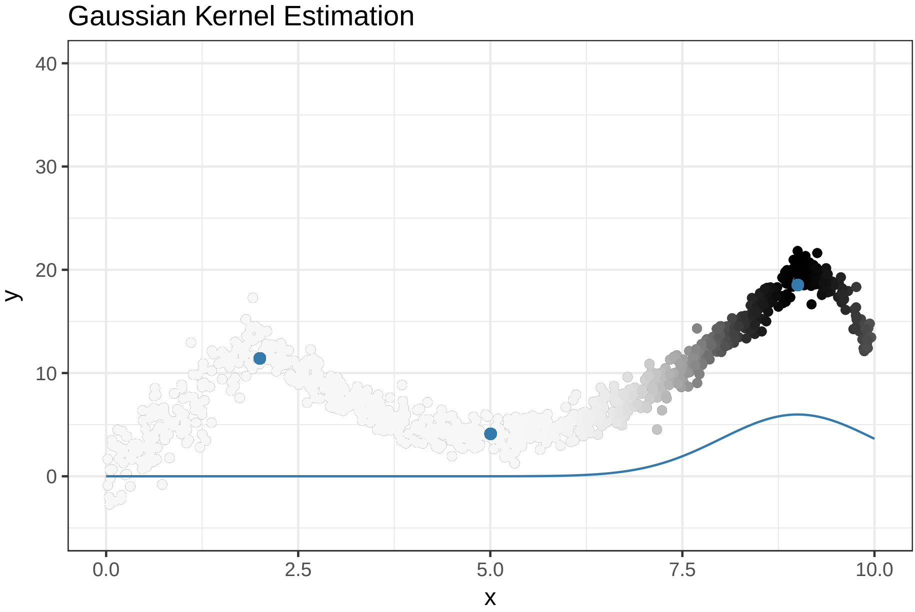

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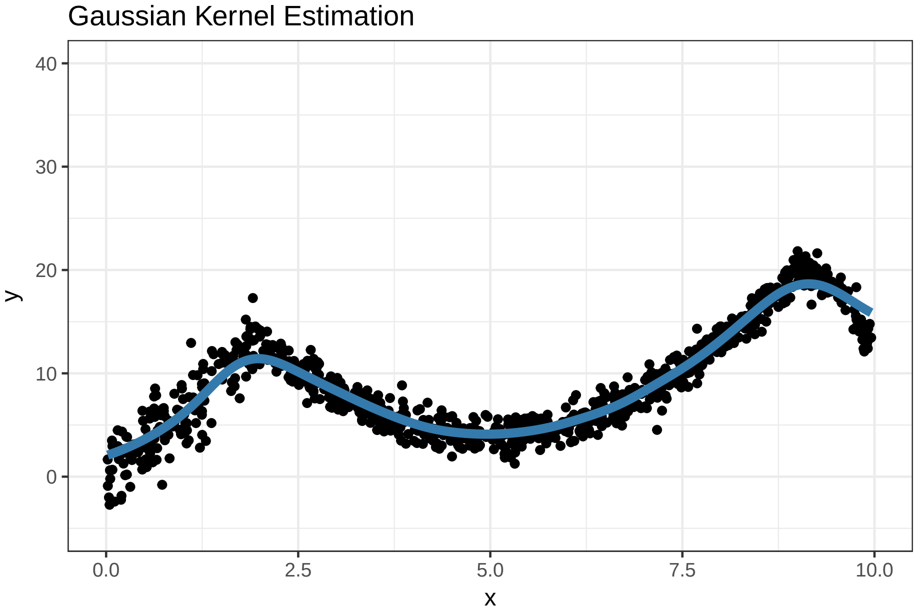

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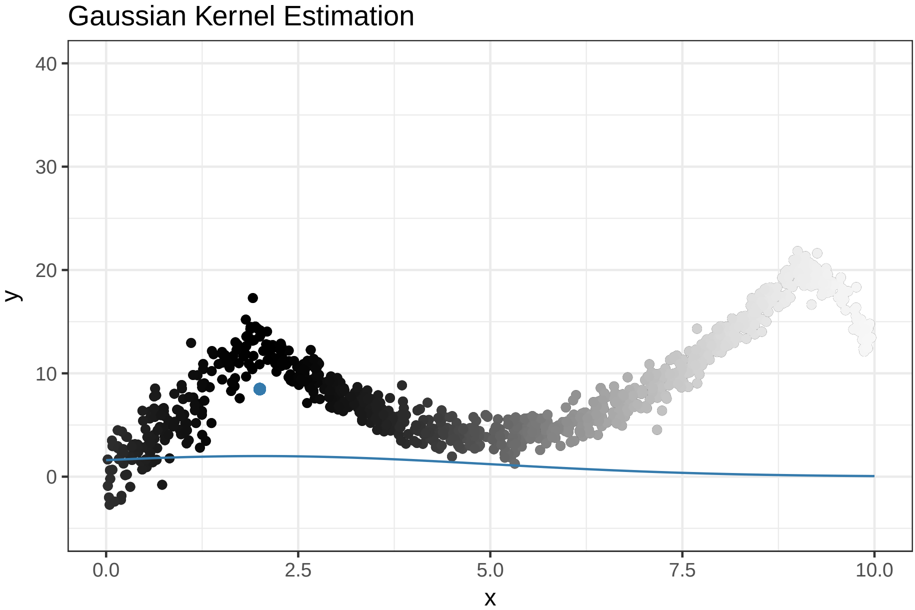

