

Week 12: Repeated Observations and Panel Data

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Princeton

November 16–20, 2020

¹These slides are heavily influenced by Matt Blackwell, Adam Glynn, Jens Hainmueller, and Erin Hartman.

Where We've Been and Where We're Going...

- Last Week
 - ▶ causal inference with unmeasured confounding
- This Week
 - ▶ panel data
 - ▶ diff-in-diff
 - ▶ fixed effects
 - ▶ wrap-up
- The Following Week
 - ▶ ?
- Long Run
 - ▶ probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causality

- 1 Differencing Models
- 2 Difference-in-Differences
- 3 Fixed Effects
- 4 Non-parametric Identification and Fixed Effects
- 5 Wrap-Up
 - Questions
 - Concluding Thoughts for the Course

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 - ▶ possess some cultural trait correlated with better health outcomes
- If we have data on countries over time, can we make any progress in spite of these problems?

Ross Data

##	cty_name	year	democracy	infmort_unicef
## 1	Afghanistan	1965	0	230
## 2	Afghanistan	1966	0	NA
## 3	Afghanistan	1967	0	NA
## 4	Afghanistan	1968	0	NA
## 5	Afghanistan	1969	0	NA
## 6	Afghanistan	1970	0	215

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- NB: we won't be using T for treatment today because it is extremely consistently used for time. We will end up using D for treatment which is another common letter for treatment.

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- **Time series, cross-sectional (TSCS) data**: smaller n , large T
- We are primarily going to focus on similarities today but there are some differences.

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We will start by considering performance of estimators assuming this model is true.

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 - ① Heteroskedasticity (see clustering from diagnostics week)
 - ② Possible violation of zero conditional mean errors
- Both problems arise out of ignoring the **unmeasured heterogeneity** inherent in a_i

Pooled OLS with Ross data

```
pooled.mod <- lm(log(kidmort_unicef) ~ democracy + log(GDPcur),
                  data = ross)
summary(pooled.mod)

##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  9.76405     0.34491   28.31  <2e-16 ***
## democracy   -0.95525     0.06978  -13.69  <2e-16 ***
## log(GDPcur) -0.22828     0.01548  -14.75  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7948 on 646 degrees of freedom
## (5773 observations deleted due to missingness)
## Multiple R-squared:  0.5044, Adjusted R-squared:  0.5029
## F-statistic: 328.7 on 2 and 646 DF,  p-value: < 2.2e-16
```

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- Ignore the heterogeneity $\rightsquigarrow$ **correlation between the combined error and the independent variables:**

$$E[v_{it}|\mathbf{X}] = E[a_i + u_{it}|\mathbf{X}] \neq 0$$

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- Pooled OLS will be **biased and inconsistent** because zero conditional mean error fails for the combined error.

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- Two time periods:

$$y_{i1} = \mathbf{x}'_{i1}\boldsymbol{\beta} + a_i + u_{i1}$$

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$$= (\mathbf{x}'_{i2}\boldsymbol{\beta} + a_i + u_{i2}) - (\mathbf{x}'_{i1}\boldsymbol{\beta} + a_i + u_{i1})$$

$$= (\mathbf{x}'_{i2} - \mathbf{x}'_{i1})\boldsymbol{\beta} + (a_i - a_i) + (u_{i2} - u_{i1})$$

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- Due to 'no perfect collinearity': $\mathbf{x}_{it}$ has to change over time for **some** units. High variance if its slow moving.
- Differencing will **reduce** the variation in the independent variables and thus **increase** standard errors.

