

Soc504: Mixtures, EM and Missing Data

Brandon Stewart¹

Princeton

March 27- April 5, 2017

¹The EM section draws on some slides from Justin Grimmer, Patrick Lam and generations of teaching assistants for Gov2001 at Harvard. The missing data section draws heavily on slides from Gary King. The measurement error section draws heavily on slides from Matt Blackwell.

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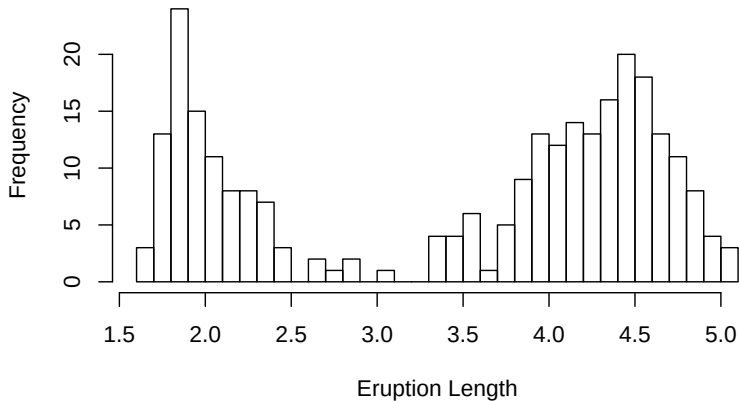
- ▶ Blackwell, Matthew, James Honaker, and Gary King. 2014. "A Unified Approach to Measurement Error and Missing Data: Overview, Details and Extensions" *Sociological Methods and Research* (Optional)

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 - Basic Mixtures
 - Application: Mixtures as Preprocessing
 - Application: Mixture of Regressions
- 2 Expectation Maximization
 - EM for Probit Regression
 - EM for Gaussian Mixtures
 - EM in General
- 3 Missing Data
 - Motivating Example
 - Overview and Assumptions
 - Existing Heuristics
 - Application Specific Approaches
 - Multiple Imputation
 - The Full Amelia Scheme
- 4 Measurement Error
- 5 Appendix: Additional Details and Examples

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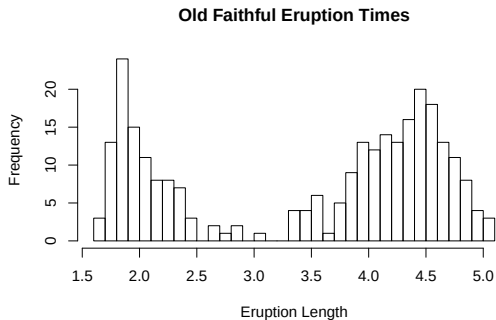
Old Faithful

Old Faithful Eruption Times



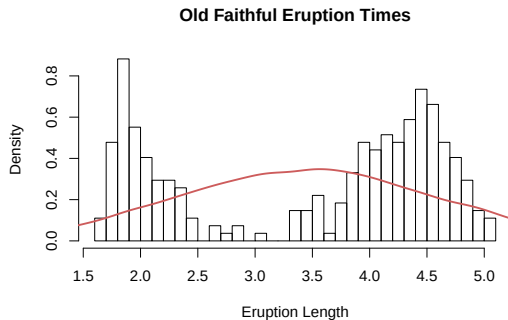
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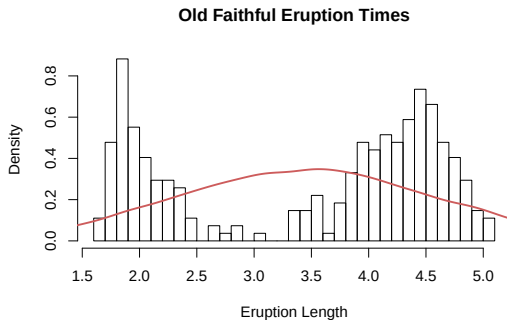
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- We can try fitting a normal but the fit is poor
- If you squint, it looks like two different normals

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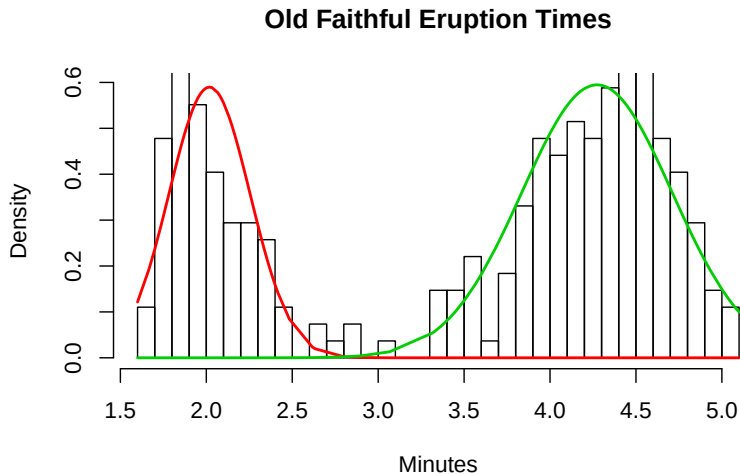
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- However, we don't observe z_i , this is a type of missing data.

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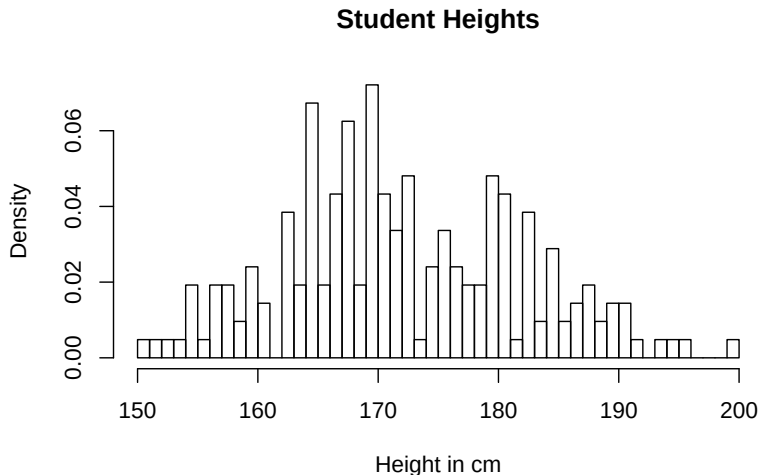
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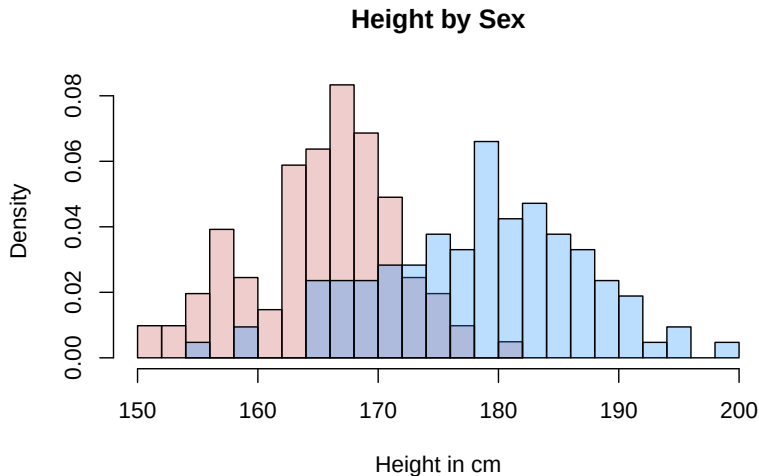
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- This problem was easy because the components are well separated.

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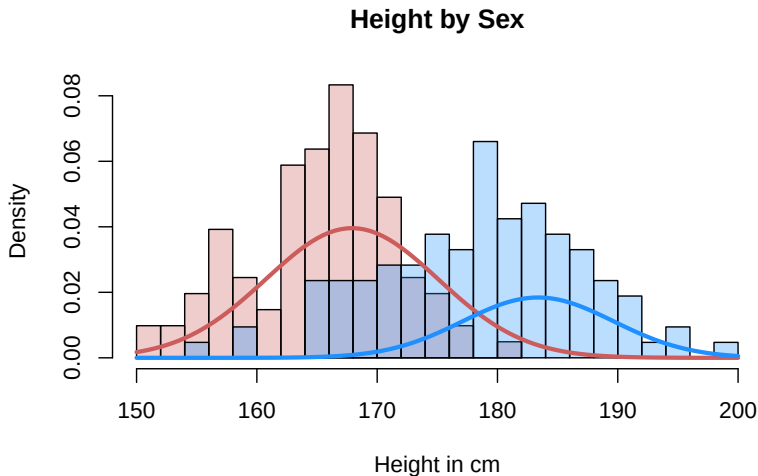
Some distributions have less clear separation

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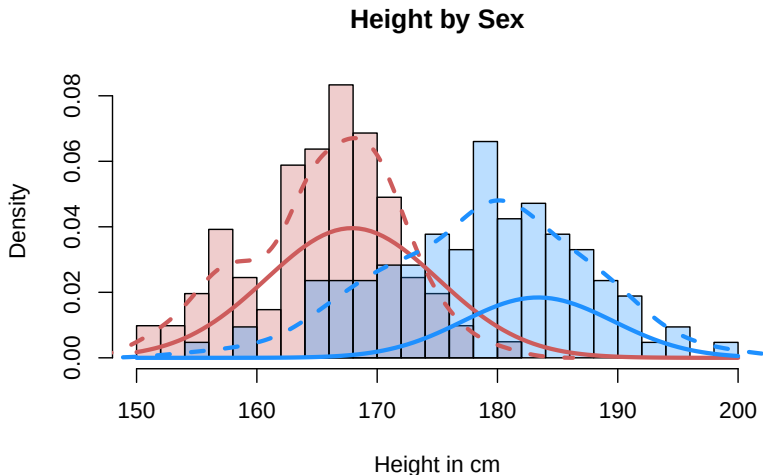
Bimodality here arises due to **gender**

A Harder Problem



The mixture model *sort of* captures this

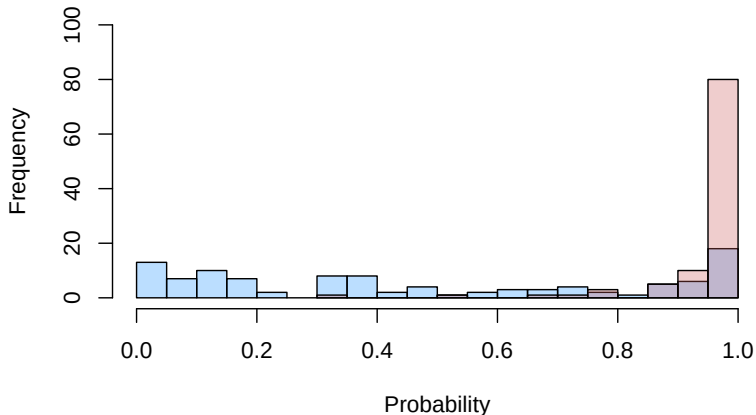
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The true distributions are more peaked with fatter tails

A Harder Problem

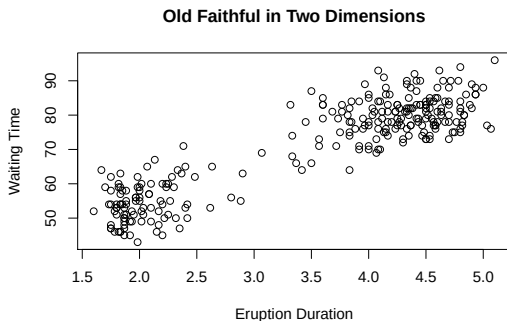
Probabilities of Membership in Cluster 1 By Sex



One component captures all the women but also many men

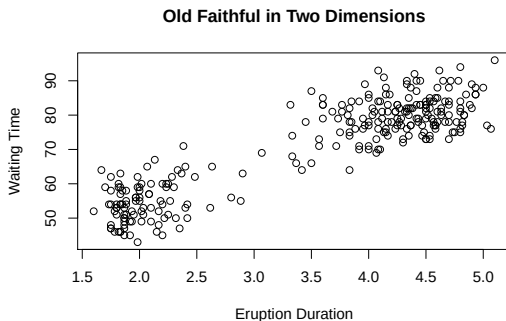
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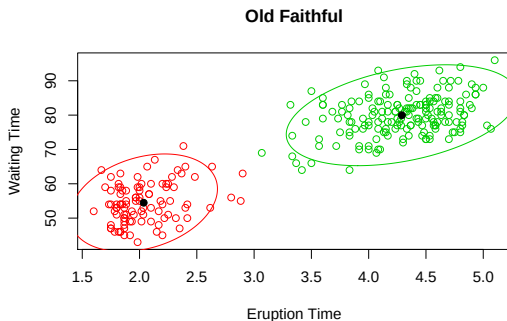
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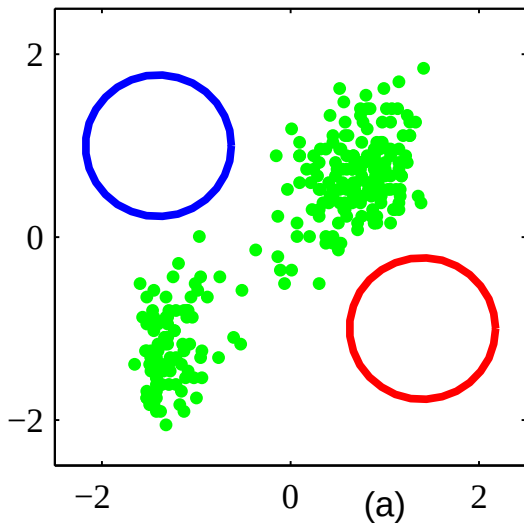
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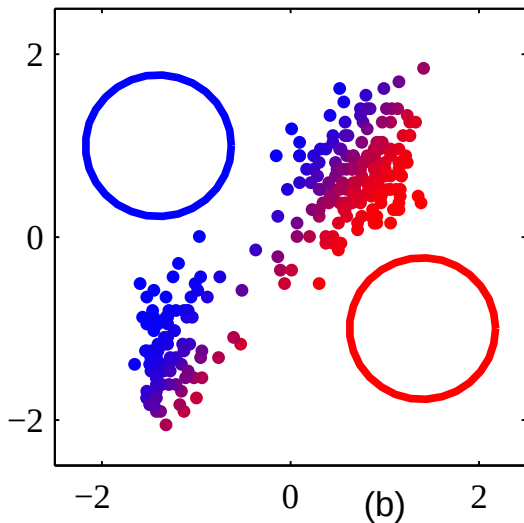
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- This fits the data reasonable well