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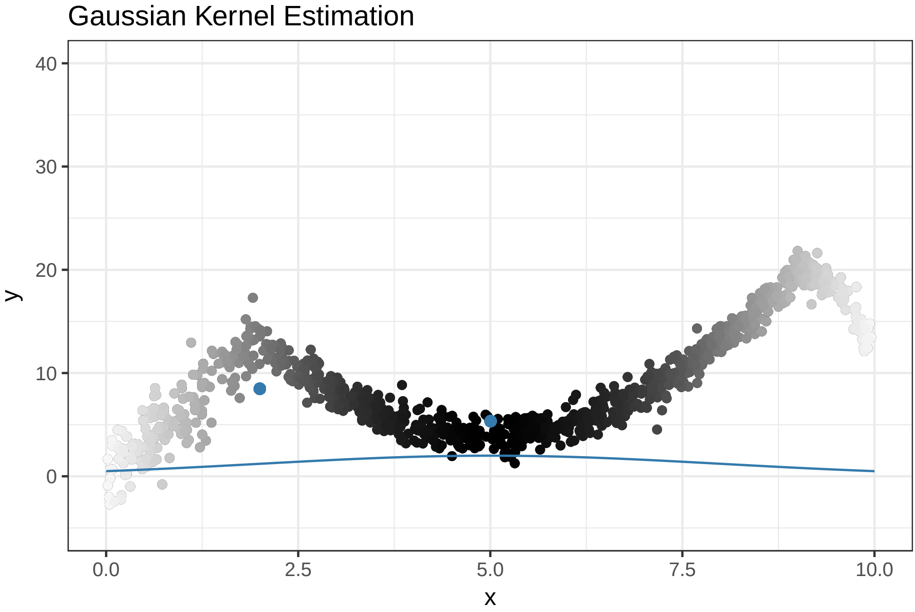

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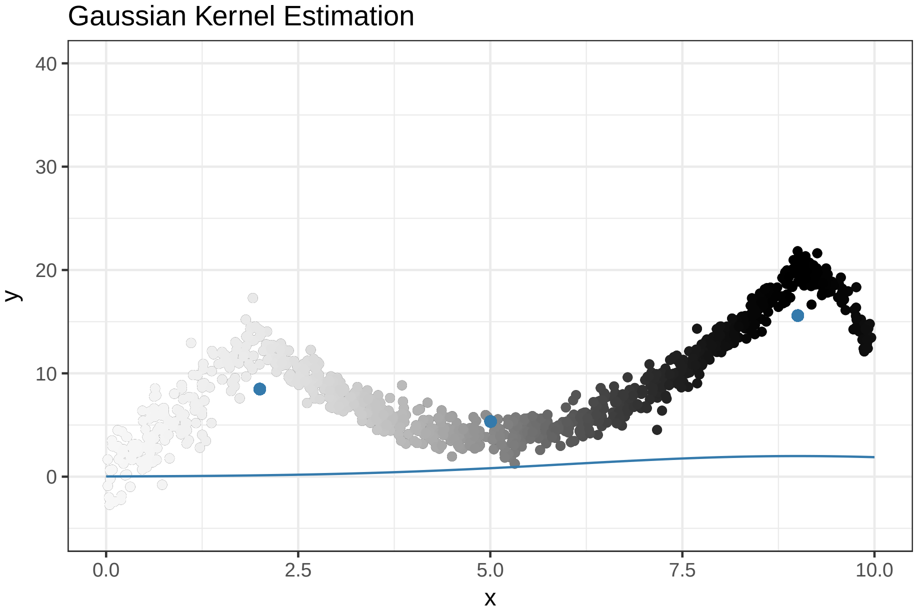

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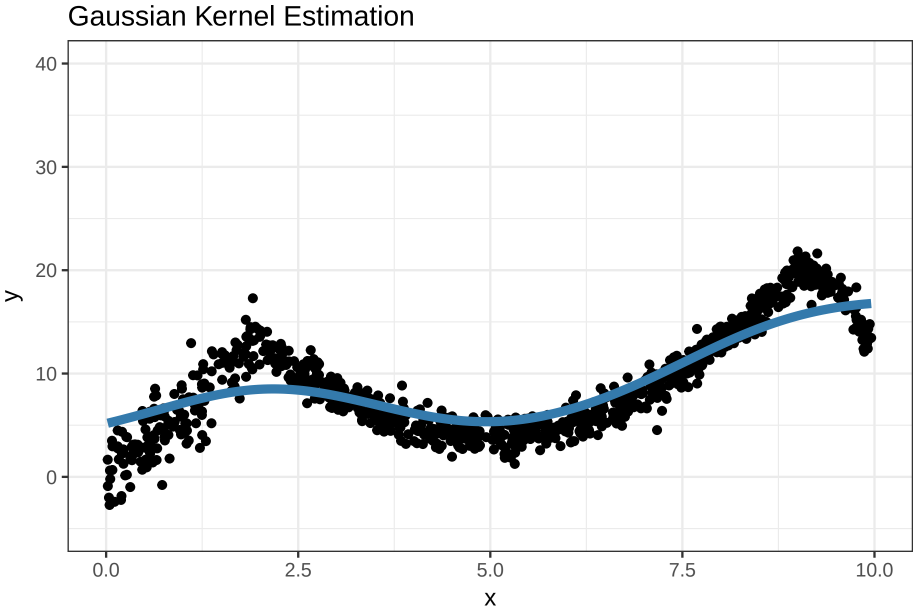