First Differences in R (via plm package)

```
library(plm)

fd.mod <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur), data = ross,
              index = c("id", "year"), model = "fd")

summary(fd.mod)

## Oneway (individual) effect First-Difference Model
##
## Call:
## plm(formula = log(kidmort_unicef) ~ democracy + log(GDPcur),
##      data = ross, model = "fd", index = c("id", "year"))
##
## Unbalanced Panel: n=166, T=1-7, N=649
##
## Residuals :
##      Min. 1st Qu.  Median 3rd Qu.    Max.
## -0.9060 -0.0956   0.0468   0.1410   0.3950
##
## Coefficients :
##              Estimate Std. Error t-value Pr(>|t|)
## (intercept) -0.149469   0.011275 -13.2567 < 2e-16 ***
## democracy   -0.044887   0.024206  -1.8544  0.06429 .
## log(GDPcur) -0.171796   0.013756 -12.4886 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Total Sum of Squares:    23.545
## Residual Sum of Squares: 17.762
## R-Squared      : 0.24561
##      Adj. R-Squared : 0.24408
## F-statistic: 78.1367 on 2 and 480 DF, p-value: < 2.22e-16
```

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Next Time: Difference-in-Differences

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Motivation: Studying the Minimum Wage

Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania

By DAVID CARD AND ALAN B. KRUEGER*

On April 1, 1992, New Jersey's minimum wage rose from \$4.25 to \$5.05 per hour. To evaluate the impact of the law we surveyed 410 fast-food restaurants in New Jersey and eastern Pennsylvania before and after the rise. Comparisons of employment growth at stores in New Jersey and Pennsylvania (where the minimum wage was constant) provide simple estimates of the effect of the higher minimum wage. We also compare employment changes at stores in New Jersey that were initially paying high wages (above \$5) to the changes at lower-wage stores. We find no indication that the rise in the minimum wage reduced employment. (JEL J30, J23)

<https://www.jstor.org/stable/2118030>

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- Based on survey data:
 - ▶ Wave 1: March 1992, one month before the minimum wage increased
 - ▶ Wave 2: December 1992, eight months after increase

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- Idea: compare employment rates in 410 fast-food restaurants in NJ and eastern PA (where there wasn't a wage increase) both before and after the change.
- Based on survey data:
 - ▶ Wave 1: March 1992, one month before the minimum wage increased
 - ▶ Wave 2: December 1992, eight months after increase
- “What would a skeptic consider convincing evidence?” David Card
- “There was a time when we thought econometric techniques would solve a lot of the data problems. Now I think the feeling is that there are a lot of problems for which it is easier to get better data.” Alan Krueger

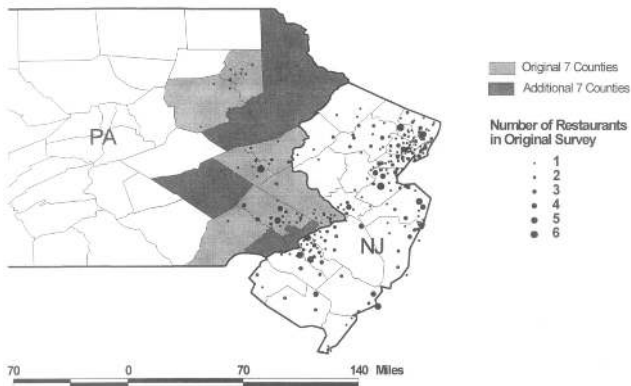


FIGURE 1. AREAS OF NEW JERSEY AND PENNSYLVANIA COVERED BY ORIGINAL SURVEY AND BLS DATA

Source: Card and Krueger 2000

Two Economists Catch Clinton's Eye By Bucking the Common Wisdom

BY DAVID CARD

ACTING PRESIDENT OF THE UNITED STATES, Bill Clinton has a reputation for being a pragmatist. He is known for his ability to listen to the views of others and to make decisions based on the facts. This is why he has been able to bring together two of the most prominent economists in the country, David Card and Alan Krueger, to serve on the President's Council of Economic Advisors.

Card and Krueger are both well-known for their work on the minimum wage. Card, who is a professor at the University of California, Berkeley, and Krueger, who is a professor at Princeton University, have both published influential studies on the topic. Their work has shown that increasing the minimum wage does not lead to the loss of jobs, as many people believe.