The Gist of Computation



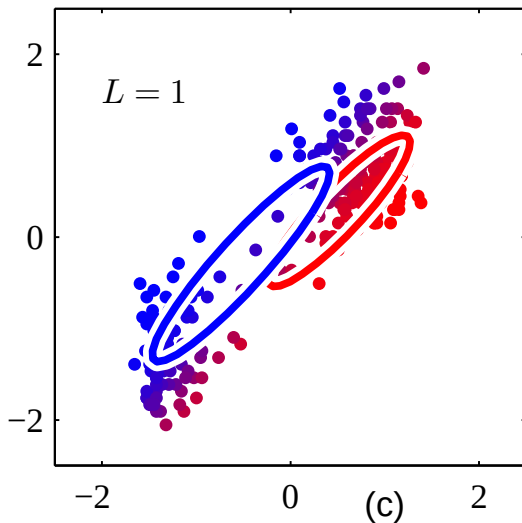
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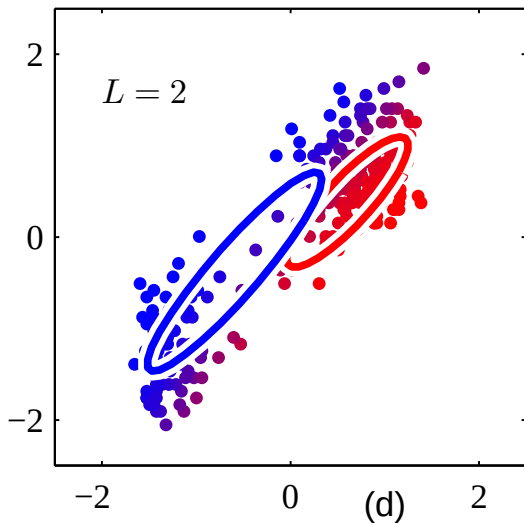
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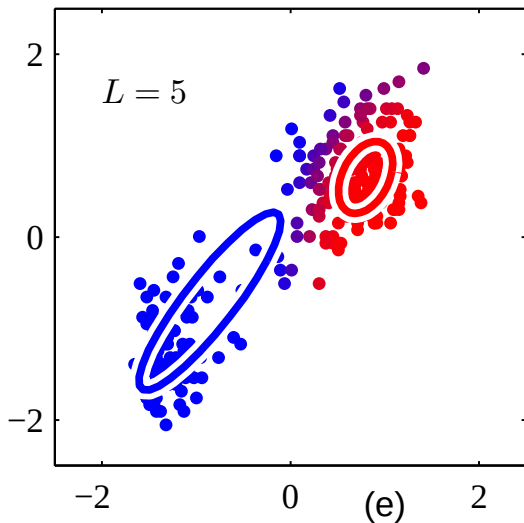
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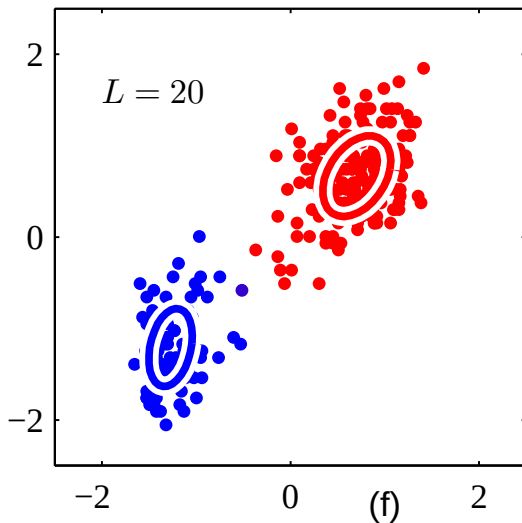
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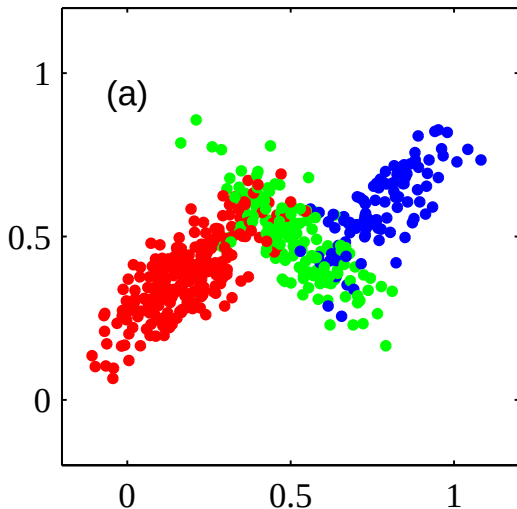
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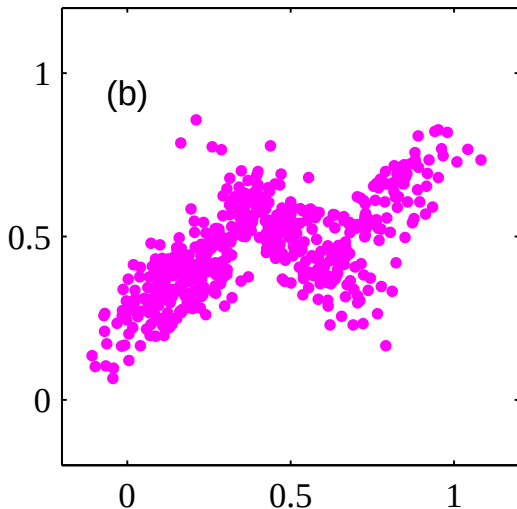
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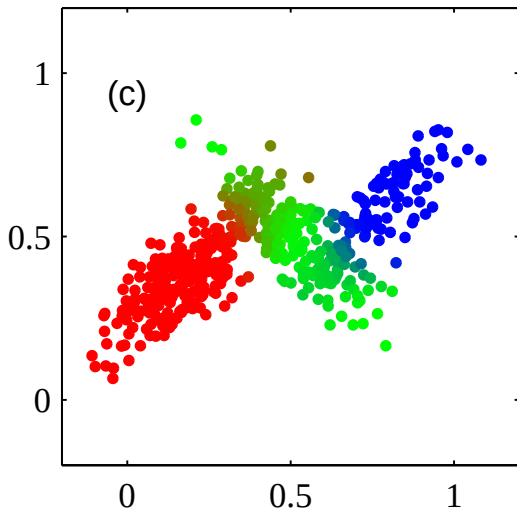
Imagine we draw from data with a 3 component mixture

Mixture Models Can Have Many Components



We observe only the data without the labels

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But we can still infer the components well

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- The mixture model framework can also be used in various other models
- For example, Latent Class Analysis is a mixture of multinomials model commonly used to analyze surveys

Two Applications

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- Two articles motivated from a common methodological place
- Both use mixtures in the context of **regression**

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- We are interested in **heterogeneity** which is masked by **missing** groups.

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- Each type of identified migrant has a distinct configuration of individual, household and community characteristics
- Garip (2012) outlines the steps in cluster analysis as choosing:
 - 1 the relevant attributes
 - 2 an algorithm
 - 3 a similarity measure
 - 4 number of clusters or mixture components
 - 5 validation strategy
- After dividing the units, separate regressions are estimated for each cluster.

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- Garip (2012) uses the “city block” or Manhattan distance which minimizes **L_1 distance** rather than the Euclidean distance

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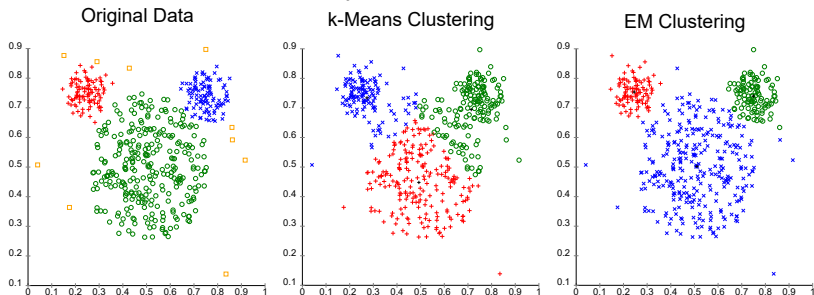
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- There is often a correspondence between **probabilistic models** and popular **distance-based algorithms**.
- This emphasizes the connections between an assumptions about a distance or loss function and an assumption about the model.

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The biggest impact is that k -means strongly prefers equal sized clusters.

Different cluster analysis results on "mouse" data set:



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- Big advance in our understanding with a data-driven approach!

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Is there a model optimized for finding heterogeneous mechanisms?

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- Thus we have the log-likelihood

$$\ell = \sum_{i=1}^N \log \left(\sum_{k=1}^K \pi_k f_k(Y_i | X_i, \theta_k) \right)$$

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- Any one division in time open to critique- can we do better?

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- We will step through a few cases to see how this works.

Review of the Probit Latent Regression Formulation

Let $Y_i^* \sim P(y_i^* | \mu_i)$ where $\mu_i = X_i \beta$ and assume that we only observe

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For the probit model, $P(\cdot) = \mathcal{N}(\mu_i, \sigma^2)$. Typically assume that $\tau = 0$ and $\sigma = 1$ in order to fit the model.

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We'll come back to that last part in a second.

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- 3 Increment until convergence.

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- 4 Repeat Steps 2-3 Until Convergence.

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The EM Algorithm

Note that this is a really high-level, heuristic view of EM. The steps are always the same though:

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- 5 Assess convergence either by changes in parameters or the log-likelihood.

Example 2: Mixtures

Single distribution data generating process:

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In words:

- Draw a cluster label
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Gaussian Mixture

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This leads to the likelihood:

$$\begin{aligned} p(\mathbf{x}) &= \sum_{\mathbf{z}} p(\mathbf{z}) p(\mathbf{x} | \mathbf{z}) \\ &= \sum_{k=1}^K \pi_k \mathcal{N}(\mathbf{x} | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \end{aligned}$$

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We know x and so we plug in our best guess of z , the expectation.

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The Slovenian Plebiscite (Rubin, Stern and Vehovar, 1995)

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Attendance	Independence		
	Yes	No	DK
Yes	1439	78	159
No	16	16	32
DK	144	54	136

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Attendance	Independence	
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Yes	θ_{11}	θ_{12}
No	θ_{21}	θ_{22}

Here the first subscript refers to the attendance question and the second to the independence question.

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4. **Imputation estimator**: assert that the missingness is determined only by the observed values and then attempt to impute the missing data.

Imputation

Here's the data again, with the proportion of observed data filled in.

Attendance	Independence		
	Yes	No	DK
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We can do exactly the same set of calculations for the other three “don't know” groups to impute the missing data.

Imputation: An Updated Sense of the Proportions?

Attendance	Independence	
	Yes	No
Yes	1439 + 150.87 + 142.42 .896	78 + 8.12 + 44.81 .066
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Table: Imputations for I-DK's in **red**; imputations based on A-DK's in **blue**.

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We have made a guess of missing values based on estimates of population parameters θ . What would be a suitable next step?

Iteration

We can now use our updated (and, in fact, improved) estimate of the population proportions in order to re-impute the missing data using the same approach as before.

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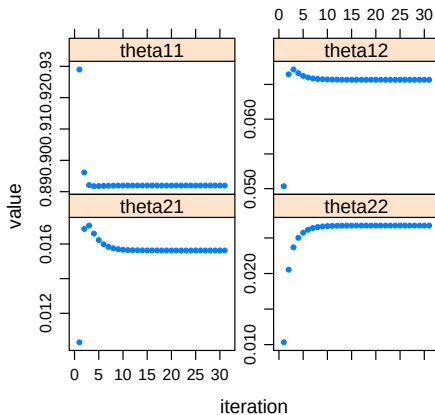
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Once we have updated our best guess of how the various DK people will vote, then we can re-estimate the population proportions.

We can iterate this approach until our estimates of the population proportions converge to a stable maximum.

Iterations

Here are the trace plots showing how the estimates of the θ evolve through the iterations:



A Final Estimate

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Neat!

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- We will (mostly) focus on **imputation**

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- In this sense- we cannot really separate the missing data procedure from the **inferential goal of the analysis**

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~> **Missing elements must exist** (what's your view on the National Helium Reserve?)

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- This only works with bounded support and becomes much harder with missingness on many variables

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- Reasons for the odd terminology are historical.

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 - ▶ Adding variables to predict income can change NI to MAR

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4. Nonresponse Weighting (including HT weights, Hajek weights)
unbiased and consistent but inefficient and high variability in small samples

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8. “Missing” Category for Categorical Variable
simple and often useful but differential rates in how missingness spreads over categories could cause bias

Random Imputation for one variable

9. Simple Random Imputation

ignores useful information, helpful as a starting point

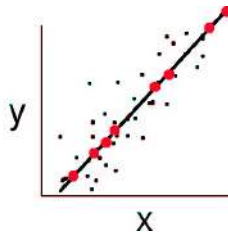
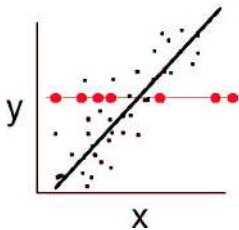
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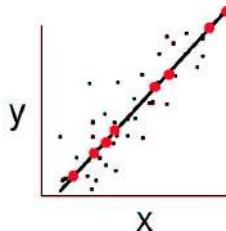
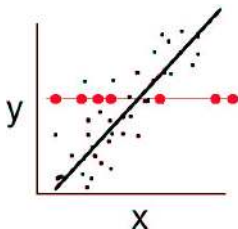
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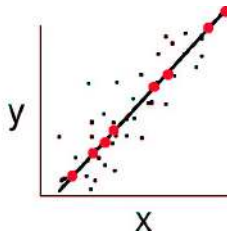
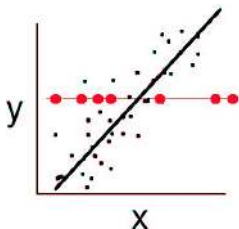
10. Hot Deck Imputation (aka matching imputation)

consistent but otherwise bad: inefficient, standard errors wrong





11. $\hat{y}$ Regression Imputation (aka regression deterministic)
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12. $\hat{y} + \epsilon$ regression imputation (aka regression predictive)
 assumes no estimation uncertainty, does not help for scattered
 missingness

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6. Very difficult with missingness scattered through X and Y

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3. **Run whatever statistical method you would have** with no missing data for each completed data set

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5. **Easier by simulation**: draw $1/m$ sims from each data set of the QOI, combine (i.e., concatenate into a larger set of simulations), and make inferences as usual.

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7. **For social science survey data**, which mostly contain ordinal scales, this is a reasonable choice for imputation, even though it may not be a good choice for analysis.

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$$\begin{aligned} D &\sim N(D|\mu, \Sigma) \\ &= N\left[\begin{pmatrix} Y \\ X \end{pmatrix} \mid \begin{pmatrix} \mu_y \\ \mu_x \end{pmatrix}, \begin{pmatrix} \sigma_y & \sigma_{xy} \\ \sigma_{xy} & \sigma_x \end{pmatrix}\right] \end{aligned}$$

4. Conditionals of bivariate normals are normal:

$$Y|X \sim N(y|E(Y|X), V(Y|X))$$

$$E(Y|X) = \mu_y + \beta(X - \mu_x) \quad (\text{a regression of } Y \text{ on all other } X\text{'s!})$$

$$\beta = \sigma_{xy}/\sigma_x$$

$$V(Y|X) = \sigma_y - \sigma_{xy}^2/\sigma_x$$

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6. In this simple example (X fully observed), this is equivalent to simulating from a linear regression of Y on X ,

$$\tilde{y}_i = x_i \tilde{\beta} + \tilde{\epsilon}_i,$$

with **estimation** and **fundamental** uncertainty

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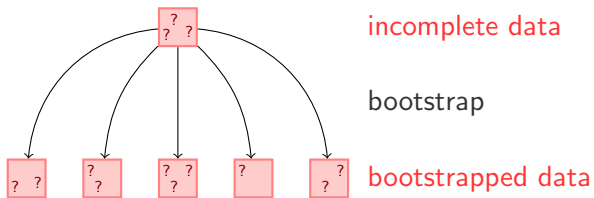
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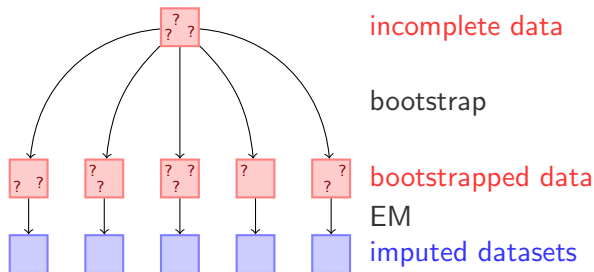


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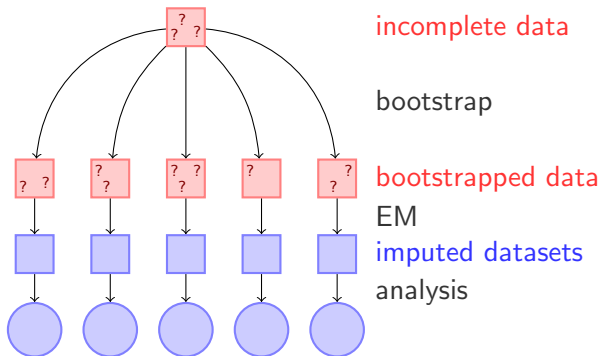
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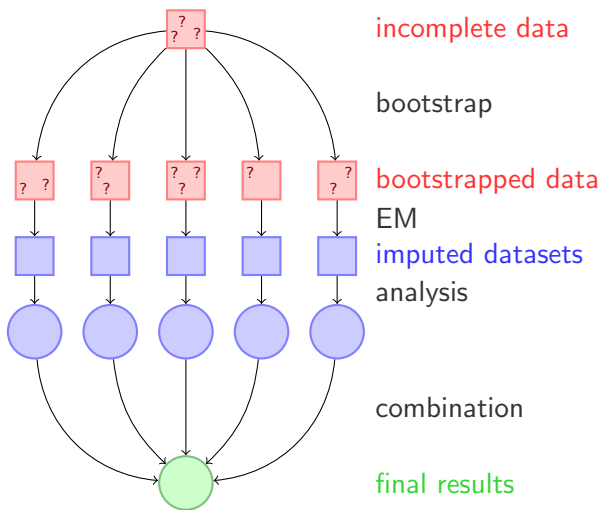
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- Code ordinal variables as close to interval as possible.

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 - ▶ Answer to both: the draws are from the joint posterior and put back into the data. Nothing is being changed.

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The EM algorithm in this case involves selecting an initial value for (μ, Σ) , using that value to impute the missing data, and then re-estimating (μ, Σ) based on the (now-complete) data.

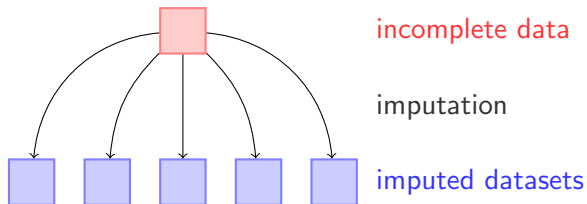
The Multiple Imputation Scheme

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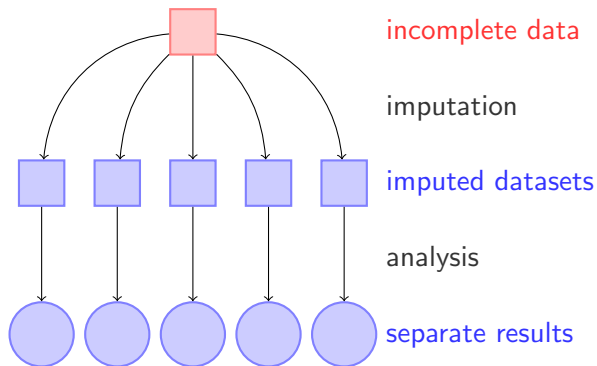


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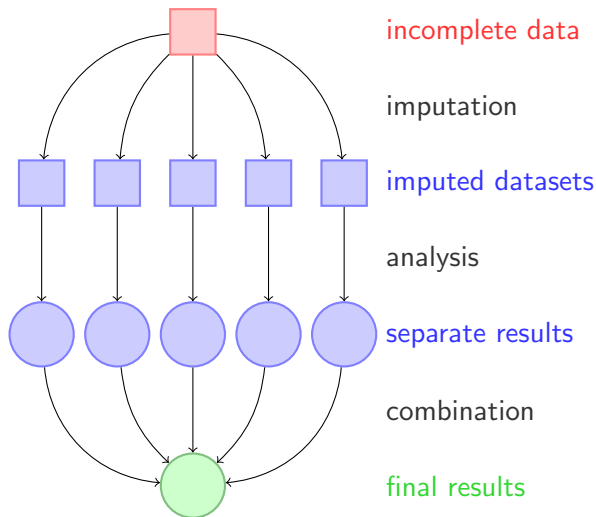
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Multiple Imputation

REGRESSION

To preserve the relationships in the data.