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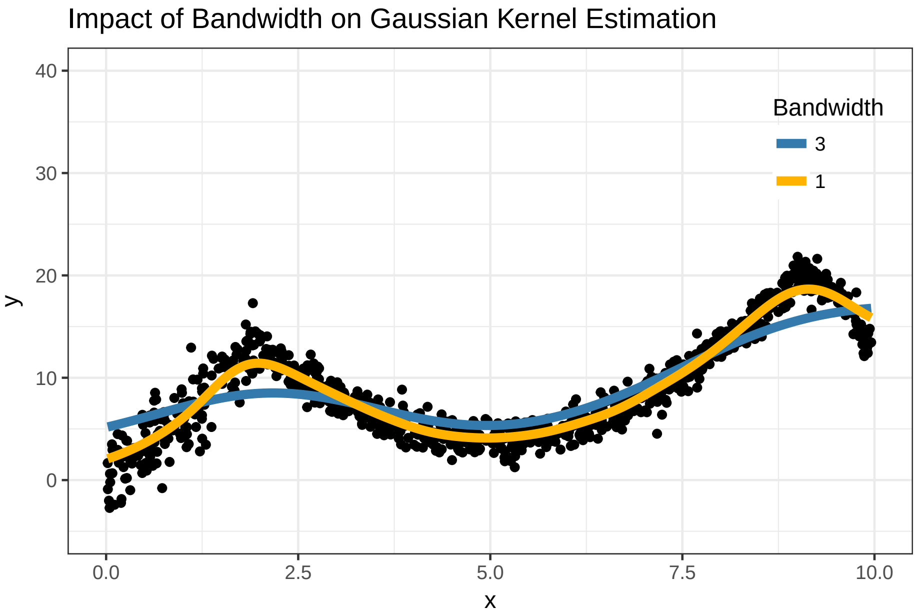

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  - A very flexible estimator allows the shape of the function to vary (e.g. a kernel regression with a small bandwidth)
  - A very inflexible estimator restricts the shape of the function to a particular form
    (e.g. a kernel regression with a very wide bandwidth)

Best Linear Predictor

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- We call this the linear regression line
- What function minimizes MSE, among this class of functional forms?

Best Linear Predictor

Theorem

For a random variable X and Y, if $V[X] > 0$, then the best linear predictor (BLP) of Y given X is $g(X) = \alpha + \beta X$ where,

$$\alpha = E[Y] - \frac{Cov[X, Y]}{V[X]} E[X]$$

$$\beta = \frac{Cov(X, Y)}{V(X)}$$

Corollary

- *The BLP is the best linear predictor of the CEF. I.e. setting $a = \alpha$ and $b = \beta$ minimizes*

$$E\left[(E[Y \mid X] - (a + bX))^2\right]$$

- *If the CEF is linear, the CEF is the BLP*

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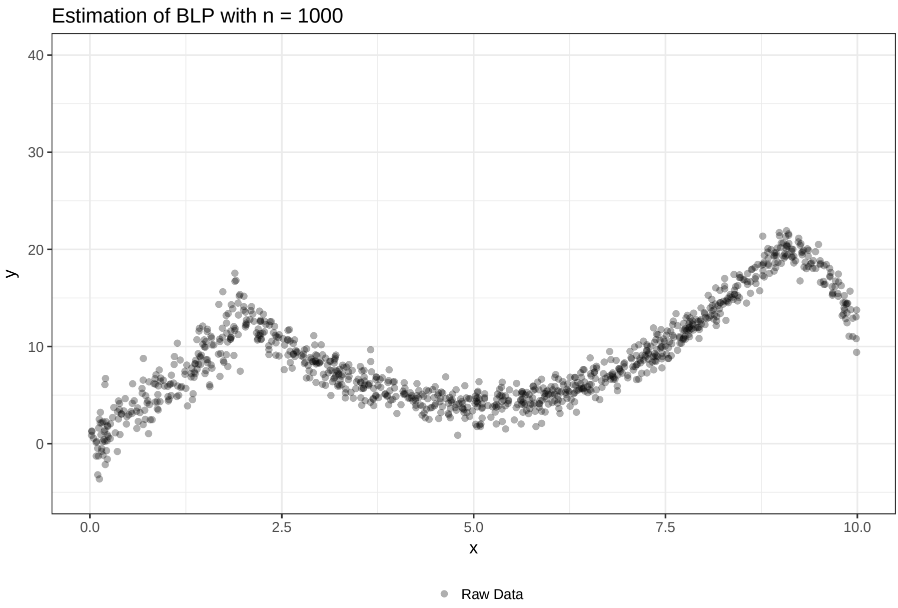