Clinton's decision to appoint Card and Krueger to his Council of Economic Advisors is a sign of his commitment to listening to the views of experts. It is also a sign of his commitment to making decisions based on the facts. Card and Krueger's work on the minimum wage is a classic example of this. They used rigorous statistical methods to show that increasing the minimum wage does not lead to the loss of jobs.

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Recently, Clinton chose David Card and Alan Krueger.

They use control groups in their research, and test minimum-wage theories by surveying the managers of fast-food restaurants.

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- ▶ δ_0 : the difference in the average outcome from period 1 to period 2 in the **untreated** group
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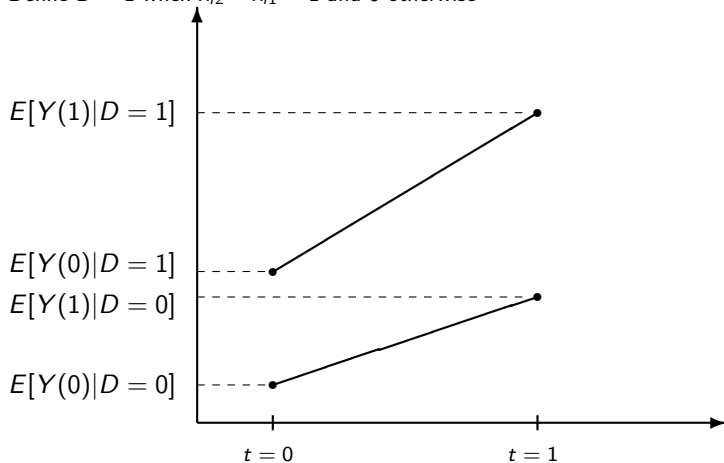
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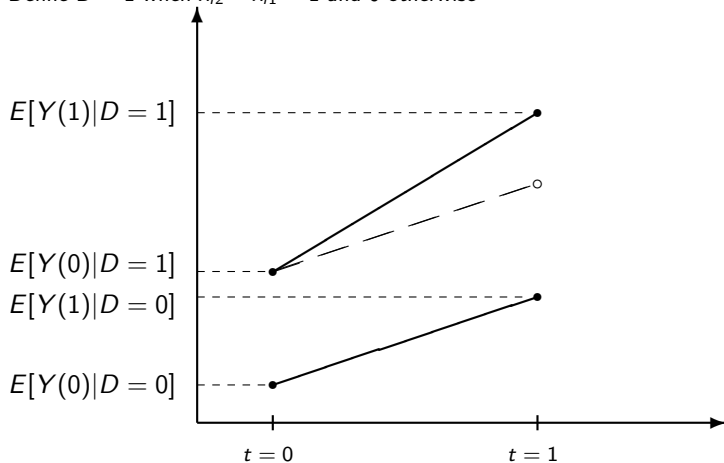
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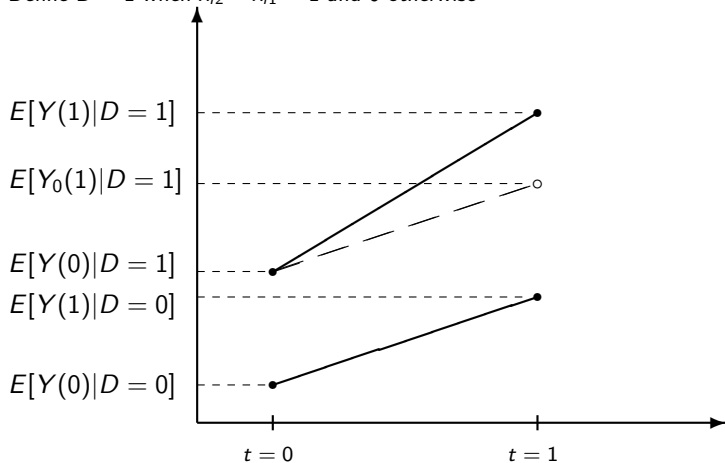
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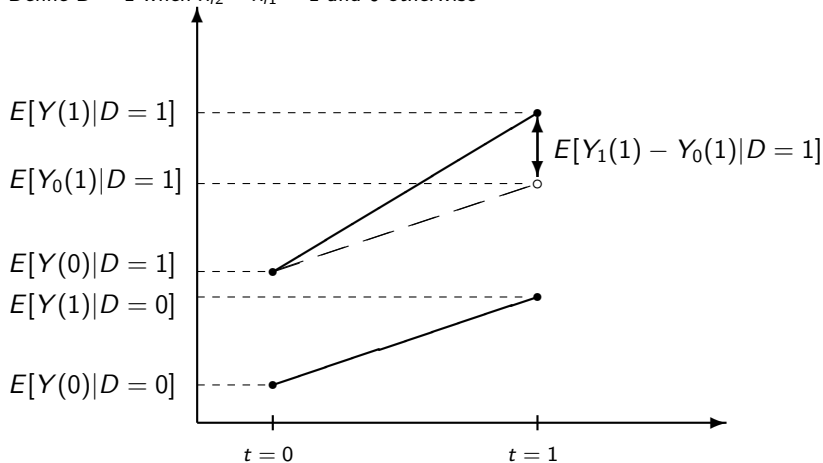
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Identification with Difference-in-Differences