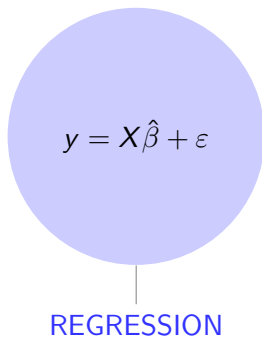
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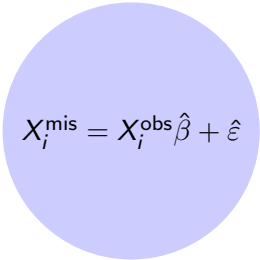
SIMULATION

To reflect the uncertainty of our imputation.

How to Impute

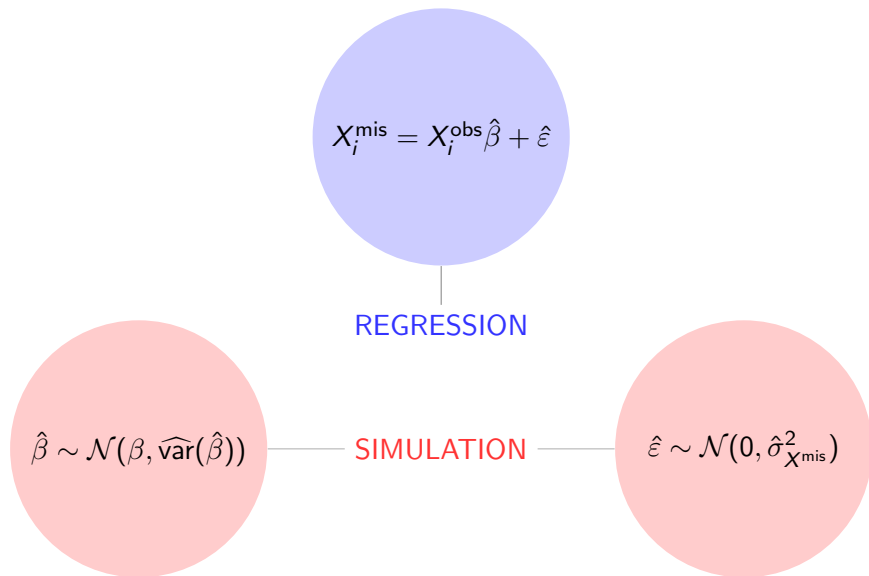


How to Impute


$$X_i^{\text{mis}} = X_i^{\text{obs}} \hat{\beta} + \hat{\varepsilon}$$

REGRESSION

How to Impute



Patterns of Missingness

	year	country	GDP	infl	trade	population
1	1972	Burkina Faso	377	-2.92	29.69	5848380
2	1973	Burkina Faso	376	7.60	31.31	5958700
3	1974	Burkina Faso	393	NA	NA	6075700
4	1975	Burkina Faso	416	18.76	40.11	6202000
5	1976	Burkina Faso	435	NA	37.76	6341030
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$$infl = \beta_0 + \beta_1 \cdot GDP + \beta_2 \cdot trade + \beta_3 \cdot population + \varepsilon$$

Any β is just (μ, Σ)

- If $X \sim \mathcal{N}(\mu, \Sigma)$, we can recover any regression from the vector of means and the covariance matrix.
- Thus, we need $(\mu, \Sigma | X^{\text{obs}})$.

A complicated likelihood

$$\mathcal{L}(\mu, \Sigma | D^{\text{obs}}) \propto \prod_{i=1}^n \mathcal{N}(D_i^{\text{obs}} | \mu_i^{\text{obs}}, \Sigma_i^{\text{obs}})$$

The EM algorithm

Turn a hard problem into a repeated easy problem.

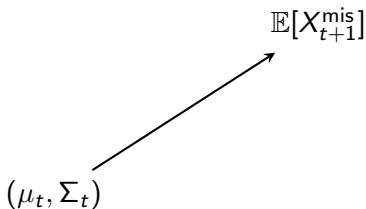
- ① Use current estimates of (μ, Σ) to estimate X^{mis} .
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$$(\mu_t, \Sigma_t)$$

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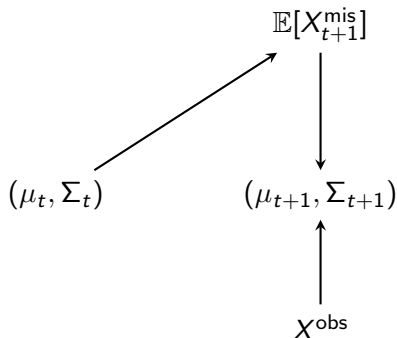
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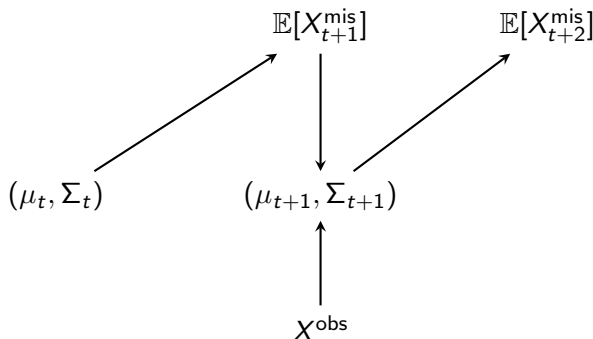
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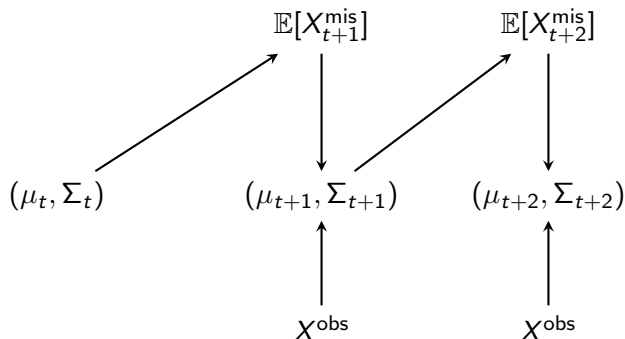
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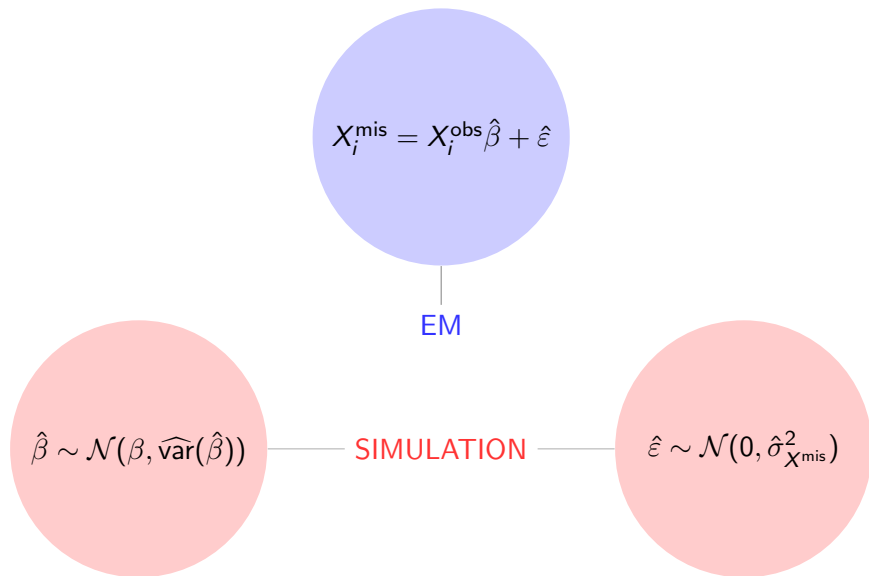
Simulation

$$\underbrace{X_i^{\text{mis}}}_{\text{missing values in row } i} = \underbrace{X_i^{\text{obs}} \beta}_{\text{observed values in row } i} + \underbrace{\varepsilon}_{\mathcal{N}(0, \sigma^2)}$$

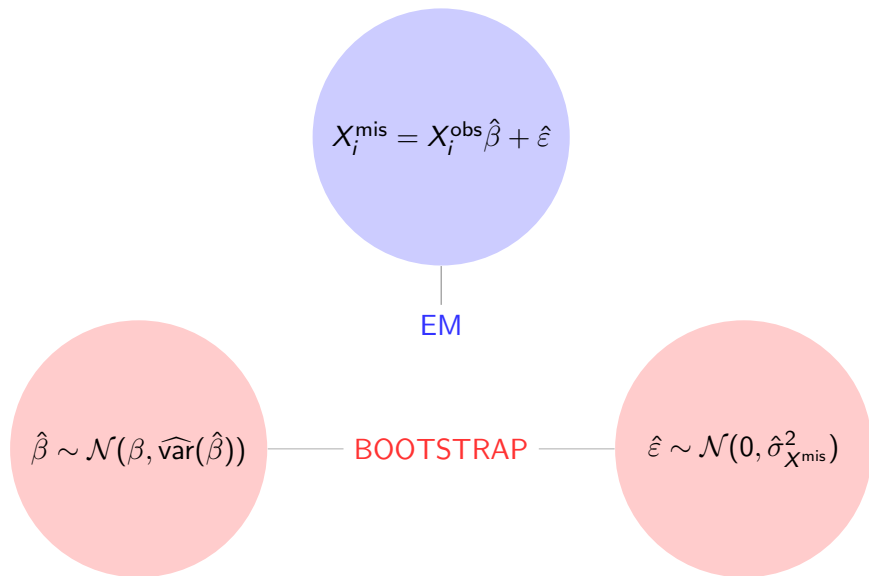
EM is a tool for **REGRESSION**. In order to **SIMULATE**, we need...

- 1 a Normal approximation.
- 2 importance sampling.
- 3 a bootstrap-based approach.

How to Impute



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REGRESSION **SIMULATION**

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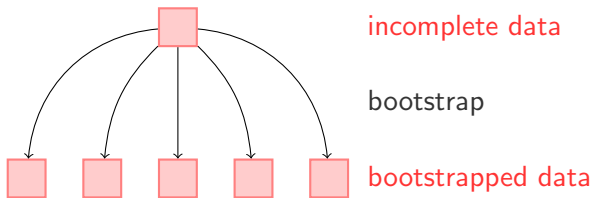
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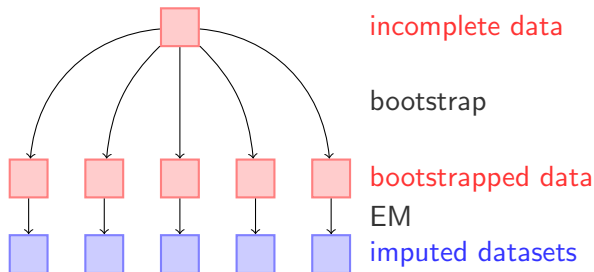


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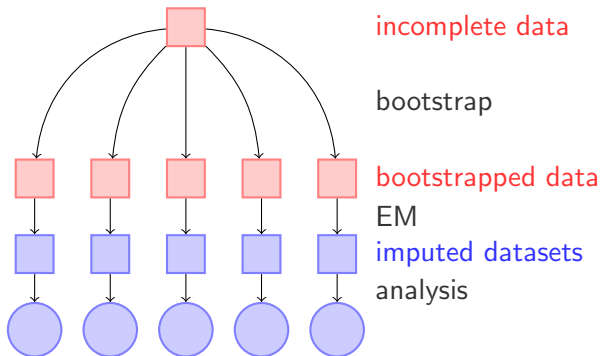
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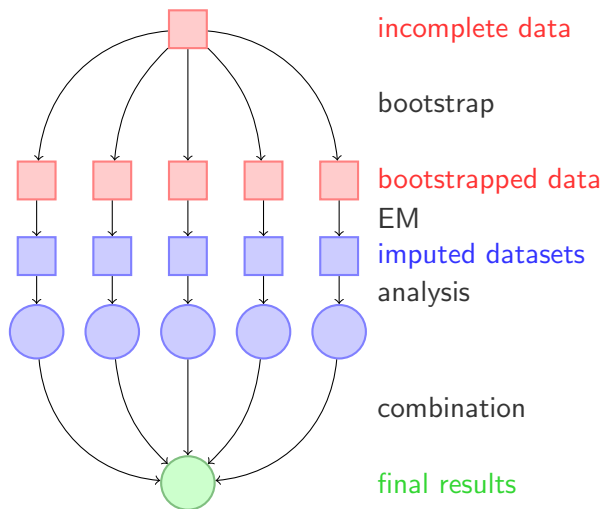
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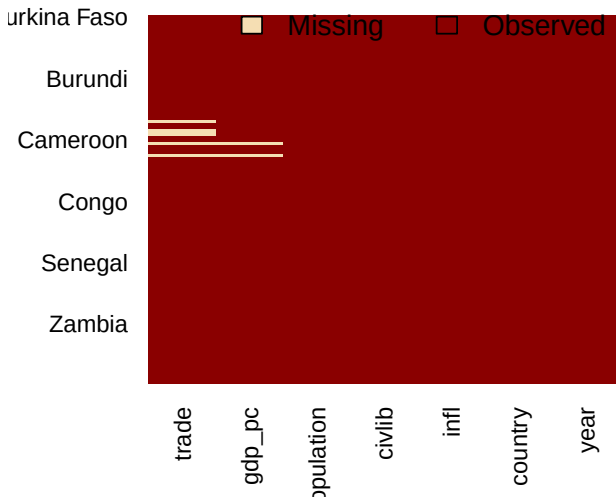
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- Not really covered here but see the Amelia vignette and the Su et al paper.

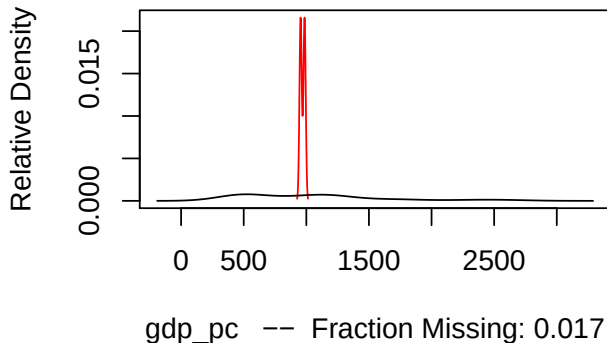
Example Amelia Diagnostics

Missingness Map



Example Amelia Diagnostics

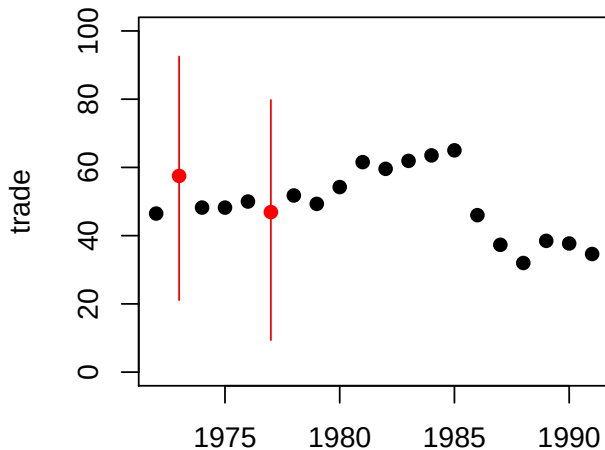
Observed and Imputed values of gdp_pc



Observed and Imputed values of trade

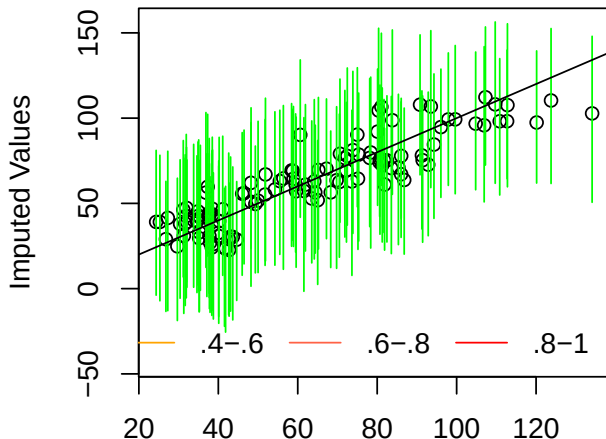
Example Amelia Diagnostics

Cameroon



Example Amelia Diagnostics

Observed versus Imputed Values of trad



Final Thoughts on Missing Data

- There is a bit of a goldilocks region here- too few observations and it doesn't matter, too many and it will give crazy answers

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- There is a bit of a goldilocks region here- too few observations and it doesn't matter, too many and it will give crazy answers
- Overimputing and observed vs. imputed distributions are helpful diagnostics but there are no hard and fast rules
- As per usual, domain knowledge here is key. The missing data literature just helps you apply that domain knowledge

- 1 Mixture Models
 - Basic Mixtures
 - Application: Mixtures as Preprocessing
 - Application: Mixture of Regressions
- 2 Expectation Maximization
 - EM for Probit Regression
 - EM for Gaussian Mixtures
 - EM in General
- 3 Missing Data
 - Motivating Example
 - Overview and Assumptions
 - Existing Heuristics
 - Application Specific Approaches
 - Multiple Imputation
 - The Full Amelia Scheme
- 4 Measurement Error
- 5 Appendix: Additional Details and Examples

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Measurement Error or, How Amelia Solves All Your Problems

Blackwell, Matthew, James Honaker, and Gary King. “Multiple Overimputation: A Unified Approach to Measurement Error and Missing Data.” *Sociological Methods and Research* 2015.

