

Soc500: Applied Social Statistics

Week 1: Introduction and Probability

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Princeton

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¹These slides are heavily influenced by Matt Blackwell, Adam Glynn and Matt Salganik. The spam filter segment is adapted from Justin Grimmer and Dan Jurafsky. Illustrations by Shay O'Brien.

Where We've Been and Where We're Going...

- Last Week
 - ▶ methods camp
 - ▶ pre-grad school life
- This Week
 - ▶ Wednesday
 - ★ welcome
 - ★ basics of probability
- Next Week
 - ▶ random variables
 - ▶ joint distributions
- Long Run
 - ▶ probability $\rightarrow$ inference $\rightarrow$ regression

Questions?

Welcome and Introductions

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- 2 Goals
- 3 Ways to Learn
- 4 Structure of Course
- 5 Introduction to Probability
 - What is Probability?
 - Sample Spaces and Events
 - Probability Functions
 - Marginal, Joint and Conditional Probability
 - Bayes' Rule
 - Independence
- 6 Fun With History

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- First in a two course sequence $\rightsquigarrow$ replication project
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- Difficult course but with many resources to support you.
- When we are done you will be able to teach **yourself** many things

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 - ▶ write **clean**, **reusable**, and **reliable** R code.
 - ▶ feel **empowered** working with data

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 - ▶ build a solid, **reproducible research pipeline** to go from raw data to final paper
 - ▶ provide you with the tools to produce your **own research** (e.g. second year empirical paper).

Why R?

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- It will help you do **research**

Why RMarkdown?

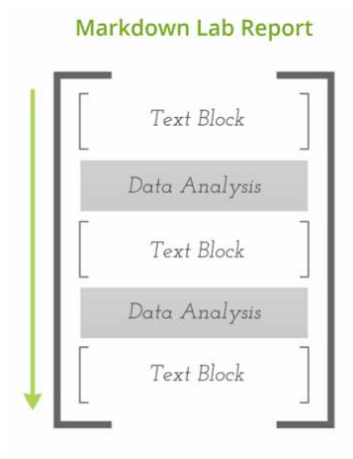
What you've done before



Baumer et al (2014)

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- We will teach you any math you need as we go along
- Crucially though- this class is **not** about statistical aptitude, it is about **effort**

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- **Lecture**
learn broad topics

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- When and how to do the reading

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Note: You may find these difficult. Start early and seek help!

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Your Job: get **help** when you need it!

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- Also thanks to those who have discussed with me at length including Dalton Conley, Chad Hazlett, Gary King, Kosuke Imai, Matt Salganik and Teppei Yamamoto.
- Shay O'Brien produced the hand-drawn illustrations used throughout.

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- Probability $\rightarrow$ Inference $\rightarrow$ Regression

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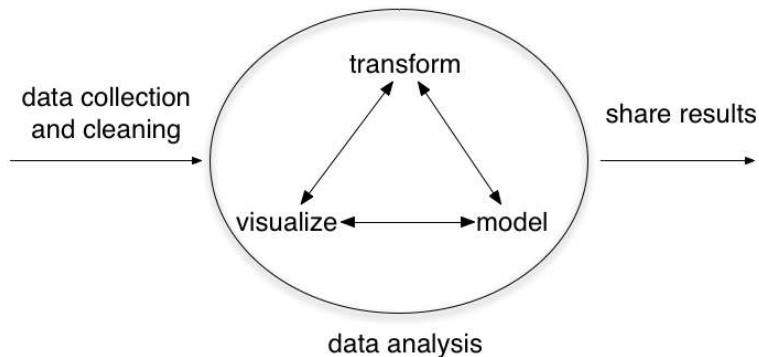
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 - 3 statisticians **generalize** and **export** the best of these methods