Identification Assumption (parallel trends)

$$E[Y_0(1) - Y_0(0)|D = 1] = E[Y_0(1) - Y_0(0)|D = 0]$$

Identification Result

Given parallel trends the ATT is identified as:

$$\begin{aligned} E[Y_1(1) - Y_0(1)|D = 1] &= \left\{ E[Y(1)|D = 1] - E[Y(1)|D = 0] \right\} \\ &\quad - \left\{ E[Y(0)|D = 1] - E[Y(0)|D = 0] \right\} \end{aligned}$$

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Proof.

Note that the identification assumption implies

$$E[Y_0(1)|D = 0] = E[Y_0(1)|D = 1] - E[Y_0(0)|D = 1] + E[Y_0(0)|D = 0]$$

plugging in we get

$$\begin{aligned} & \{E[Y(1)|D = 1] - E[Y(1)|D = 0]\} - \{E[Y(0)|D = 1] - E[Y(0)|D = 0]\} \\ = & \{E[Y_1(1)|D = 1] - E[Y_0(1)|D = 0]\} - \{E[Y_0(0)|D = 1] - E[Y_0(0)|D = 0]\} \\ = & \{E[Y_1(1)|D = 1] - (E[Y_0(1)|D = 1] - E[Y_0(0)|D = 1] + E[Y_0(0)|D = 0])\} \\ - & \{E[Y_0(0)|D = 1] - E[Y_0(0)|D = 0]\} \\ = & E[Y_1(1) - Y_0(1)|D = 1] + \{E[Y_0(0)|D = 1] - E[Y_0(0)|D = 0]\} \\ - & \{E[Y_0(0)|D = 1] - E[Y_0(0)|D = 0]\} \\ = & E[Y_1(1) - Y_0(1)|D = 1] \end{aligned}$$



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- DiD works for **additive** and **time-invariant** confounding (i.e. satisfies parallel trends)

Does Indiscriminate Violence Incite Insurgent Attacks?

Evidence from Chechnya

Jason Lyall

*Department of Politics and the Woodrow Wilson School
Princeton University, New Jersey*

Journal of Conflict Resolution

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June 2009 331-362

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10.1177/0022002708330881

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- Counterintuitive findings: shelled villages experience a 24% reduction in insurgent attacks relative to controls.