Three New Things

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- ② Missing data is the limiting, most extreme form of measurement error.
- ③ We can rework the multiple imputation framework to simultaneously correct for both missing data and measurement error.

	country	polityiv	f-house	log-gdppc	primary
1	BUKINA FASO	5	6	6.23	5.92
2	LIBERIA	NA	3	NA	NA
3	SIERRA LEONE	3	3	6.60	NA
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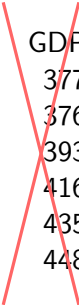
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One Solution:

	country	GDP	infl	trade	population
1	Ghana	377	-2.92	29.69	5848380
2	Ivory Coast	376	7.60	31.31	5958700
3	Kenya	393	8.72	35.22	6075700
4	Nigeria	416	18.76	40.11	6202000
5	Uganda	435	-8.40	37.76	6341030
6	Burkina Faso	448	29.99	41.11	6486870

One Solution: Change research agendas



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- Denial

The current approaches in the literature

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- application-specific.
- model dependent.
- difficult to implement.
- inapplicable with multiple variables.
- invalid with heteroskedastic errors.
- unusable with missing data.

Why is this the state of the art?

Why is this the state of the art?
It's easy and tolerated.

Why is this the state of the art?
It's easy and tolerated.
But it's make believe.

A Brief Review of Measurement Error

$$x_i = x_i^* + u_i$$

A Brief Review of Measurement Error

observed



$$x_i = x_i^* + u_i$$

A Brief Review of Measurement Error

observed

latent

$$x_i = x_i^* + u_i$$

The diagram illustrates the measurement error model. The word "observed" is positioned above the variable x_i in the equation, with a black arrow pointing down to it. The word "latent" is positioned above the variable x_i^* in the equation, with a red arrow pointing down to it. The equation itself is $x_i = x_i^* + u_i$, where x_i is the observed variable, x_i^* is the latent variable, and u_i is the measurement error term.

A Brief Review of Measurement Error

The diagram illustrates the measurement error model equation $x_i = x_i^* + u_i$. It features three labels with arrows pointing to the corresponding terms in the equation: 'observed' points to x_i , 'latent' points to x_i^* , and 'measurement error' (in red) points to u_i .

$$\begin{array}{ccccc} \text{observed} & & \text{latent} & & \text{measurement error} \\ & \searrow & \downarrow & & \swarrow \\ & x_i & = & x_i^* & + & u_i \end{array}$$

A Brief Review of Measurement Error

[illegible]

$$u_i|x_i^* \sim \mathcal{N}(0, \sigma_u^2)$$

A Brief Review of Measurement Error

A diagram illustrating the measurement error model. Three labels are positioned at the top: "observed", "latent", and "measurement error". Arrows point from each label to a corresponding term in the equation $x_i = x_i^* + u_i$. The arrow from "observed" points to x_i , the arrow from "latent" points to x_i^* , and the arrow from "measurement error" points to u_i .

$$\begin{array}{ccc} \text{observed} & & \text{latent} & & \text{measurement error} \\ & \searrow & \downarrow & & \swarrow \\ & x_i & = & x_i^* & + & u_i \end{array}$$

$$u_i | x_i^* \sim \mathcal{N}(0, \sigma_u^2)$$

unbiased
independent

Two red arrows originate from the text "unbiased independent" and point towards the u_i and x_i^* terms in the equation above, indicating that the measurement error is unbiased and independent of the latent variable.

A Brief Review of Measurement Error

[illegible]

The diagram shows the equation $u_i | x_i^* \sim \mathcal{N}(0, \sigma_u^2)$. Two arrows point from the text "unbiased independent" to the terms x_i^* and $\mathcal{N}$. A red arrow points from the text "measurement error variance" to the term σ_u^2 .

Want to run:

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$$y_i = \beta_0 + \beta_1 x_i^* + \epsilon_i$$

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Can only run:

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Leads to:

ATTENUATION

Want to run:

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Can only run:

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Leads to:

ATTENUATION

(But ONLY in linear models with one bad variable)

Want to run:

$$y_i = \beta_0 + \beta_1 x_i^* + \beta_2 w_i^* + \beta_3 z_i^* + \epsilon_i$$

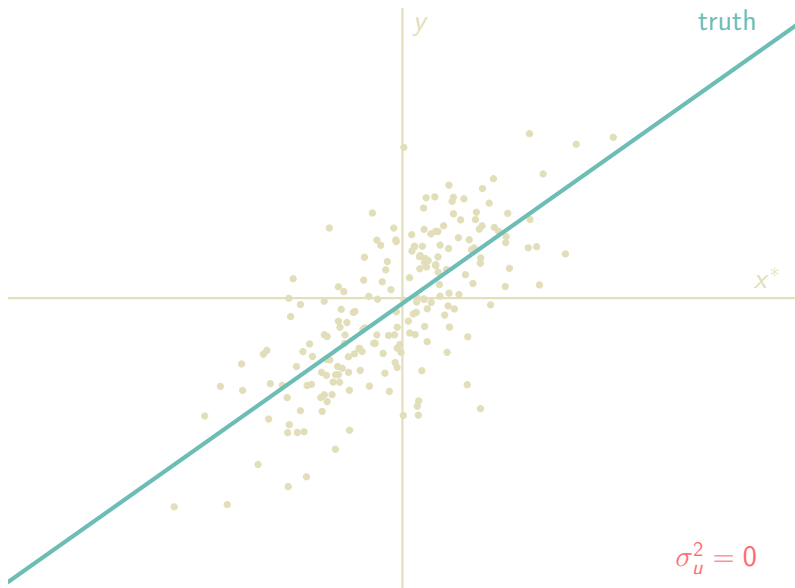
Can only run:

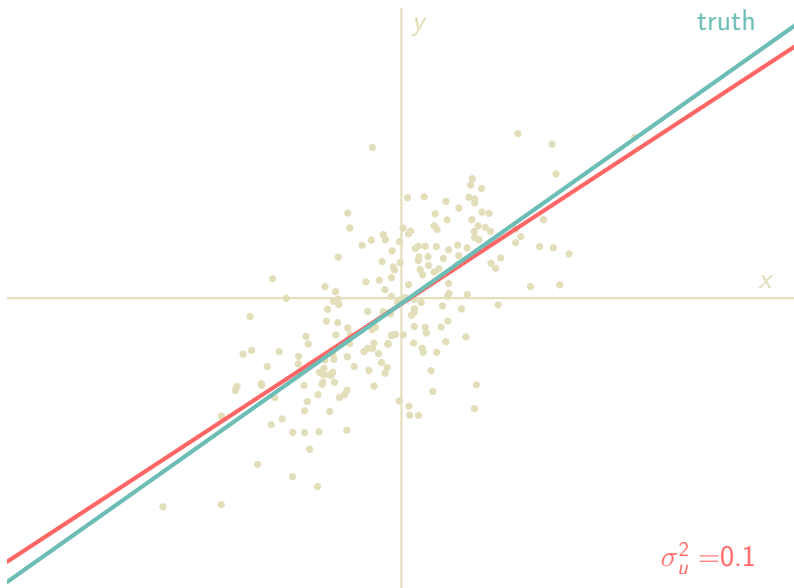
$$y_i = \alpha_0 + \alpha_1 x_i + \alpha_2 w_i + \alpha_3 z_i + \nu_i$$

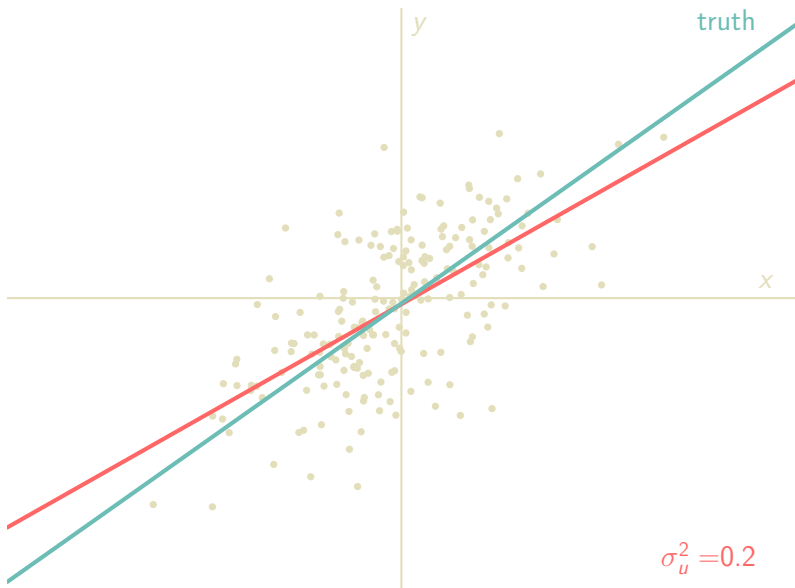
Leads to:

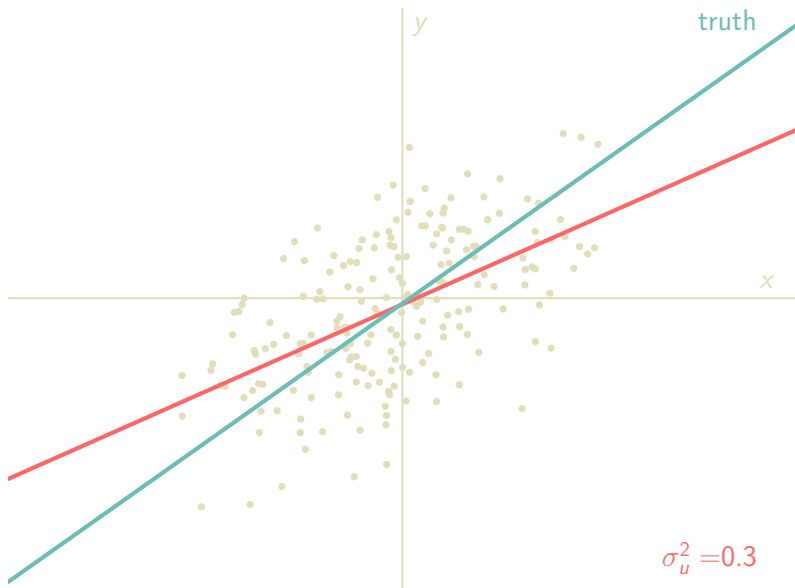
UNKNOWN

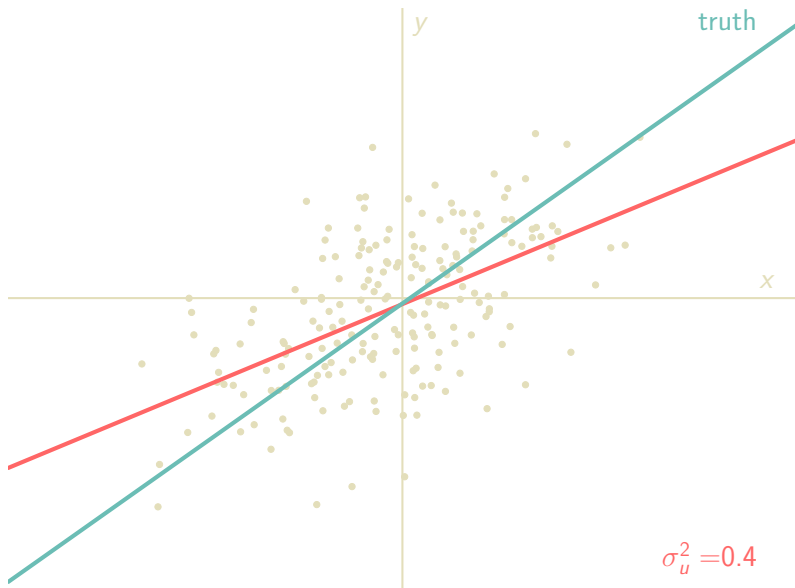
(No guarantees with more mismeasured variables)

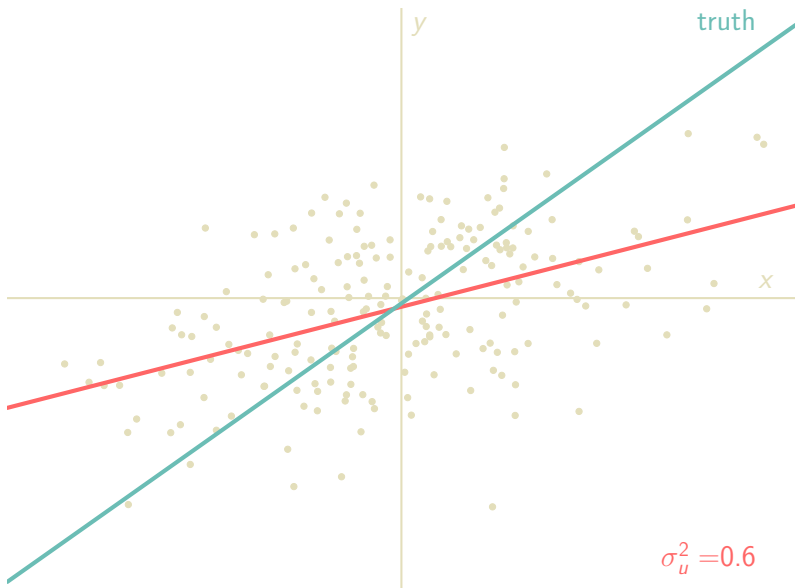


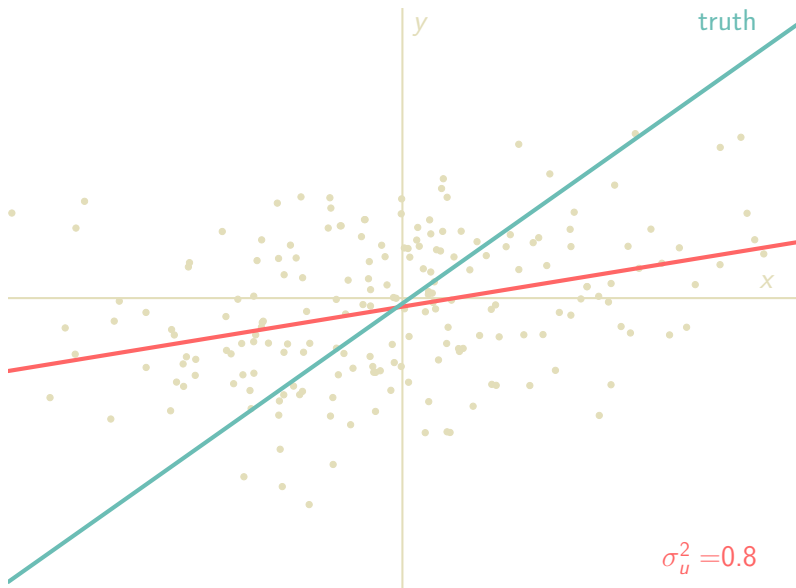


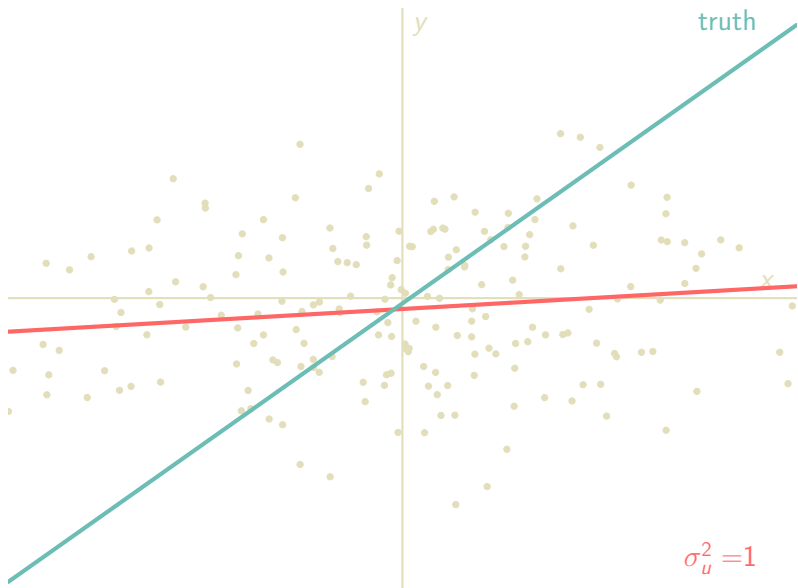


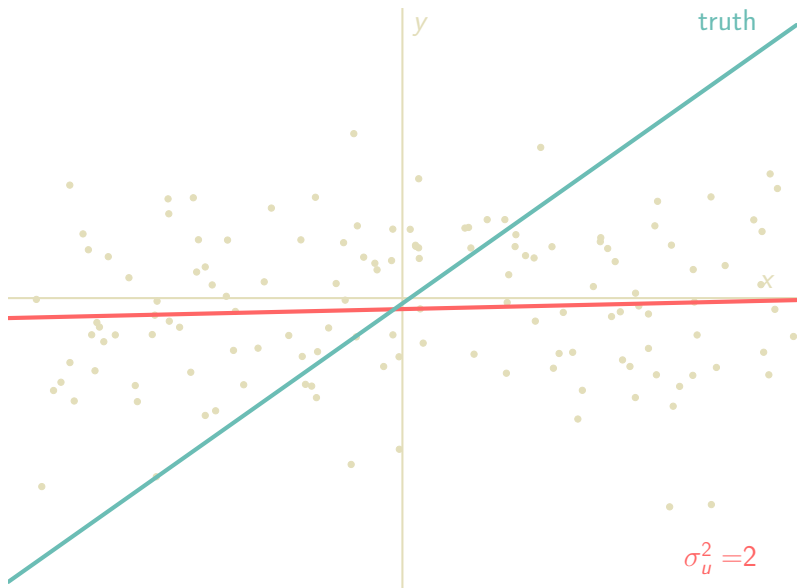












ATTENUATION

...only guaranteed in the simplest of cases:

ATTENUATION

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linear model

ATTENUATION

...only guaranteed in the simplest of cases:

linear model
one mismeasured variable

ATTENUATION

...only guaranteed in the simplest of cases:

linear model

one mismeasured variable

measurement error unrelated to other variables and x^* .