Quantitative Research in Theory

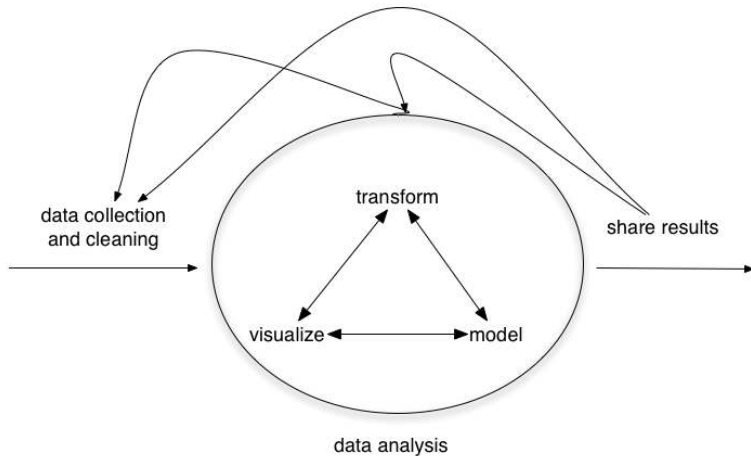
Quantitative Research in Theory



Inspiration: Hadley Wickham, Image: Matt Salganik

Quantitative Research in Practice

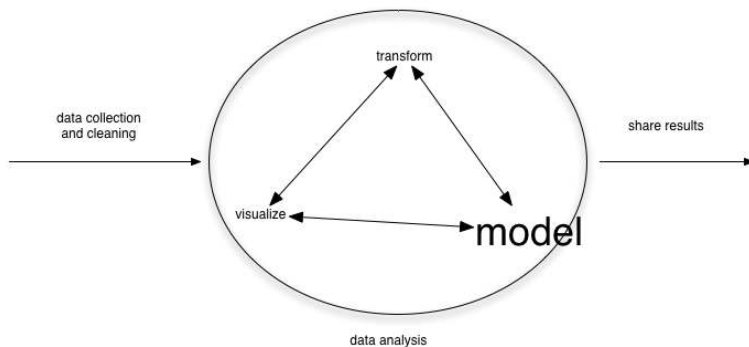
Quantitative Research in Practice



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Traditional Statistics Class

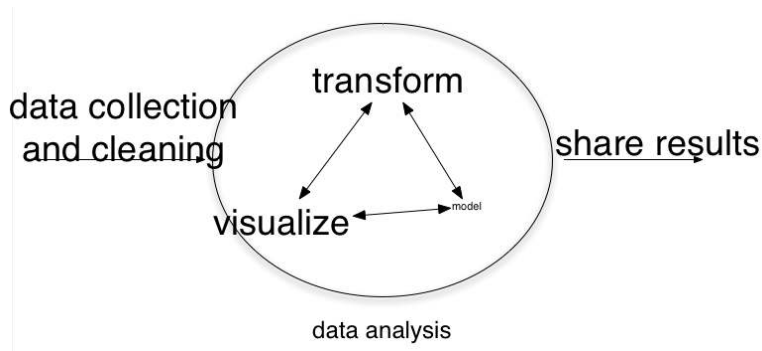
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Time Actually Spent

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 - ▶ problem sets with real data
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- Teaching select key principles from statistics

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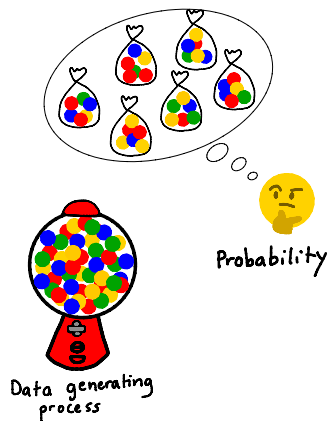
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 - ▶ This allows us to have uncertainty over outcomes given our inputs

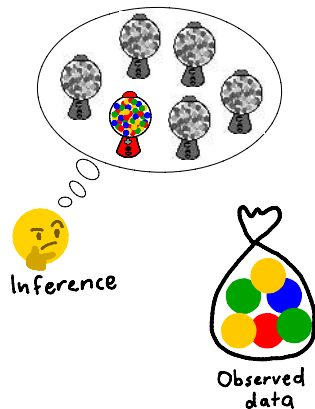
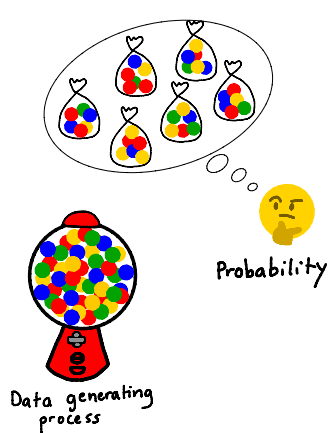
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 - ▶ This allows us to have uncertainty over outcomes given our inputs
- Our way of talking about stochastic outcomes is **probability**.