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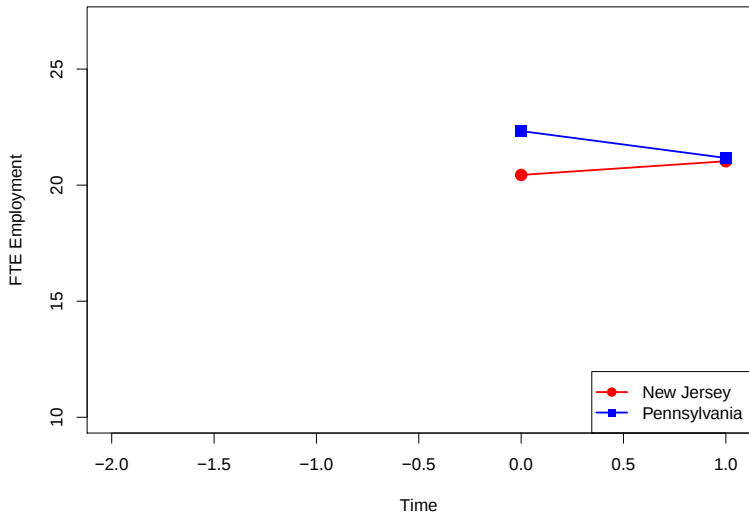
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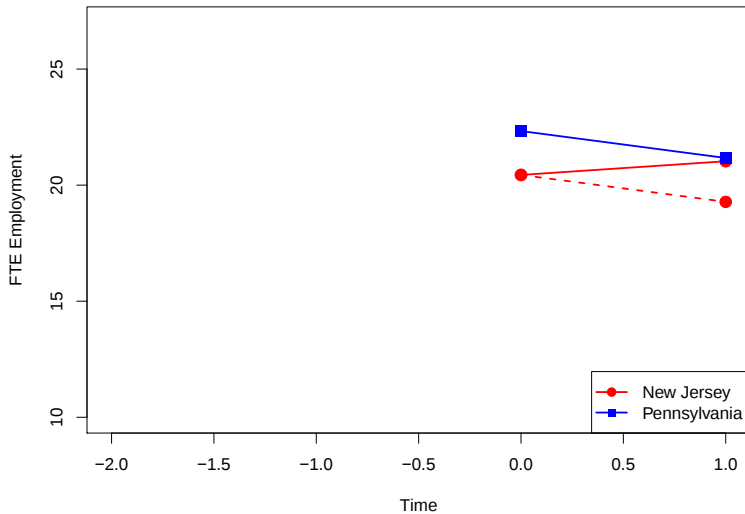
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- NJ_i indicates which stores received the treatment of a higher minimum wage at time period $t = 2$