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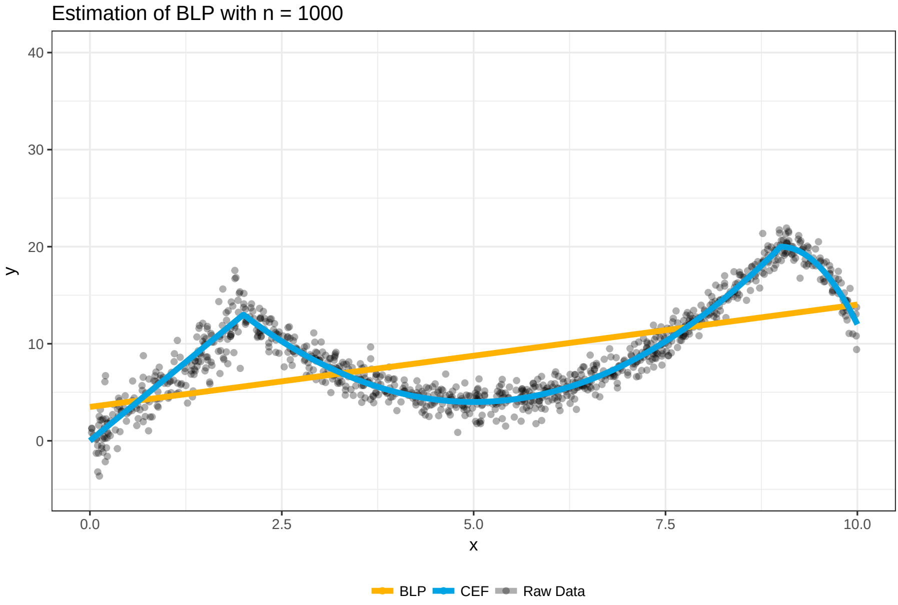

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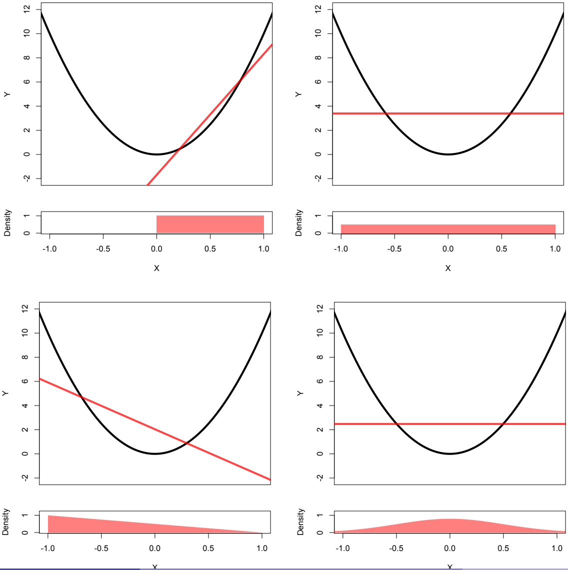
  - Linear projection is the best predictor among linear functions.
- The nice thing about the linear projection is that it exists and is well-defined even if the CEF is non-linear.

BLP Approximations Depend on the Marginal Distribution of X



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The BLP is always a line regardless of the data.

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- This is called the ordinary least squares (OLS) estimator.

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Noting we can rewrite $\frac{Cov[X,Y]}{V[X]} = \frac{E[XY]-E[X]E[Y]}{E[X^2]-E[X]^2}$, and using the plug-in principle, we can estimate the parameters of the BLP as:

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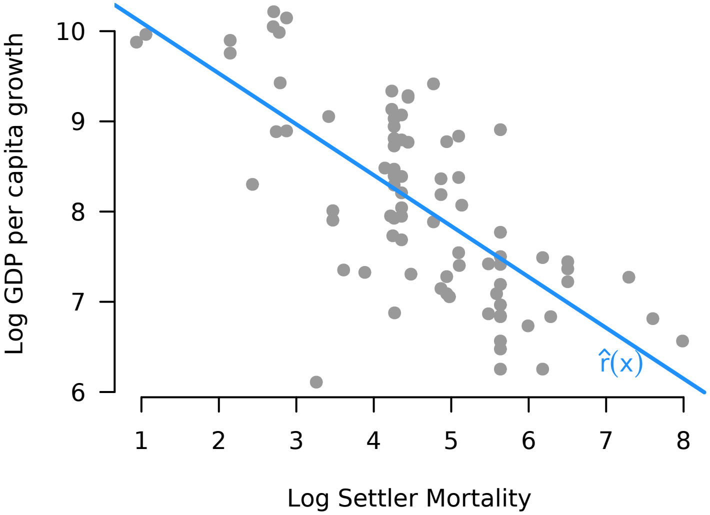
This corresponds to the linear projection which minimizes the sum of squared errors.