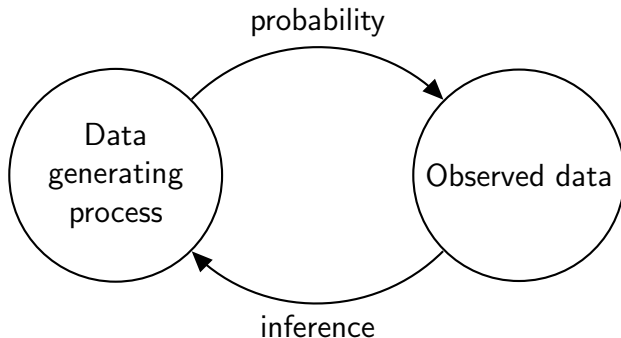
In Picture Form



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Statistical Thought Experiments

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- Allows us to contemplate world under hypothetical scenarios
 - ▶ hypotheticals let us ask- is the observed relationship happening by chance or is it systematic?
 - ▶ it tells us what the world would look like under a certain assumption
- We will review probability today, but feel free to ask questions as needed.

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- The Story Setup



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- The Story Setup
(lady discerning about tea)



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- **The Experiment**
(perform a taste test)



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- The Story Setup
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- The Hypothetical

Tea-Tasting Distribution		
Success count	Permutations of selection	Number of permutations
0	OOOO	$1 \times 1 = 1$
1	OXXX, OXOX, OXOX, XOXO	$4 \times 4 = 16$
2	OOXX, OXOX, OXOX, XOXO, XXOO, XOOX	$6 \times 6 = 36$
3	OOXX, XOXO, XOXO, XXOO	$4 \times 4 = 16$
4	XXXX	$1 \times 1 = 1$
Total		70

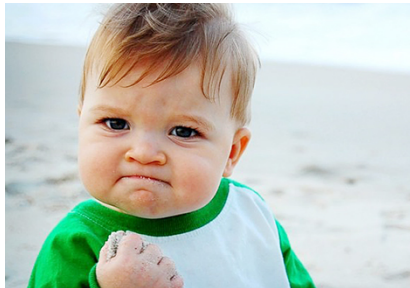
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- The Story Setup
(lady discerning about tea)
- The Experiment
(perform a taste test)
- **The Hypothetical**
(count possibilities)

Success count	Permutations of selection	Number of permutations
0	0000	$1 \times 1 = 1$
1	000X, 00X0, 0X00, X000	$4 \times 4 = 16$
2	00XX, 0X0X, 0XX0, X0X0, XX00, X00X	$6 \times 6 = 36$
3	0XXX, X0XX, XX0X, XXX0	$4 \times 4 = 16$
4	XXXX	$1 \times 1 = 1$
Total		70

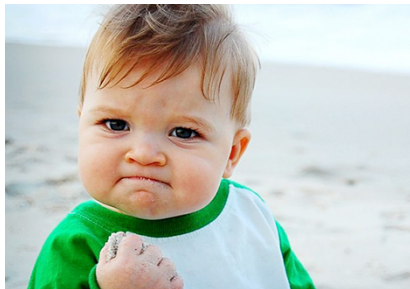
Example: Fisher's Lady Tasting Tea

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(lady discerning about tea)
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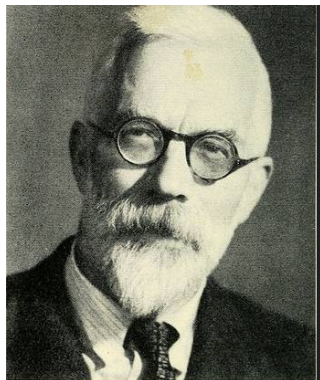
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This became the Fisher Exact Test.

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- 2 Goals
- 3 Ways to Learn
- 4 Structure of Course
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- Data Analysis: estimate probability that something will happen
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All the rules of probability can be derived from these axioms.

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(Note we defined illogical guesses to be prob= 0)

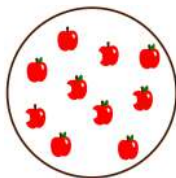
A Running Visual Metaphor

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Imagine that we sample an apple from a bag.