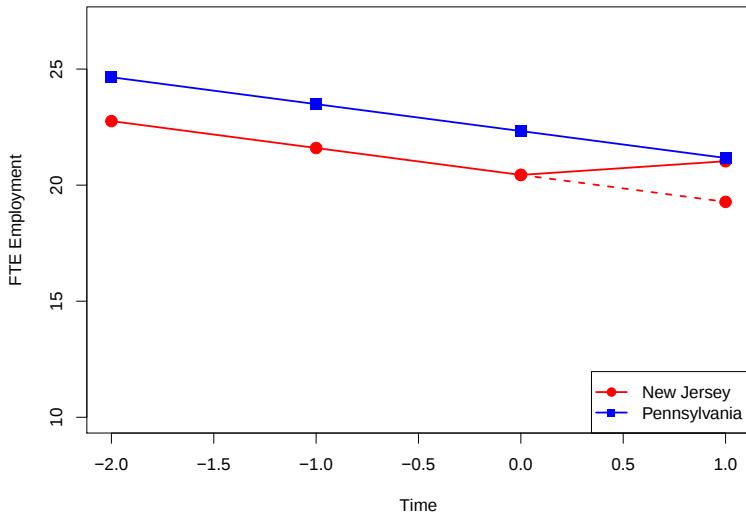
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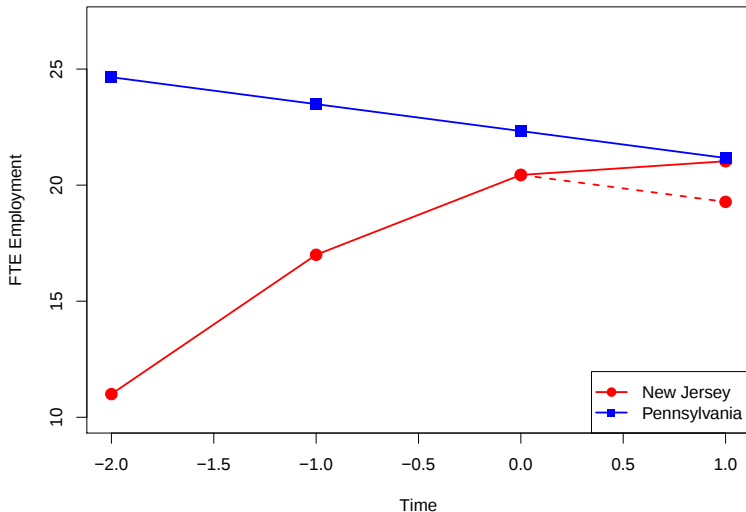
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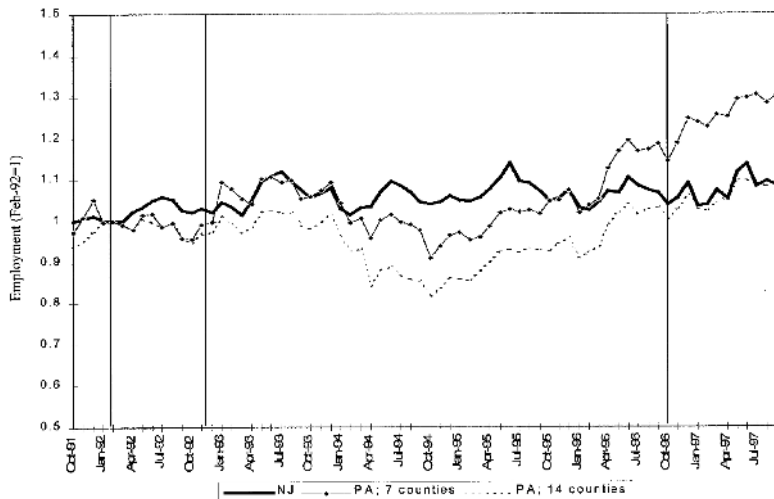
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Longer Trends in Employment (Card and Krueger 2000)



First two vertical lines indicate the dates of the Card-Krueger survey. October 1996 line is the federal minimum wage hike which was binding in PA but not NJ

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4) Long-term vs. Short-term Effects

parallel trends are less credible over a long time horizon, but many policies need time to take effect.

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6) Endogenous Control Variables

can add (time-varying) covariates to help with some of above concerns $\rightsquigarrow$ “regression diff-in-diff”

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but need to be careful that they aren't affected by the treatment.

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What to read next?

- Angrist and Pischke Chapter 5 Parallel Worlds: Fixed Effects, Differences-in-Differences and Panel Data
- Morgan and Winship Chapter 11 Repeated Observations and the Estimation of Causal Effects

We Covered

- Difference-in-Differences
- Parallel Trends Assumption

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- Parallel Trends Assumption

Next Time: Fixed Effects

Where We've Been and Where We're Going...

- Last Week
 - ▶ causal inference with unmeasured confounding
- This Week
 - ▶ panel data
 - ▶ diff-in-diff
 - ▶ fixed effects
 - ▶ wrap-up
- The Following Week
 - ▶ ?
- Long Run
 - ▶ probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causality

- 1 Differencing Models
- 2 Difference-in-Differences
- 3 Fixed Effects
- 4 Non-parametric Identification and Fixed Effects
- 5 Wrap-Up
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 - Concluding Thoughts for the Course

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- We discussed a **differencing** approach to this model
- The **Fixed effects model** is an alternative way to remove time-invariant unmeasured confounding
- We will start by assuming the model and discussing properties and in the next section, we will consider non-parametric identification.