BIAS FROM MEASUREMENT ERROR

In unpredictable directions with most realistic models.

The strict dichotomy of data.

observed

missing

(fully) observed

(fully) missing

(fully) observed

(fully) missing

The false dichotomy of data.



fully
observed



fully
observed



fully
missing



fully
observed



partially
missing



fully
missing



But what is this continuum?

[illegible]

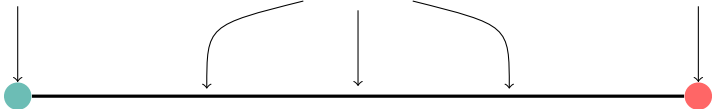
$$u_i | x_i^* \sim \mathcal{N}(0, \sigma_u^2)$$

measurement
error
variance

fully
observed

partially
missing

fully
missing



fully
observed

partially
missing

fully
missing

$$\sigma_u^2 = 0$$

observed latent measurement
error

$$x_i = x_i^* + u_i$$

$$u_i \sim \mathcal{N}(0, 0)$$

measurement
error
variance

observed latent measurement
error

$$x_i = x_i^* + 0$$

$$u_i \sim \mathcal{N}(0, 0)$$

measurement
error
variance

fully
observed

partially
missing

fully
missing

$$\sigma_u^2 = 0$$

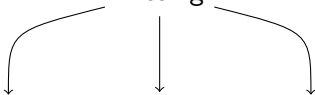


fully
observed



$$\sigma_u^2 = 0$$

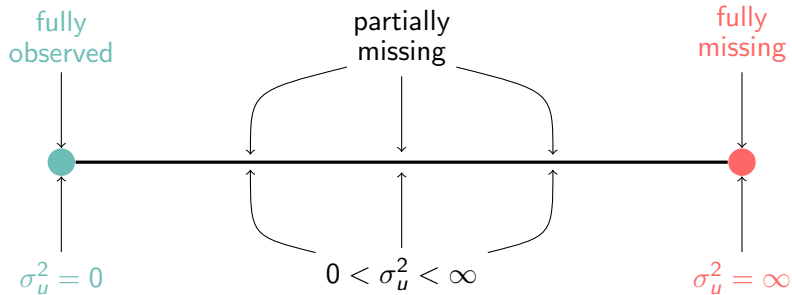
partially
missing



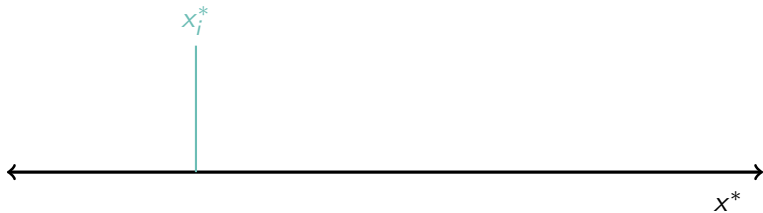
fully
missing

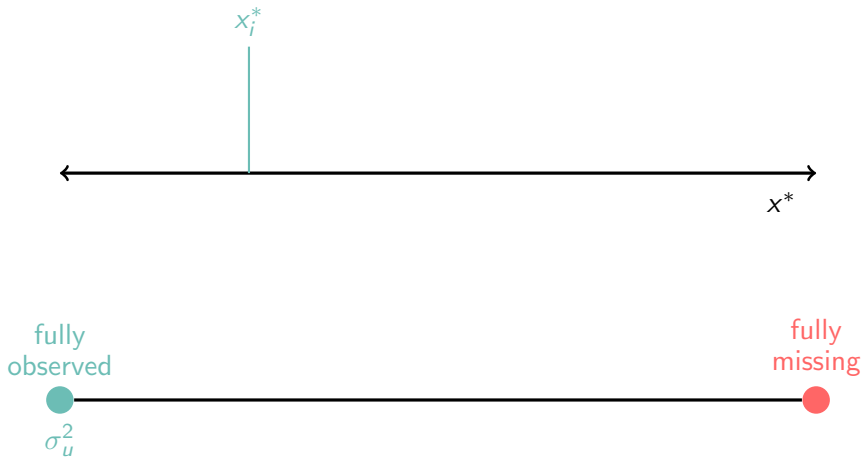


$$\sigma_u^2 = \infty$$

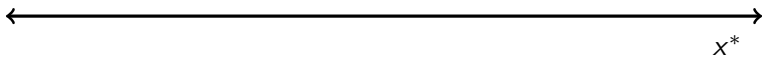


Missing data is the most
extreme case of measurement error.





x_i



x^*

fully
observed

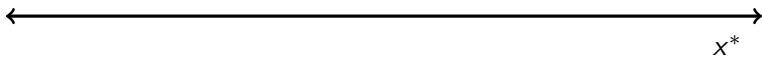


σ_u^2

fully
missing



x_i



x^*

fully
observed



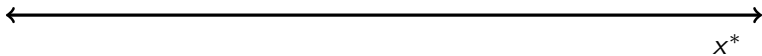
σ_u^2



fully
missing



x_i



x^*

fully
observed

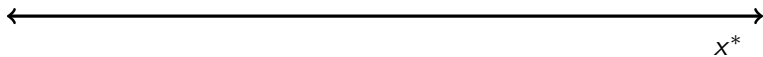


σ_u^2

fully
missing



x_i



x^*

fully
observed



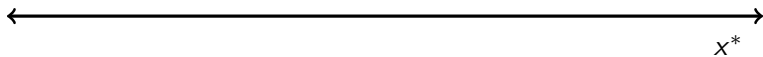
σ_u^2



fully
missing



x_i



x^*

fully
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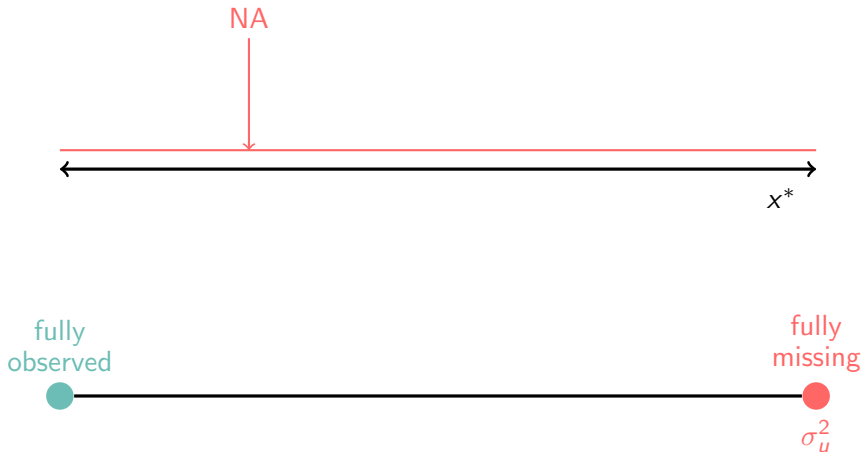


fully
missing



σ_u^2





Multiple imputation:

observed

missing

Multiple imputation:

(fully) observed (fully) missing

Multiple overimputation:

x_i^* fully observed | fully missing

Multiple overimputation:

x_i^* fully observed | partially missing | fully missing

Multiple overimputation:

x_i^*	fully observed perfectly measured	partially missing measured with error	fully missing infinite error
---------	--------------------------------------	--	---------------------------------

Multiple overimputation:

x_i^*	fully observed	partially missing	fully missing
	perfectly measured	measured with error	infinite error
$p(x_i x_i^*)$	$\mathcal{N}(x_i^*, 0)$	$\mathcal{N}(x_i^*, \sigma_u^2)$	$\mathcal{N}(x_i^*, \infty)$

Multiple Overimputation
extends the multiple imputation framework
to correct for measurement error.

Missing Data and Measurement Error

APPLICATION-SPECIFIC METHODS:

Missing Data and Measurement Error

APPLICATION-SPECIFIC METHODS:

incomplete
mismeasured
dataset

Missing Data and Measurement Error

APPLICATION-SPECIFIC METHODS:

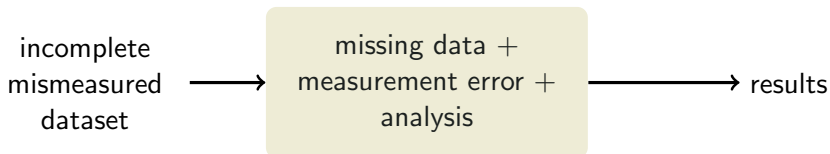
incomplete
mismeasured
dataset



missing data +
measurement error +
analysis

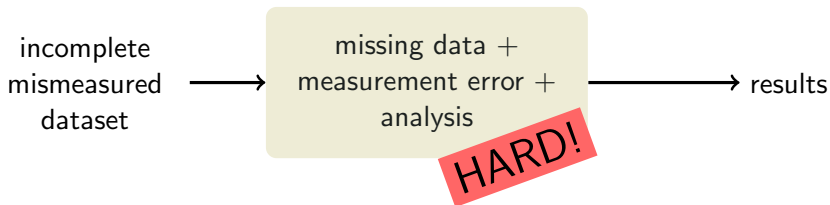
Missing Data and Measurement Error

APPLICATION-SPECIFIC METHODS:



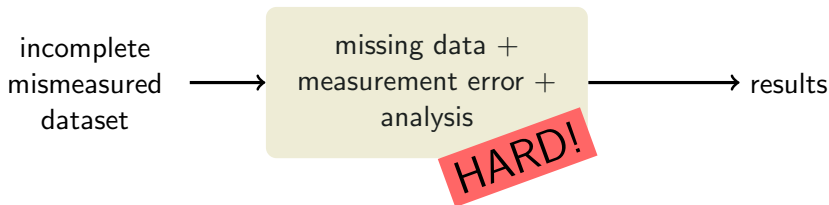
Missing Data and Measurement Error

APPLICATION-SPECIFIC METHODS:



Missing Data and Measurement Error

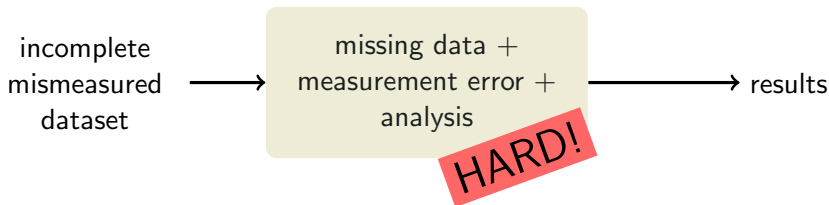
APPLICATION-SPECIFIC METHODS:



MULTIPLE OVERIMPUTATION:

Missing Data and Measurement Error

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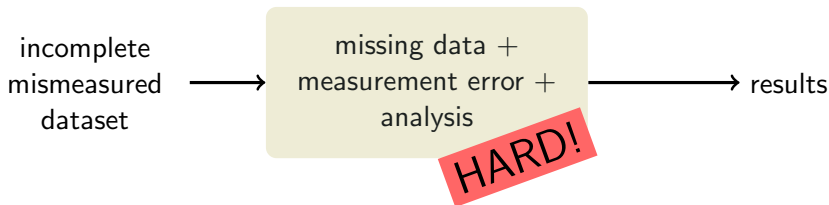


MULTIPLE OVERIMPUTATION:

incomplete
mismeasured
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Missing Data and Measurement Error

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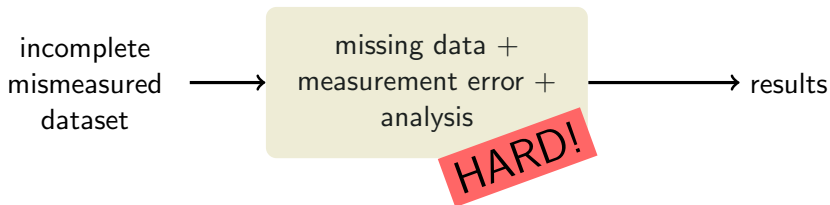


MULTIPLE OVERIMPUTATION:

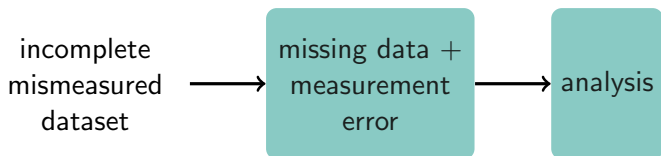


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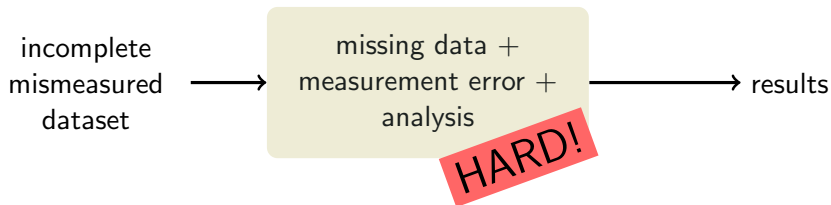


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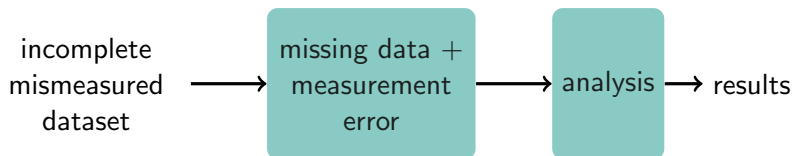


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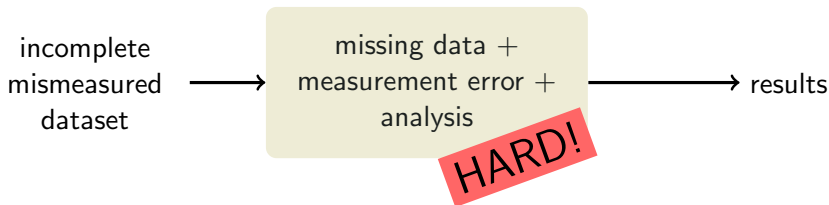


MULTIPLE OVERIMPUTATION:

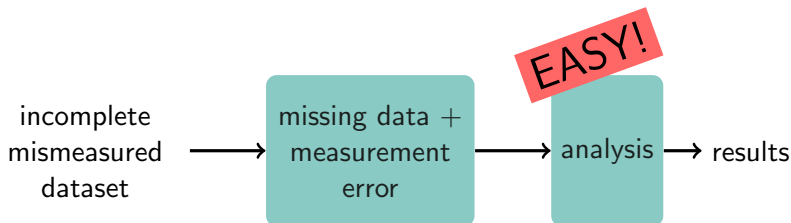


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APPLICATION-SPECIFIC METHODS:

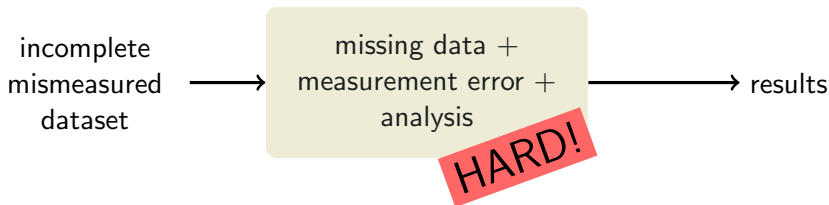


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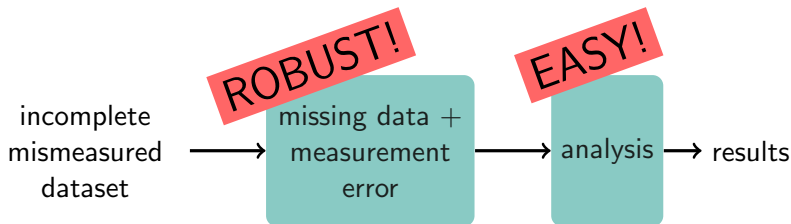


Missing Data and Measurement Error

APPLICATION-SPECIFIC METHODS:



MULTIPLE OVERIMPUTATION:



What MO allows you to do:

What MO allows you to do:
social science.