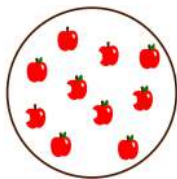
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and

$$\{ \text{apple} \}$$

are both **events**.

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$$\text{🍏} \in A, \quad \text{🍏} \in A, \quad \text{🍏} \notin A, \quad \text{🍏} \notin A$$

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A **complement** of event A is a set: A^c , is collection of all of the outcomes not in A . That is, it is “everything else” in the sample space.

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Important complement: $\Omega^c = \emptyset$, where $\emptyset$ is the **empty set**—it’s just the event that nothing happens.

Operations on Events

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The **union** of two events, A and B is the event that A or B occurs:

$$\text{🍷} \cup \text{🍎} =$$

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Sample spaces can have infinite outcomes $A_1, A_2, \dots$

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Intuition: probability as allocating chunks of a unit-long stick.

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- ▶ Example: The probability of drawing 5 red cards out of 10 drawn from a deck of cards is the frequency with which this event occurs in repeated samples of 10 cards.

Marginal and Joint Probability

So far we have only considered situations where we are interested in the probability of a single event A occurring. We've denoted this $P(A)$. $P(A)$ is sometimes called a **marginal probability**.

Marginal and Joint Probability

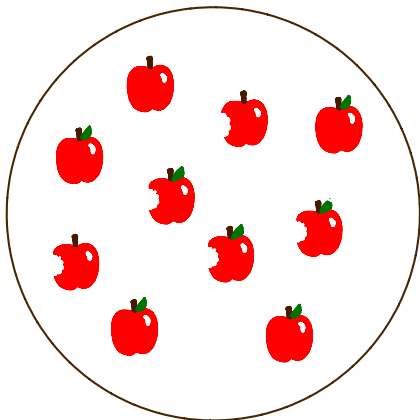
So far we have only considered situations where we are interested in the probability of a single event A occurring. We've denoted this $P(A)$. $P(A)$ is sometimes called a **marginal probability**.

Suppose we are now in a situation where we would like to express the probability that an event A and an event B occur. This quantity is written as $P(A \cap B)$, $P(B \cap A)$, $P(A, B)$, or $P(B, A)$ and is the **joint probability** of A and B .

$$P(\text{🍌}, \text{🍎}) = P(\text{🍎}) = P(\text{🍌} \cap \text{🍎})$$

$$P(\text{brown and green}) = ?$$

$$P(\text{brown and green}, \text{red}) = ?$$



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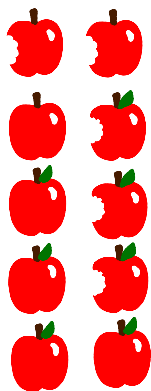
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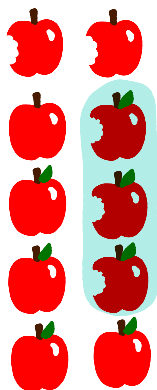
Conditional Probability: A Visual Example

$$P(\text{brown leaf} \mid \text{red apple}) = \frac{P(\text{brown leaf, red apple})}{P(\text{red apple})}$$



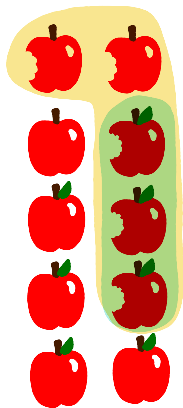
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$$P(\text{green apple} \mid \text{red apple}) = \frac{P(\text{green apple}, \text{red apple})}{P(\text{red apple})}$$



Conditional Probability: A Visual Example

$$P(\text{green apple} \mid \text{bitten apple}) = \frac{P(\text{green apple, bitten apple})}{P(\text{bitten apple})}$$



A Card Player's Example

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$A = \{\text{King on Draw 1}\}$ and $B = \{\text{King on Draw 2}\}$, then

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Law of Total Probability (LTP)