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- This regression is sometimes called the “between regression”

Within Transformation

Within Transformation

- The “fixed effects,” “within,” or “time-demeaning” transformation is when we subtract off the over-time means from the original data:

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- We are now modeling observations as deviation from their group mean.
- NB: you **must** demean the X variables not just the Y variables.

Fixed Effects with Ross data

```
fe.mod <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur), data = ross, index = c("id", "year"),
  model = "within")
summary(fe.mod)

## Oneway (individual) effect Within Model
##
## Call:
## plm(formula = log(kidmort_unicef) ~ democracy + log(GDPcur),
##     data = ross, model = "within", index = c("id", "year"))
##
## Unbalanced Panel: n=166, T=1-7, N=649
##
## Residuals :
##      Min.   1st Qu.   Median   3rd Qu.    Max.
## -0.70500 -0.11700  0.00628  0.12200  0.75700
##
## Coefficients :
##              Estimate Std. Error t-value Pr(>|t|)
## democracy    -0.143233   0.033500  -4.2756 2.299e-05 ***
## log(GDPcur)  -0.375203   0.011328 -33.1226 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Total Sum of Squares:    81.711
## Residual Sum of Squares: 23.012
## R-Squared      : 0.71838
##      Adj. R-Squared : 0.53242
## F-statistic: 613.481 on 2 and 481 DF, p-value: < 2.22e-16
```

Strict Exogeneity

- FE models are valid if $E[\mathbf{u}|\mathbf{X}] = 0$: all errors are uncorrelated with covariates in every period:

$$E[\ddot{u}_{it}|\ddot{\mathbf{X}}] = E[u_{it}|\ddot{\mathbf{X}}] - E[\bar{u}_i|\ddot{\mathbf{X}}] = 0 - 0 = 0$$

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- This is because the composite errors, $\ddot{u}_{it}$ are function of the errors in every time period through the average, $\bar{u}_i$
- This rules out, for instance, lagged dependent variables, since $y_{i,t-1}$ has to be correlated with $u_{i,t-1}$. Thus it can't be a covariate for y_{it} .

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- Basic message: any time-constant variable gets “absorbed” by the fixed effect. It has nothing to contribute because the comparison is **within the units**.

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- Basic message: any time-constant variable gets “absorbed” by the fixed effect. It has nothing to contribute because the comparison is **within the units**.
- Can include interactions between time-constant and time-varying variables, but lower order term of the time-constant variables get absorbed by fixed effects too

Time-constant variables

- Pooled model with a time-constant variable, proportion Islamic:

```
library(lmtest)
p.mod <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur) + islam,
             data = ross, index = c("id", "year"), model = "pooling")
coeftest(p.mod)

##
## t test of coefficients:
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 10.30607817  0.35951939  28.6663 < 2.2e-16 ***
## democracy   -0.80233845  0.07766814 -10.3303 < 2.2e-16 ***
## log(GDPcur) -0.25497406  0.01607061 -15.8659 < 2.2e-16 ***
## islam        0.00343325  0.00091045   3.7709 0.0001794 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Time-constant variables

- FE model, where the islam variable drops out, along with the intercept:

```
fe.mod2 <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur) + islam,  
               data = ross, index = c("id", "year"), model = "within")  
coeftest(fe.mod2)
```

```
##  
## t test of coefficients:  
##  
##           Estimate Std. Error  t value  Pr(>|t|)  
## democracy   -0.129693   0.035865  -3.6162 0.0003332 ***  
## log(GDPcur) -0.379997   0.011849 -32.0707 < 2.2e-16 ***  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Alternate Computation: Least Squares Dummy Variable