Fitted linear CEF/regression function



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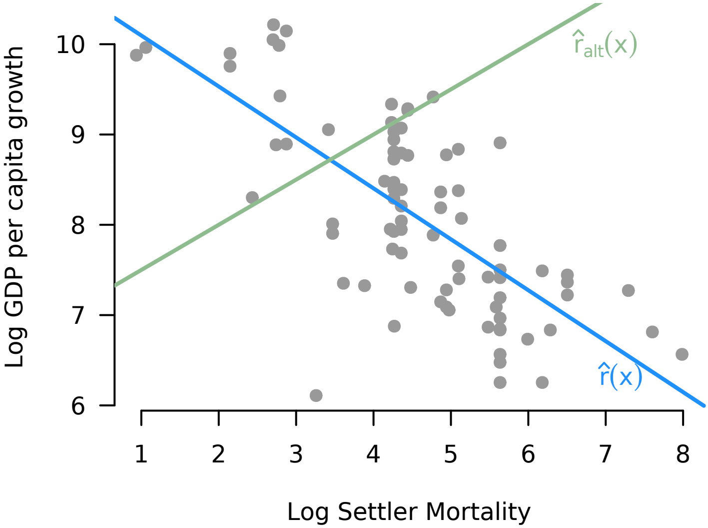

Fitted values and residuals

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A **fitted value** or **predicted value** is the estimated conditional mean of Y_i for a particular observation with independent variable X_i:

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Definition (Residual)

The **residual** is the difference between the actual value of Y_i and the predicted value, $\widehat{Y}_i$:

$$\widehat{u}_i = Y_i - \widehat{Y}_i = Y_i - \widehat{\beta}_0 - \widehat{\beta}_1 X_i$$

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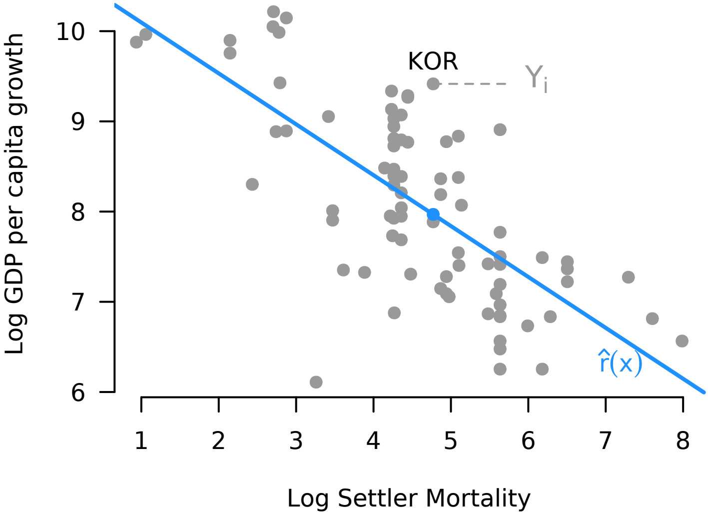

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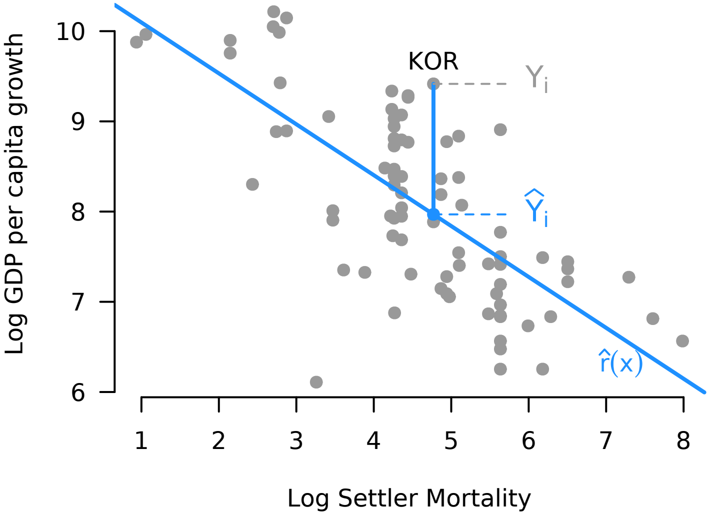