With 2 Events:

$$\begin{aligned}P(B) &= P(B, A) + P(B, A^c) \\&= P(B|A) \times P(A) + P(B|A^c) \times P(A^c)\end{aligned}$$

$$\begin{aligned}P(\text{🍏}) &= P(\text{🍏} | \text{🌿}) \times P(\text{🌿}) + P(\text{🍏} | \text{🍷}) \times P(\text{🍷}) \\&= P(\text{🍏} | \text{🌿}) \times P(\text{🌿}) + P(\text{🍏} | \text{🍷}) \times P(\text{🍷})\end{aligned}$$

Recall, if we randomly draw two cards from a standard 52 card deck and define the events $A = \{\text{King on Draw 1}\}$ and $B = \{\text{King on Draw 2}\}$, then

- $P(A) = 4/52$
- $P(B|A) = 3/51$
- $P(A, B) = P(A) \times P(B|A) = 4/52 \times 3/51$

Question: $P(B) = ?$

Confirming Intuition with the LTP

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$$\begin{aligned}P(B) &= 3/51 \times 1/13 + 4/51 \times 12/13 \\&= \frac{3 + 48}{51 \times 13} = \frac{1}{13} = \frac{4}{52}\end{aligned}$$

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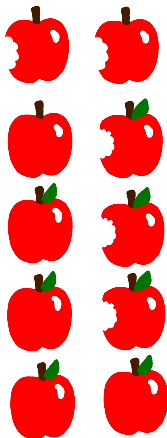
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- Proof: combine the multiplication rule $\Pr(B|A) \Pr(A) = P(A \cap B)$, and the definition of conditional probability

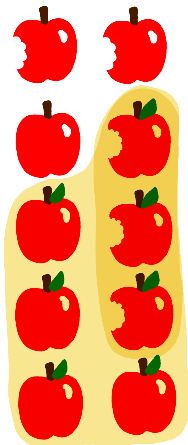
Bayes' Rule Mechanics

$$P(\text{worm} | \text{apple}) = \frac{P(\text{apple} | \text{worm}) P(\text{worm})}{P(\text{apple})}$$



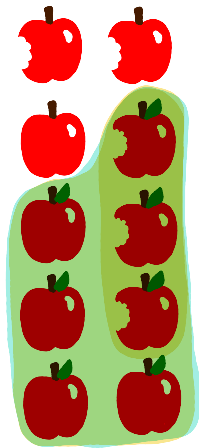
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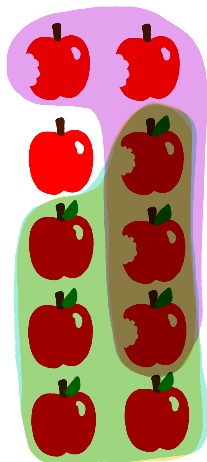
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U.S. Billionaires, 2014



Women



Men

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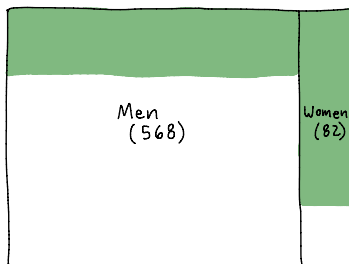
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$$P(W|I) = \frac{P(I|W)P(W)}{P(I)}$$

$$= \frac{.765 \left(\frac{82}{568+82} \right)}{.765(82) + .245(568)}$$

$$= .31$$



*Data source = Billionaires characteristics database

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$$\Pr(\text{AfAm}|\text{Wash}) = \frac{\Pr(\text{Wash}|\text{AfAm}) \Pr(\text{AfAm})}{\Pr(\text{Wash})}$$

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Independence is a massively important concept in statistics.

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- A word from your preceptors

Fun With



Fun with

Fun with History

Fun with History



Legendre

Fun with History



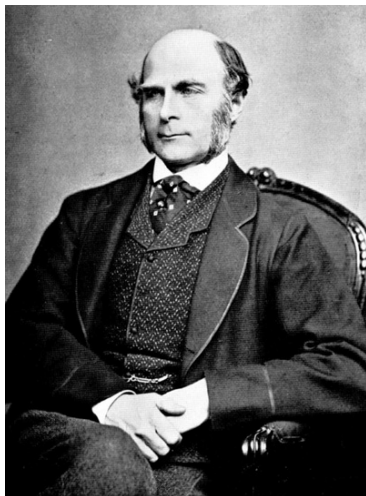
Gauss

Fun with History



Quetelet

Fun with History



Galton

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