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	country	polityiv	f-house	log-gdppc	primary
1	BUKINA FASO		6	6.23	5.92
2	LIBERIA		3		
3	SIERRA LEONE		3	6.60	
4	GHANA		6	6.86	12.68
5	TOGO		5	6.27	17.34
6	CAMEROON		5	6.93	15.47
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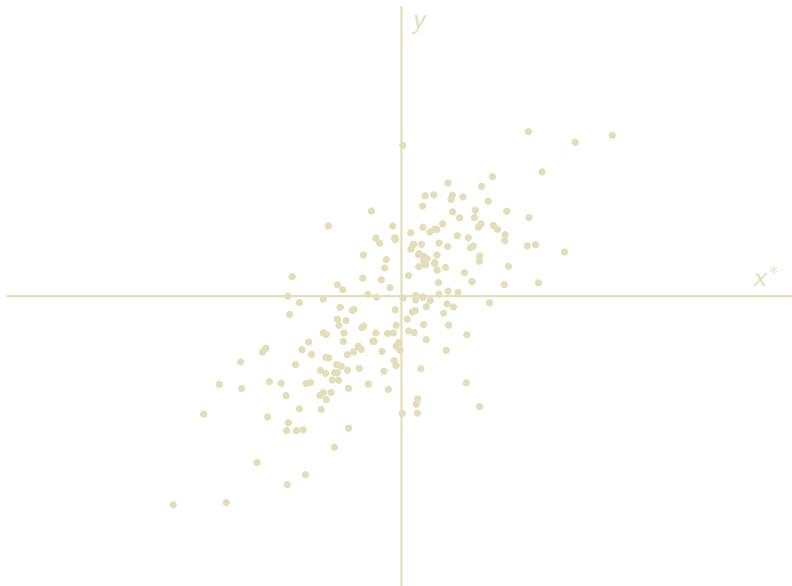
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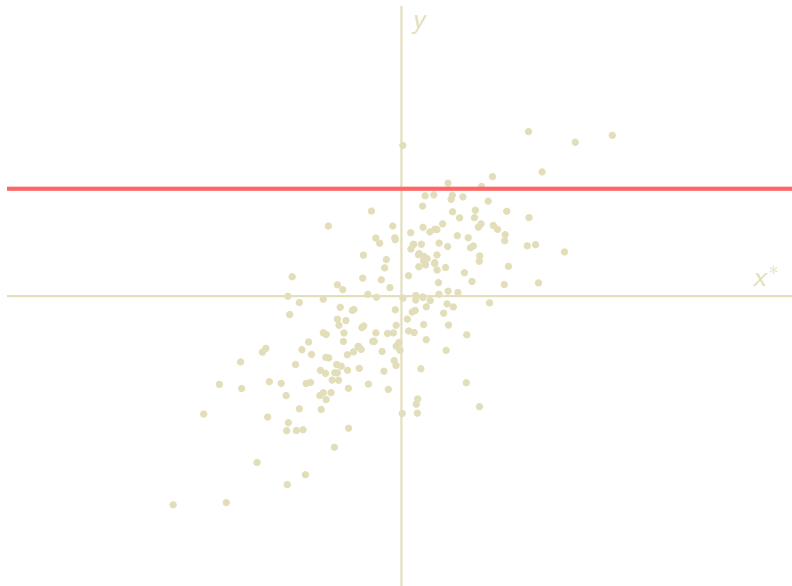
Run whatever analysis model you wanted to run.

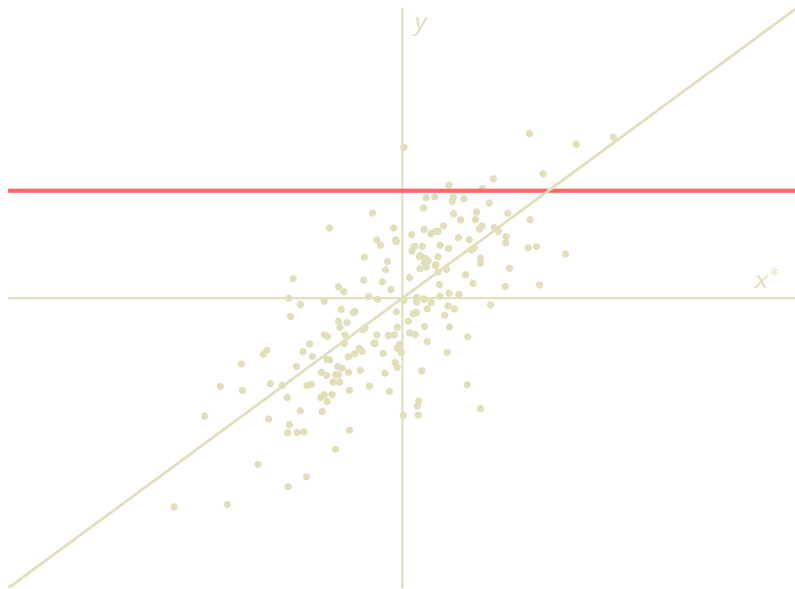
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($\times 5$)

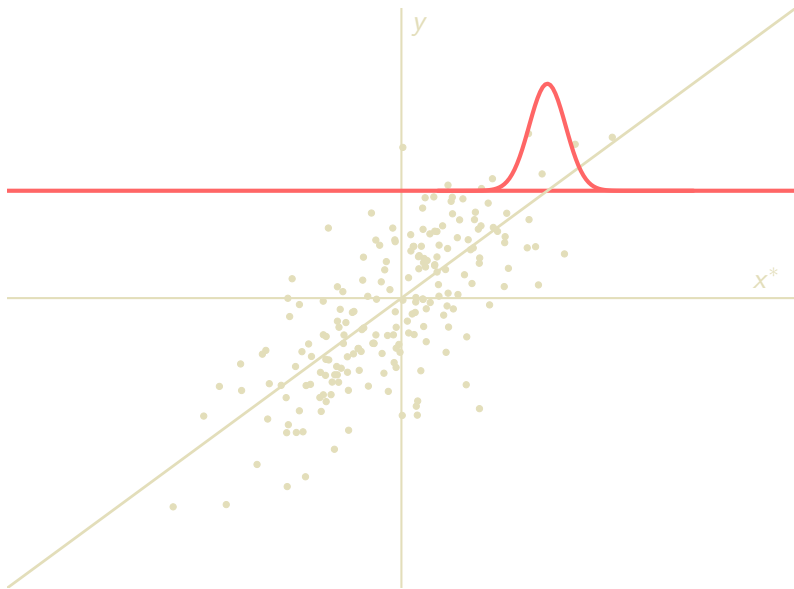
But how does it work?

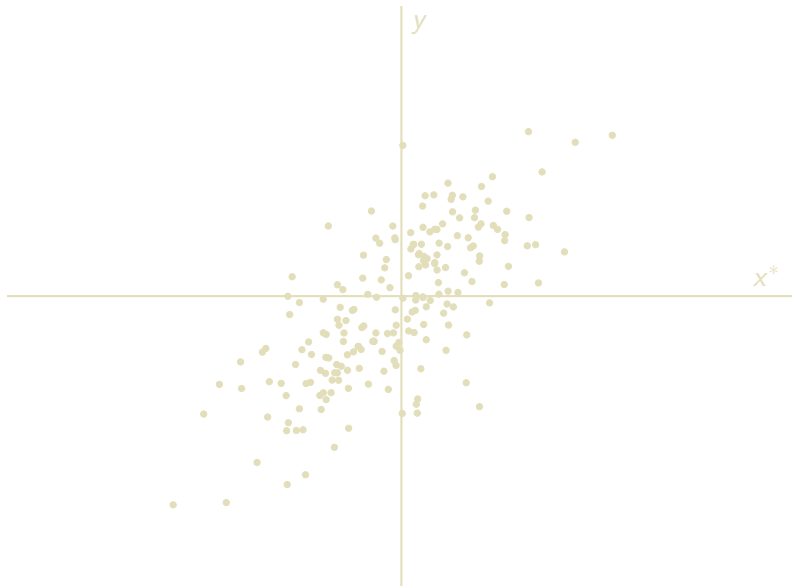
Let's look at the extreme case first.

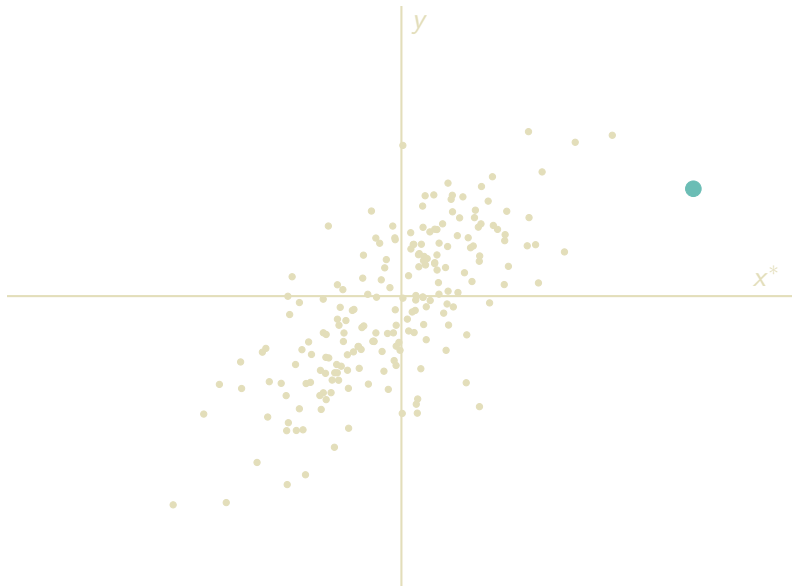


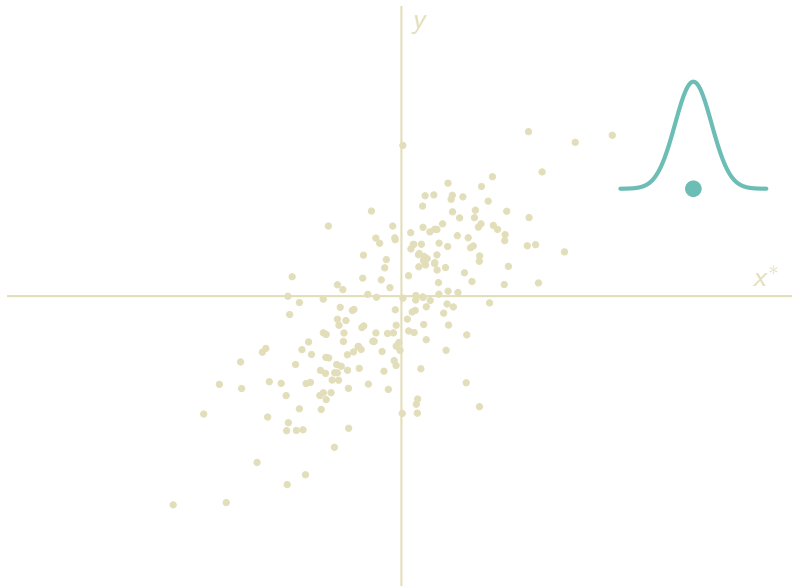


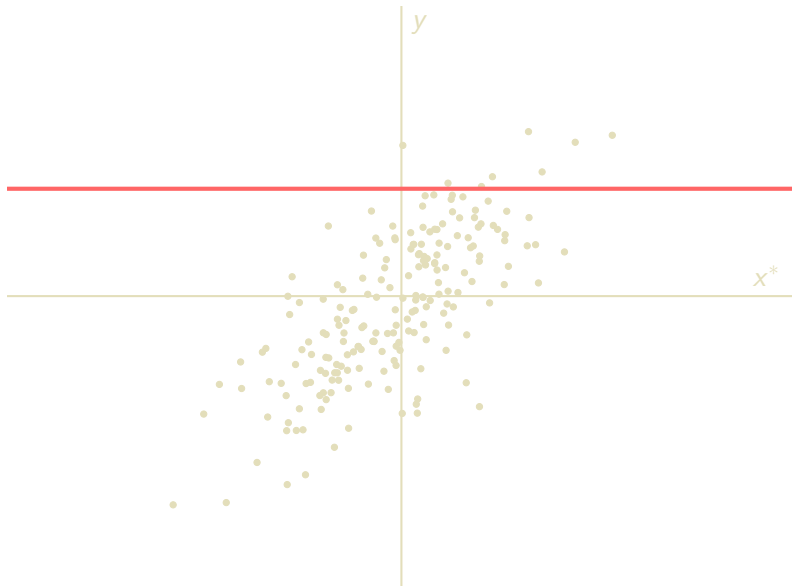


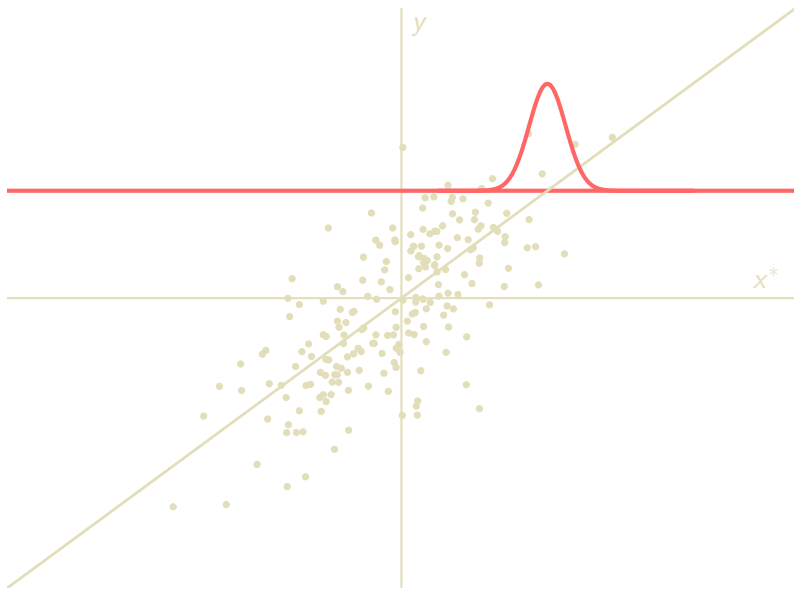


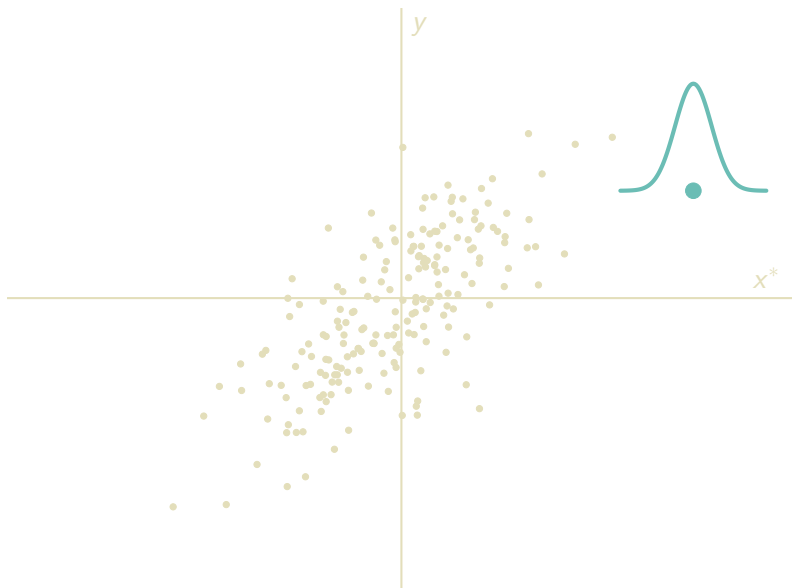


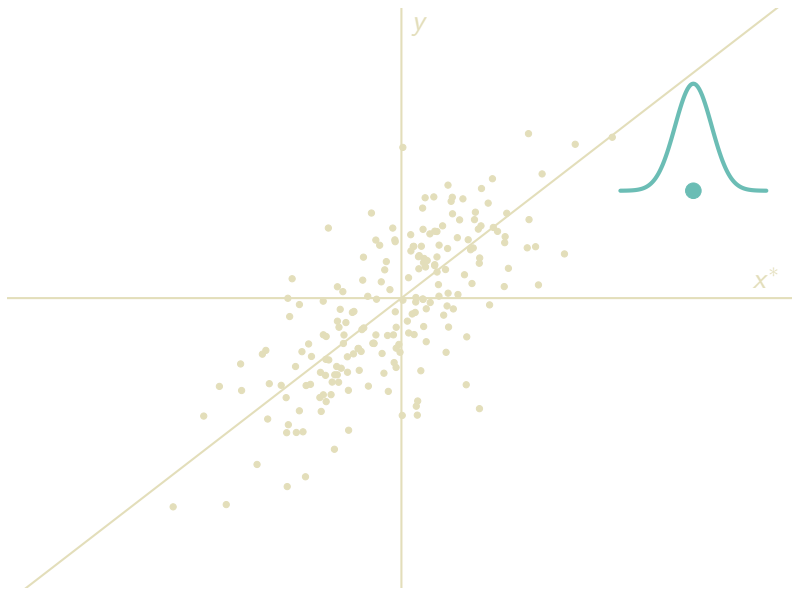


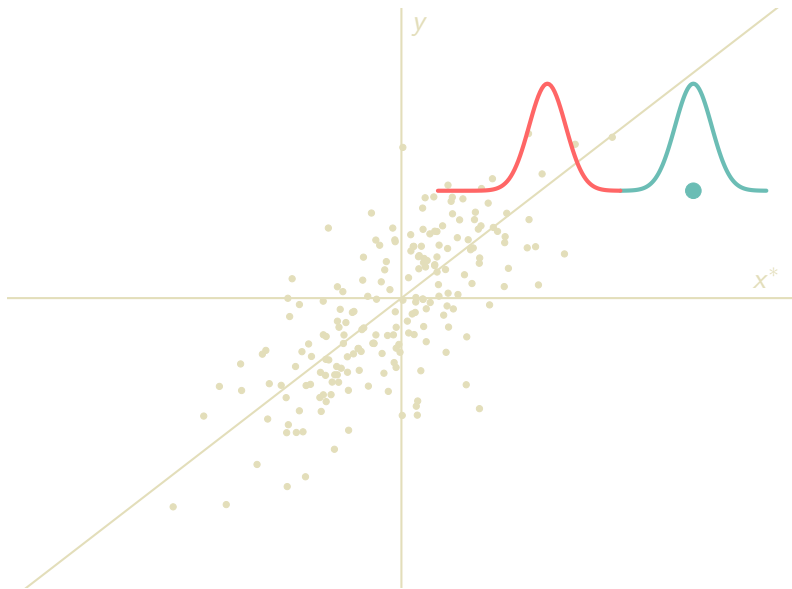


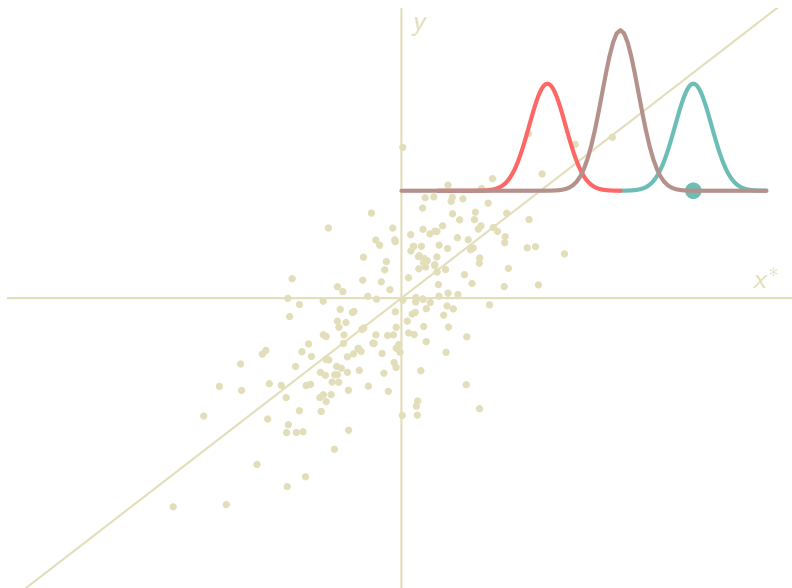












ARBITRARY PATTERNS OF MISMEASUREMENT & MISSINGNESS:

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	country	polityiv	f-house	log-gdppc	primary
1	BUKINA FASO	4	≈ 6	6.23	5.92
2	LIBERIA	NA	3	NA	NA
3	SIERRA LEONE	3	3	6.60	NA
4	GHANA	≈ 9	6	6.86	12.68
5	TOGO	NA	5	6.27	17.34
6	CAMEROON	≈ 6	5	6.93	15.47
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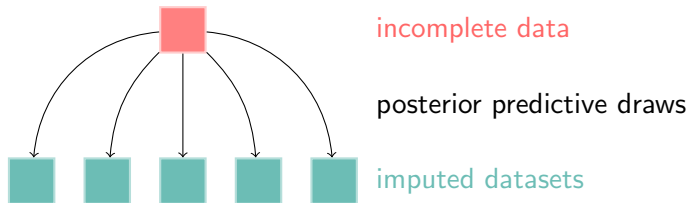
The Multiple Imputation Scheme

The Multiple Imputation Scheme

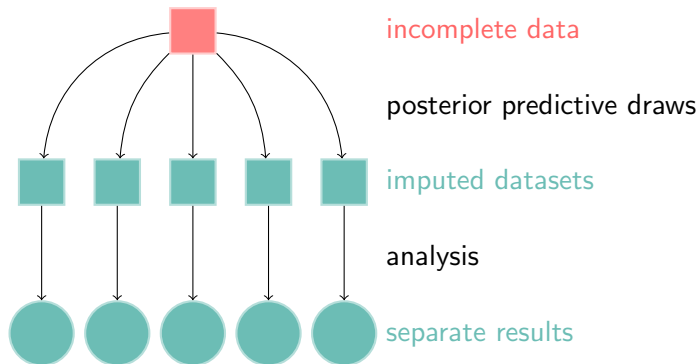


incomplete data

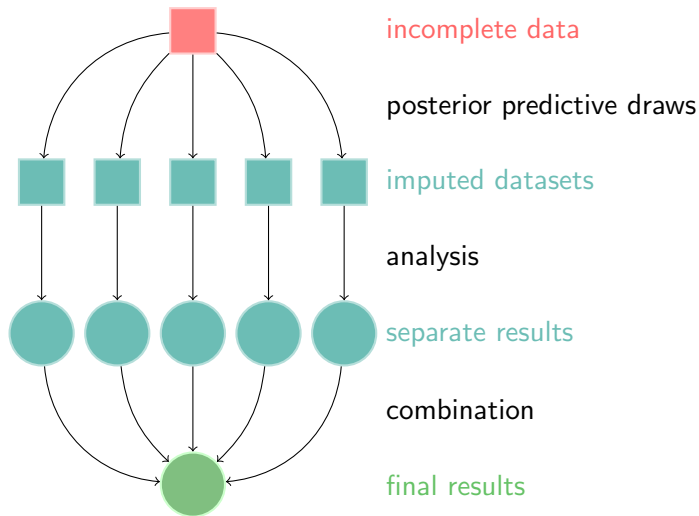
The Multiple Imputation Scheme



The Multiple Imputation Scheme



The Multiple Imputation Scheme



(1)

Mismeasured at random (MMAR).

(2)

You have to know how things were mismeasured.

(2)

You have to know $f(x_i|x_i^*)$.

(3*)

Measurement error and ideal data are statistically dual.

OUR SPECIFIC MODEL

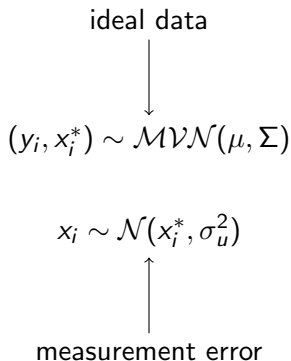
OUR SPECIFIC MODEL

ideal data



$$(y_i, x_i^*) \sim \mathcal{MVN}(\mu, \Sigma)$$

OUR SPECIFIC MODEL







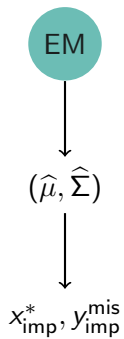
EM



$(\hat{\mu}, \hat{\Sigma})$



x_{imp}^*



CHOOSE A VALUE OF σ_u^2

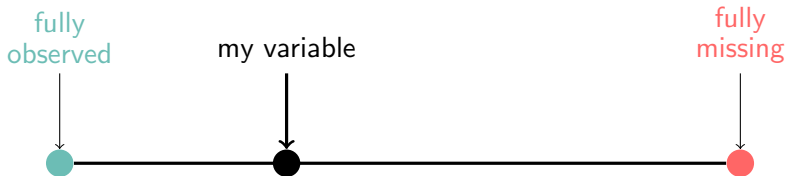
fully
observed



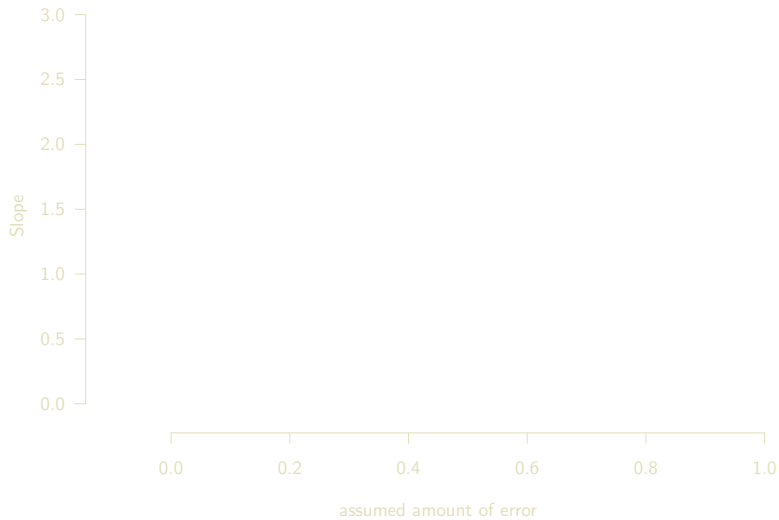
fully
missing

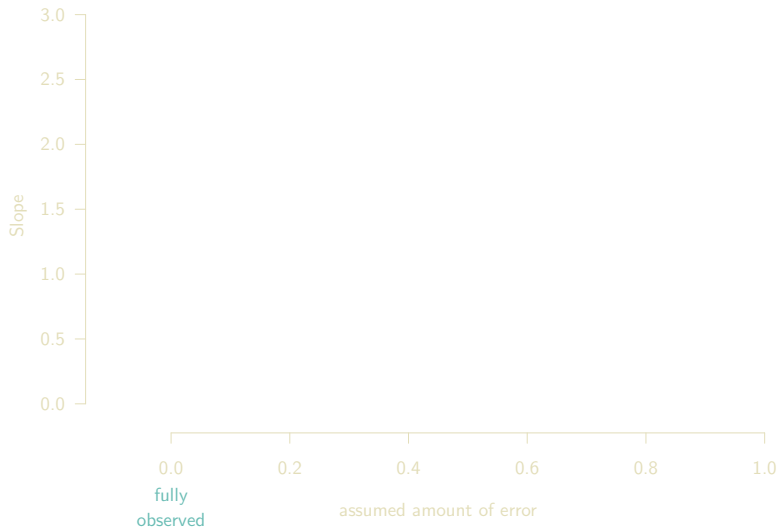


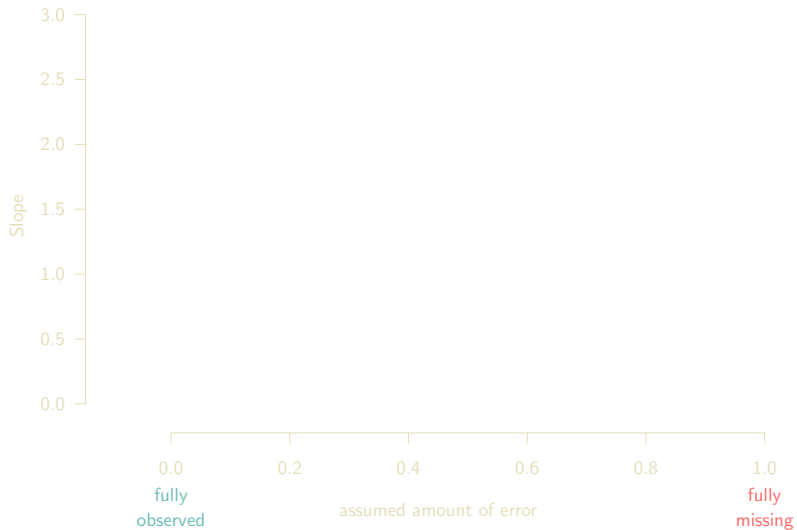
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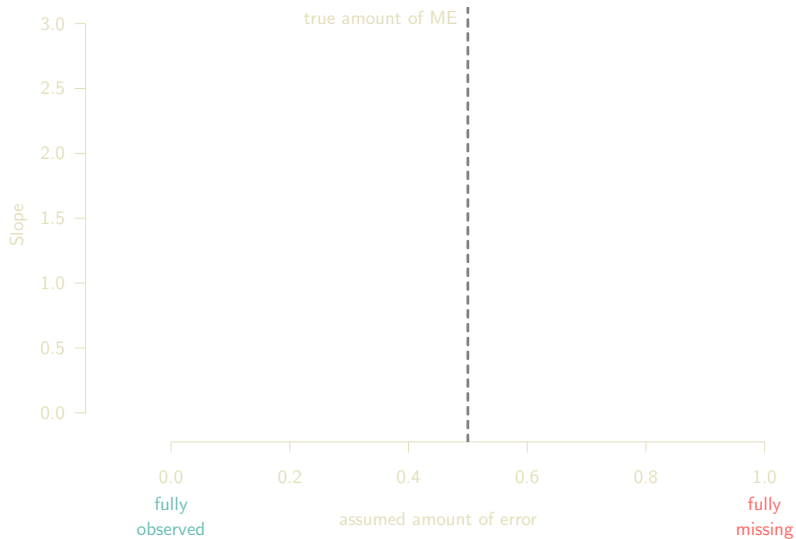


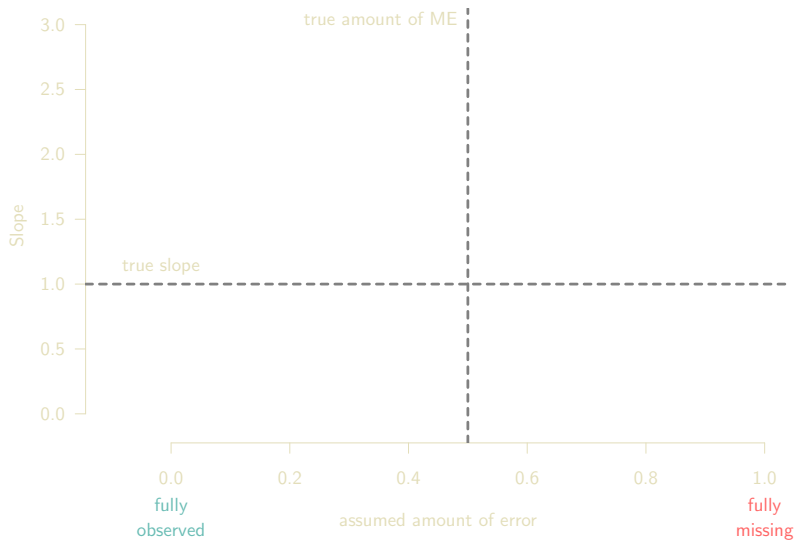
Some simulations.