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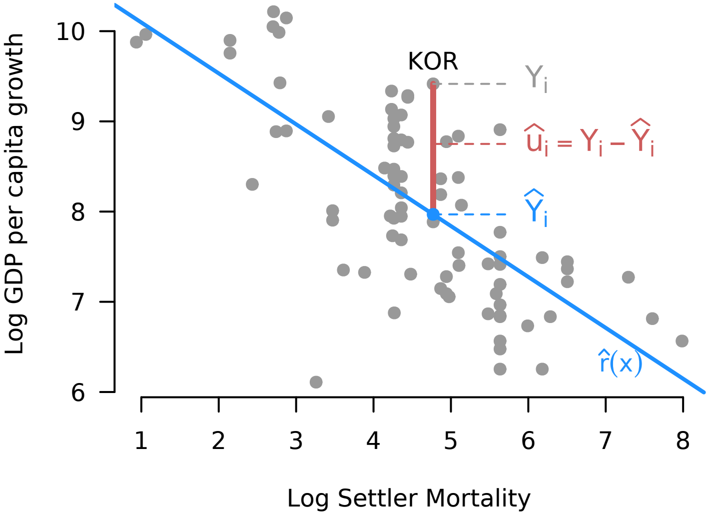
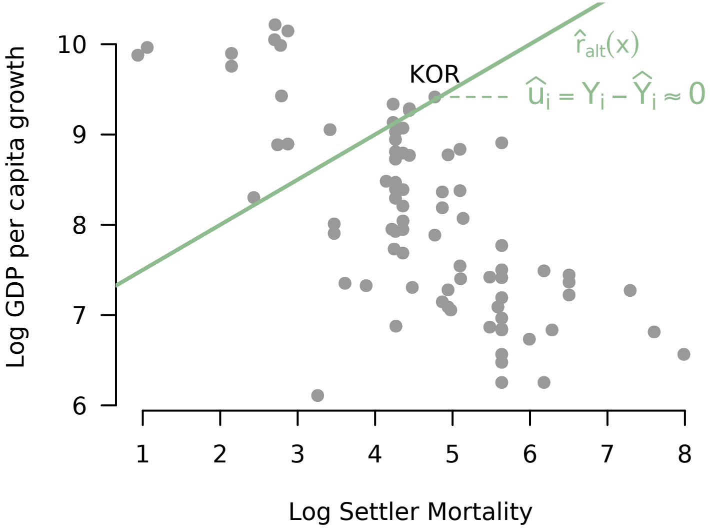


Why not this line?



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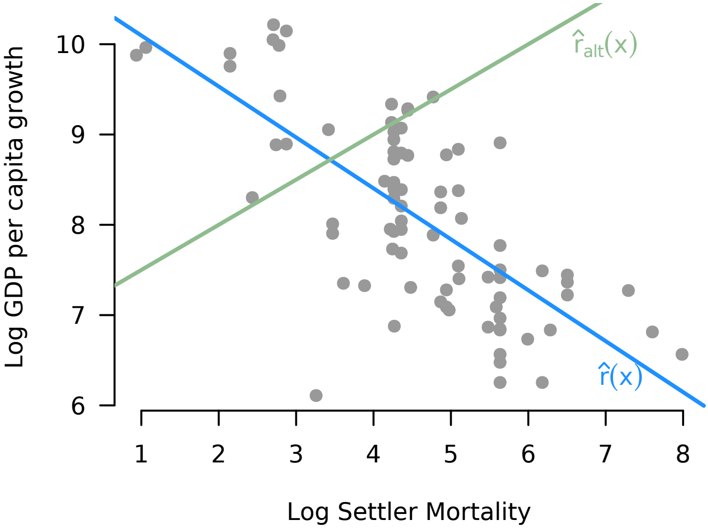
- Choose the line that minimizes the residuals

Which is better at minimizing residuals?



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- Perhaps the biggest reason is that it extends easily to the case where X is a vector of random variables.

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- This helps clear that it is an approximation to the CEF and that the units being described are different.

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  - extrapolation
  - predicting in ranges of X with sparse data

Which Predictions Do You Trust?

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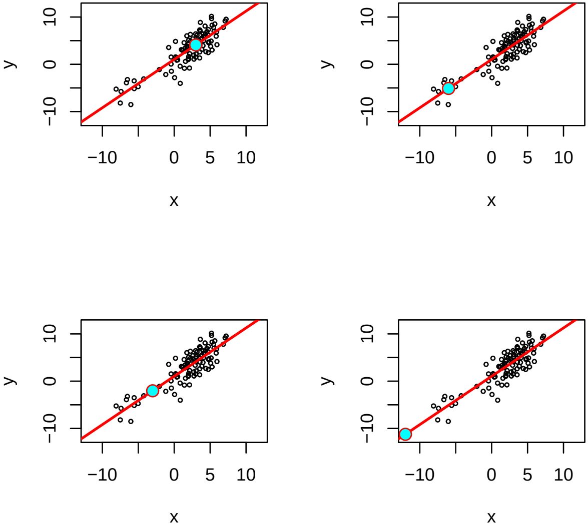

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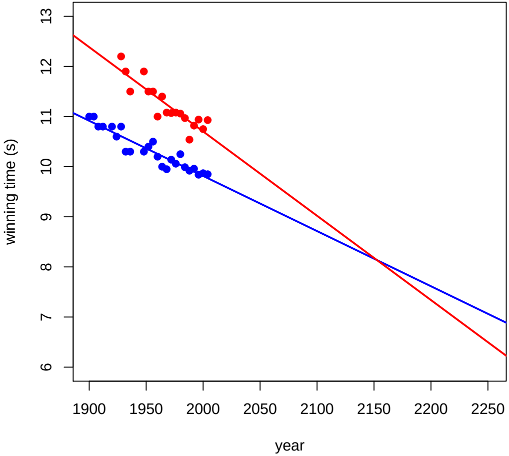
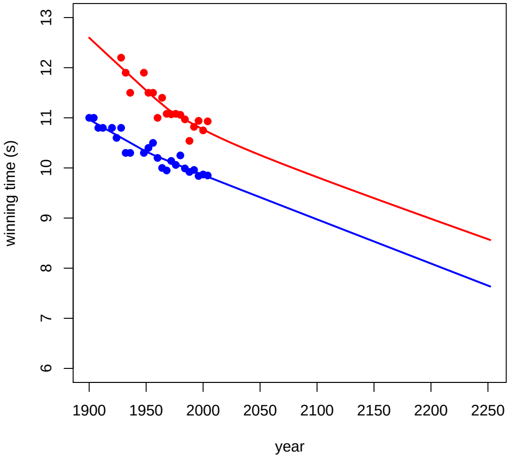
Using data from 1900 to 2004, they fit linear regression models of the winning 100 meter time on year for both men and women. They then use the estimates from these models to extrapolate 152 years into the future.

Tatem et al. Extrapolation



Tatem et al.'s predictions. Men's times are in blue, women's times are in red.

Alternate Models Fit Well, Yield Different Predictions



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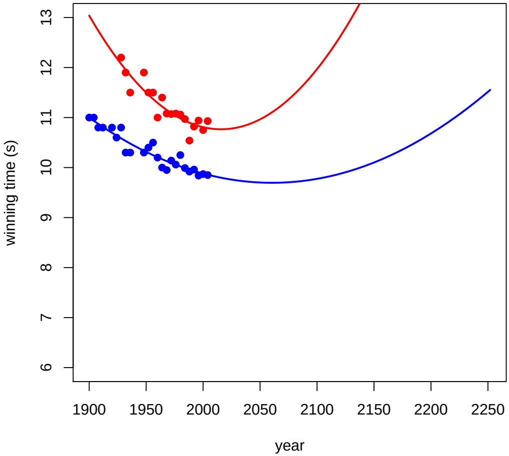

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A More Subtle Example

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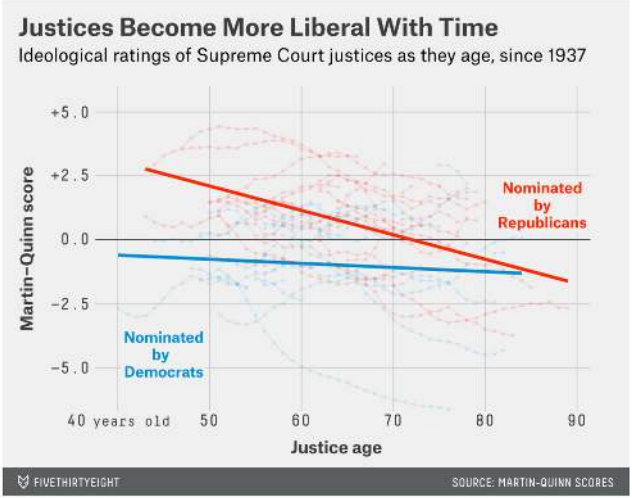

A More Subtle Example

Nate Silver @NateSilver538 · Oct 5

So, basically, John Roberts is going to be Ruth Bader Ginsburg by 2036.

53eig.ht/1GsI2u6

Supreme Court Justices Get More Liberal As They Get Older

The Supreme Court justices are back from vacation. They've picked up their robes from the cleaners — Alito's had a pesky mustard stain — and are ...

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Fun with Linearity

"The Siren's Song of Linearity"

Fun with Linearity

Psychonomic Bulletin & Review
2007, 14 (2), 288-294

Iterated learning: Intergenerational knowledge transmission reveals inductive biases

Michael L. Kalish
University of Louisiana, Lafayette, Louisiana

Thomas L. Griffiths
University of California, Berkeley, California

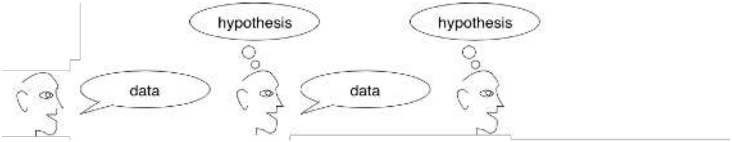
and

Stephan Lewandowsky
University of Western Australia, Perth, Australia

Cultural transmission of information plays a central role in shaping human knowledge. Some of the most complex knowledge that people acquire, such as languages or cultural norms, can only be learned from other people, who themselves learned from previous generations. The prevalence of this process of "iterated learning" as a mode of cultural transmission raises the question of how it affects the information being transmitted. Analyses of iterated learning utilizing the assumption that the learners are Bayesian agents predict that this process should converge to an equilibrium that reflects the inductive biases of the learners. An experiment in iterated function learning with human participants confirmed this prediction, providing insight into the consequences of intergenerational knowledge transmission and a method for discovering the inductive biases that guide human inferences.