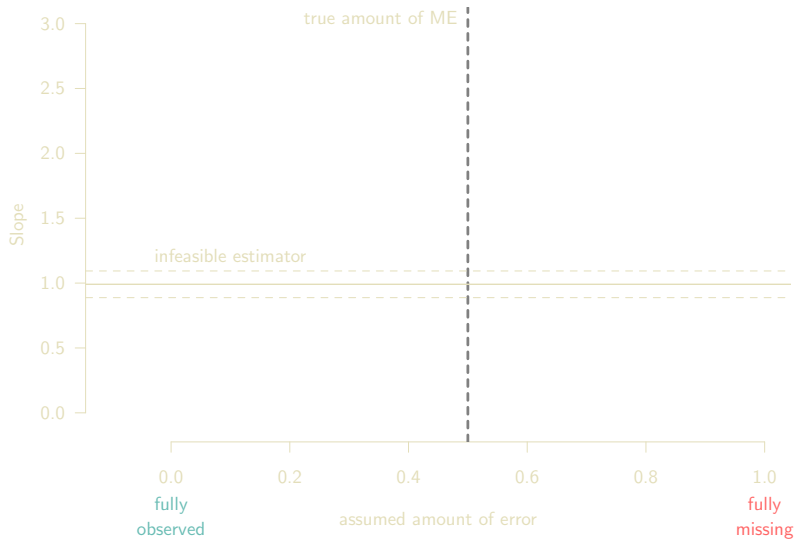


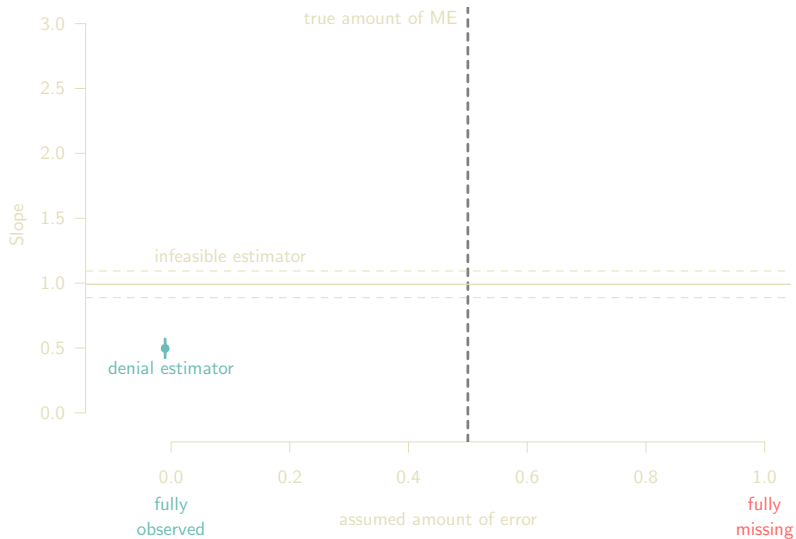


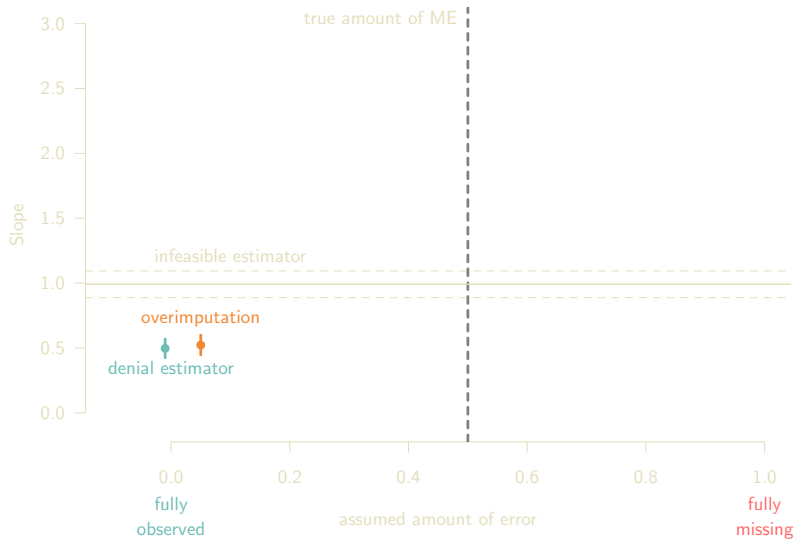


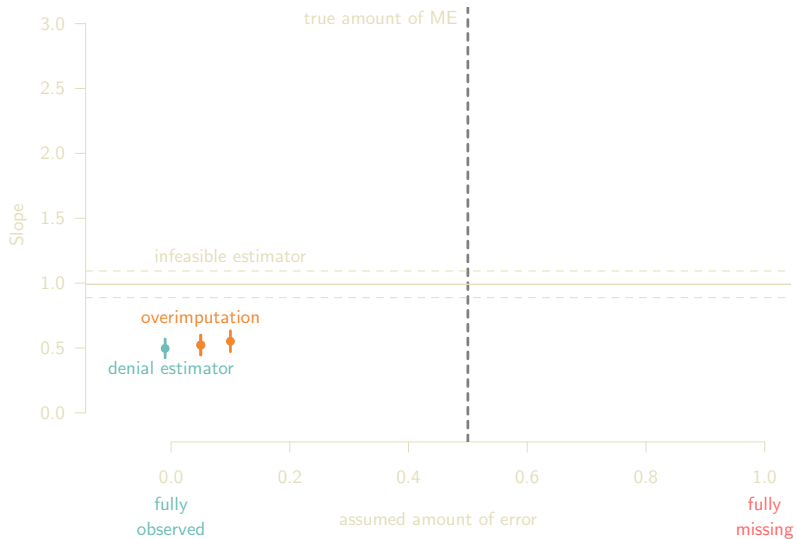


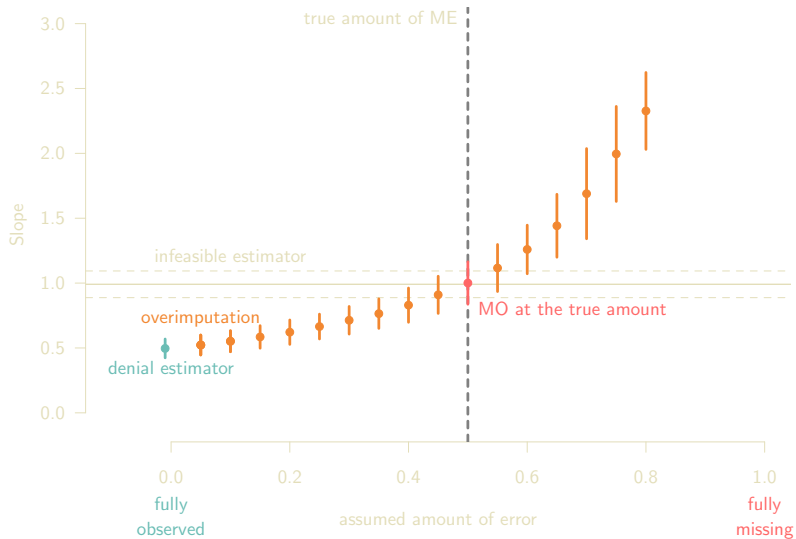


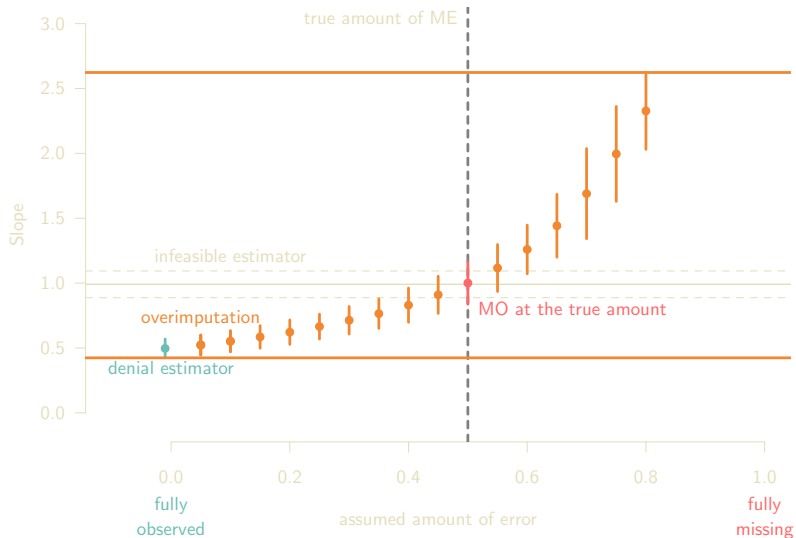






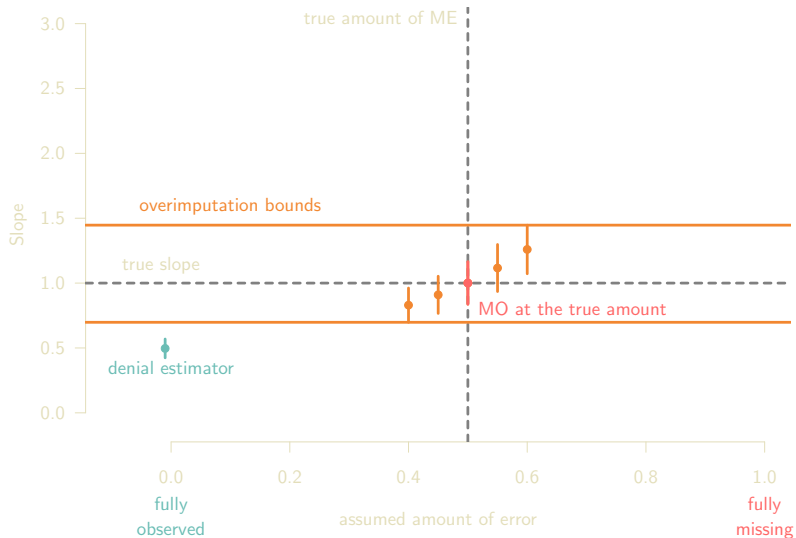






CHOOSE A RANGE OF σ_u^2





Missing Data

Measurement Error

20 Years Ago

Today

Missing Data

Measurement Error

TAILORED METHODS:

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20 Years Ago

Today

Missing Data

Measurement Error

20 Years Ago

TAILORED METHODS:
Model dependent

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Today

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MULTIPLE IMPUTATION:

	Missing Data	Measurement Error
20 Years Ago	TAILORED METHODS: Model dependent Difficult to implement Dubious assumptions	TAILORED METHODS: Model dependent Difficult to implement Dubious assumptions
Today	MULTIPLE IMPUTATION: Broadly applicable	

Missing Data

Measurement Error

20 Years Ago

TAILORED METHODS:

Model dependent

Difficult to implement

Dubious assumptions

TAILORED METHODS:

Model dependent

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Dubious assumptions

Today

MULTIPLE IMPUTATION:

Broadly applicable

Easy to implement

	Missing Data	Measurement Error
20 Years Ago	TAILORED METHODS: - Model dependent - Difficult to implement - Dubious assumptions	TAILORED METHODS: - Model dependent - Difficult to implement - Dubious assumptions
Today	MULTIPLE IMPUTATION: - Broadly applicable - Easy to implement - Widely used.	

Missing Data

Measurement Error

20 Years Ago

TAILORED METHODS:

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Difficult to implement
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TAILORED METHODS:

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Today

MULTIPLE IMPUTATION:

Broadly applicable
Easy to implement
Widely used.

MULTIPLE OVERIMPUTATION

- 1 Mixture Models
 - Basic Mixtures
 - Application: Mixtures as Preprocessing
 - Application: Mixture of Regressions
- 2 Expectation Maximization
 - EM for Probit Regression
 - EM for Gaussian Mixtures
 - EM in General
- 3 Missing Data
 - Motivating Example
 - Overview and Assumptions
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- 5 Appendix: Additional Details and Examples

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How Bad Is Listwise Deletion?

[◀ Return](#)

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Goal: estimate β_1 , where X_2 has λ missing values (y , X_1 are fully observed).

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[◀ Return](#)

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The choice in real research:

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The choice in real research:

Infeasible Estimator Regress y on X_1 and a fully observed X_2 , and use b_1' , the coefficient on X_1 .

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The choice in real research:

Infeasible Estimator Regress y on X_1 and a fully observed X_2 , and use b_1' , the coefficient on X_1 .

Omitted Variable Estimator Omit X_2 and estimate β_1 by b_1^O , the slope from regressing y on X_1 .

Goal: estimate β_1 , where X_2 has λ missing values (y , X_1 are fully observed).

$$E(y) = X_1\beta_1 + X_2\beta_2$$

The choice in real research:

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Omitted Variable Estimator Omit X_2 and estimate β_1 by b_1^O , the slope from regressing y on X_1 .

Listwise Deletion Estimator Perform listwise deletion on $\{y, X_1, X_2\}$, and then estimate β_1 as b_1^L , the coefficient on X_1 when regressing y on X_1 and X_2 .

In the best case scenerio for listwise deletion (MCAR), should we delete listwise or omit the variable?

Mean Square Error as a measure of the badness of an estimator $\hat{a}$ of a .

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$$\text{MSE}(\hat{a}) = E[(\hat{a} - a)^2]$$

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$$\begin{aligned}\text{MSE}(\hat{a}) &= E[(\hat{a} - a)^2] \\ &= V(\hat{a}) + [E(\hat{a} - a)]^2\end{aligned}$$

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To compare, compute

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To compare, compute

$$\text{MSE}(b_1^L) - \text{MSE}(b_1^O) = \begin{cases} > 0 & \text{when omitting the variable is better} \\ < 0 & \text{when listwise deletion is better} \end{cases}$$

Derivation and Implications

Derivation and Implications

$$\text{MSE}(b_1^L) - \text{MSE}(b_1^O)$$

Derivation and Implications

$$\text{MSE}(b_1^L) - \text{MSE}(b_1^O) = \left(\frac{\lambda}{1 - \lambda} V(b_1^I) \right) + F[V(b_2^I) - \beta_2 \beta_2'] F'$$

Derivation and Implications

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6. **Conclusion:** Listwise deletion is often as bad a problem as the much better known omitted variable bias — in the best case scenerio (MCAR)

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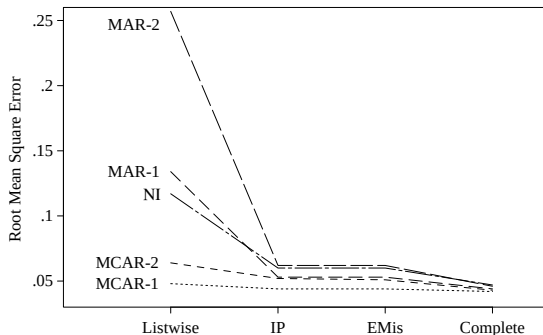
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I.e., you don't trust data to impute D_{mis} but still trust it to analyze D_{obs}

Root Mean Square Error Comparisons



Each point is RMSE averaged over two regression coefficients in each of 100 simulated data sets. (IP and EMis have the same RMSE, which is lower than listwise deletion and higher than the complete data; its the same for EMB.)

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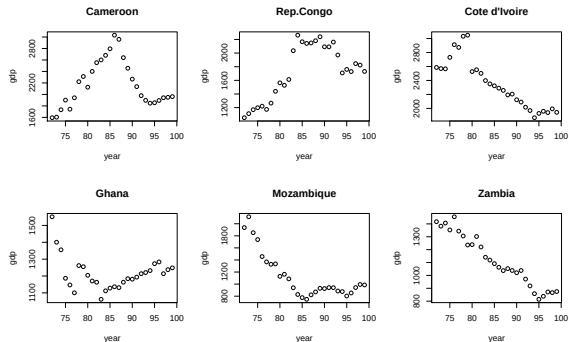
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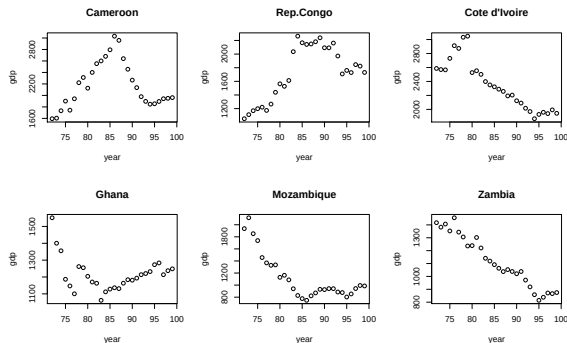
- (a) MI estimator is more efficient, with a smaller SE
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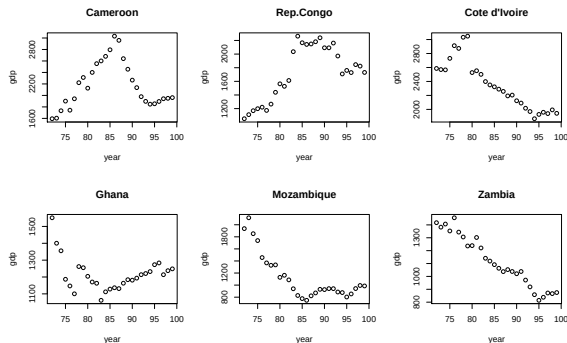


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Include: (1) fixed effects, (2) time trends, and (3) priors for cells

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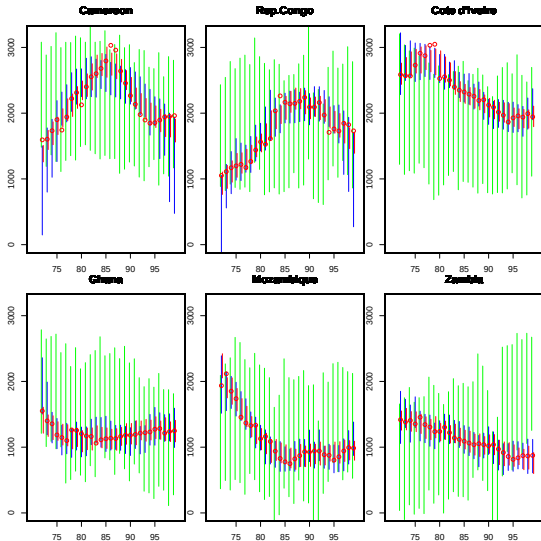


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Read: James Honaker and Gary King, "What to do About Missing Values in Time Series Cross-Section Data,"

<http://gking.harvard.edu/files/abs/pr-abs.shtml>

Imputation one Observation at a time



Circles=true GDP; green=no time trends; blue=polynomials; red=LOESS

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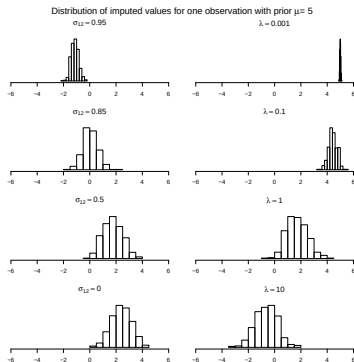
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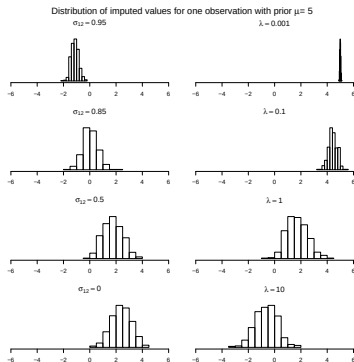
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- Honaker and King show how to modify these “data augmentation priors” to put priors on missing values rather than on μ and σ (or β).

Posterior imputation: mean=0, prior mean=5



Left column: holds prior $N(5, \lambda)$ constant ($\lambda = 1$) and changes predictive strength (the covariance, σ_{12}).

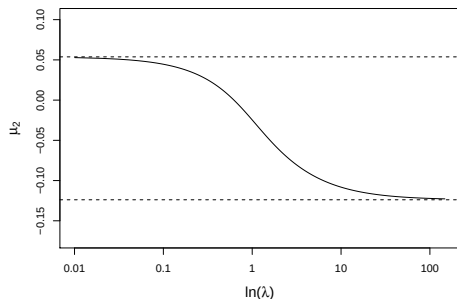
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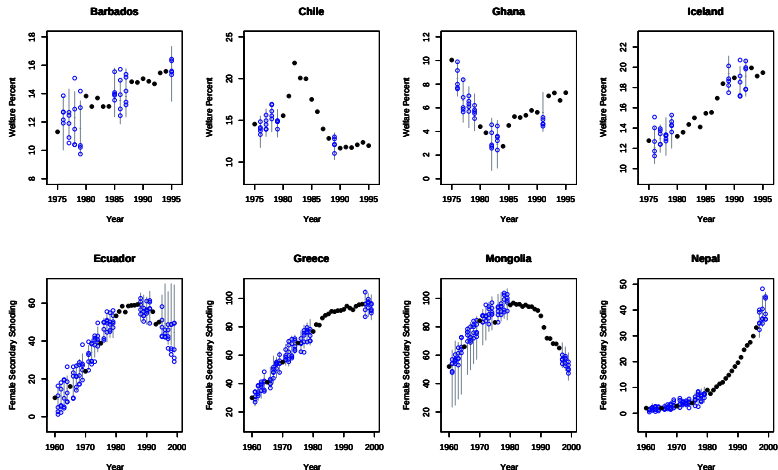
Right column: holds predictive strength of data constant (at $\sigma_{12} = 0.5$) and changes the strength of the prior (λ).

Model Parameters Respond to Prior on a Cell Value



Prior: $p(x_{12}) = N(5, \lambda)$. The parameter approaches the theoretical limits (dashed lines), upper bound is what is generated when the missing value is filled in with the expectation; lower bound is the parameter when the model is estimated without priors. The overall movement is small.

Replication of Baum and Lake; Imputation Model Fit



Black = observed. Blue circles = five imputations; Bars = 95% CIs

	Listwise Deletion	Multiple Imputation
Life Expectancy		
Rich Democracies	-.072 (.179)	.233 (.037)
Poor Democracies	-.082 (.040)	.120 (.099)
N	1789	5627
Secondary Education		
Rich Democracies	.948 (.002)	.948 (.019)
Poor Democracies	.373 (.094)	.393 (.081)
N	1966	5627

Replication of Baum and Lake; the effect of being a democracy on life expectancy and on the percentage enrolled in secondary education (with p-values in parentheses).