Images on following slides courtesy of Tom Griffiths

Fun with Linearity



The Design

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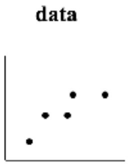

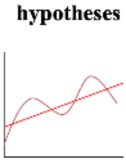

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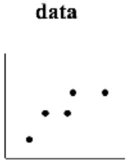

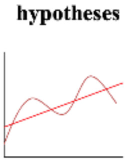

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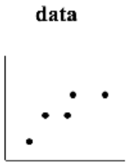

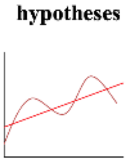

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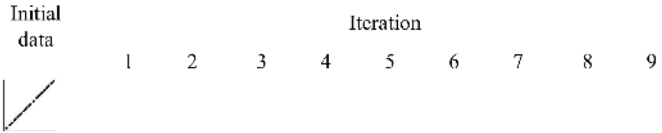
- Predictions are data for the next learner

Results



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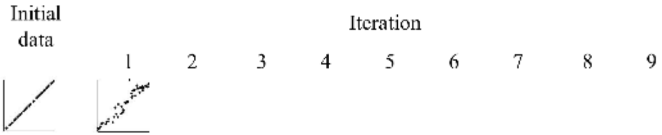

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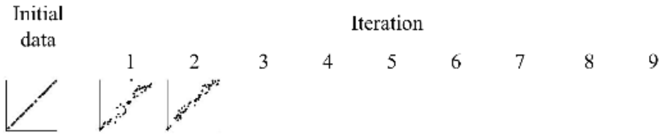

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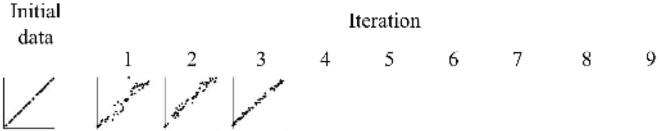

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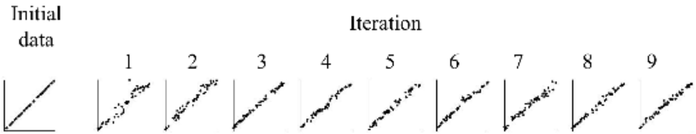

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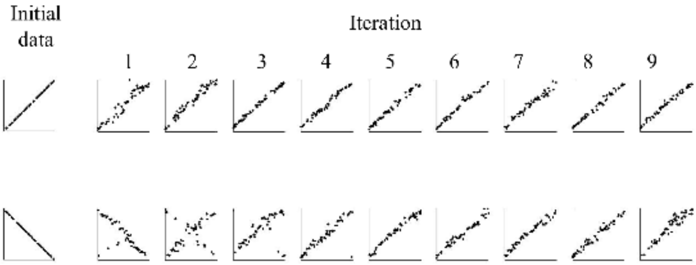

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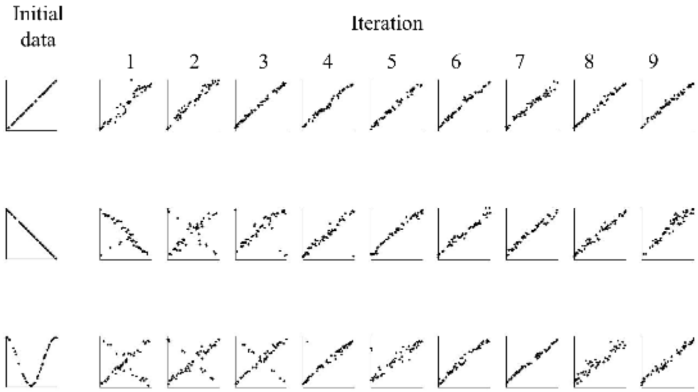

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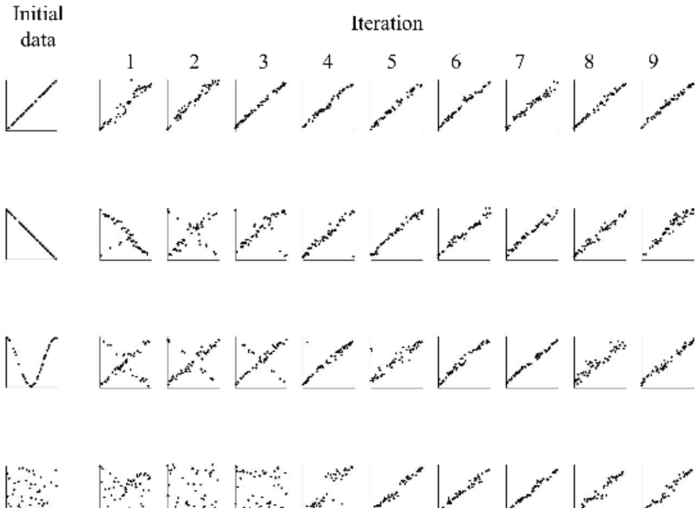

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Next week: Properties of Linear Regression with One Explanatory Variable.