- As an alternative to the within transformation, we can also include a series of $n - 1$ dummy variables for each unit:

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- Why are these equivalent? (remember partialing out strategy and Frisch-Waugh-Lovell theorem)

Example with Ross data

```
library(lmtest)
lsdv.mod <- lm(log(kidmort_unicef) ~ democracy + log(GDPcur) +
               as.factor(id), data = ross)
coeftest(lsdv.mod)[1:6,]
coeftest(fe.mod)[1:2,]
```


##		Estimate	Std. Error	t value	Pr(> t)
##	(Intercept)	13.7644887	0.26597312	51.751427	1.008329e-198
##	democracy	-0.1432331	0.03349977	-4.275644	2.299393e-05
##	log(GDPcur)	-0.3752030	0.01132772	-33.122568	3.494887e-126
##	as.factor(id)AGO	0.2997206	0.16767730	1.787485	7.448861e-02
##	as.factor(id)ALB	-1.9309618	0.19013955	-10.155498	4.392512e-22
##	as.factor(id)ARE	-1.8762909	0.17020738	-11.023558	2.386557e-25

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 - ▶ Cluster fixed effects in hierarchical data
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- Note that when the second dimension isn't time, fixed effects will be relevant more often.

We Covered

- Fixed Effects!
- Computation for Fixed Effects!

Next Time: Non-parametric Identification and Fixed Effects

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 - ▶ strict exogeneity
- We've seen the trouble with constant effects before, it goes back to Lecture 10 and results on regression with heterogeneous treatment effects more generally.

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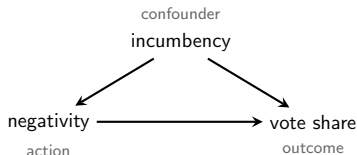
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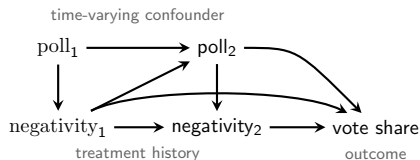
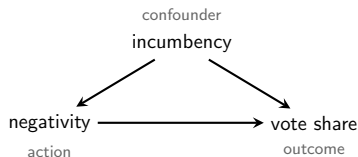
Examples of static and dynamic causal inference problems:



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A Framework for Dynamic Causal Inference in Political Science

Nathaniel Blackhall *University of Tennessee*

Abstract: This paper develops a framework for dynamic causal inference in political science. It begins by reviewing the literature on causal inference in political science, and then develops a framework for dynamic causal inference. The framework is based on the idea that causal inference in political science is a dynamic process, and that the causal relationships between treatment and outcomes are dynamic. The framework is then applied to the study of the relationship between the treatment of the Vietnam War and the outcomes of the Vietnam War.

When we think about the relationship between treatment and outcomes, we often think of it as a static relationship. We think of a treatment as a single point in time, and we think of an outcome as a single point in time. But in reality, the relationship between treatment and outcomes is dynamic. The treatment is a process that unfolds over time, and the outcome is a process that unfolds over time. This paper develops a framework for dynamic causal inference in political science, and then applies it to the study of the relationship between the treatment of the Vietnam War and the outcomes of the Vietnam War.

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When we think about the causal processes in political science, we often think of them as static. We think of a causal process as a set of variables that are related to each other in a fixed way. We think of a causal process as a set of variables that are related to each other in a fixed way. We think of a causal process as a set of variables that are related to each other in a fixed way.

But what if we think of causal processes in political science as dynamic? What if we think of them as a set of variables that are related to each other in a way that changes over time?

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How to Make Causal Inferences with Time-Series Cross-Sectional Data under Selection on Observables

MATTHEW BLACKWELL, Harvard University
ADAM N. GLYNN, Emory University

Repeated measurement of the same countries, people, or groups over time are often a major source of data in political science. These measurements, sometimes called time series (or cross-sectional) TSCS data, allow researchers to estimate a broad set of causal quantities, including counterfactual effects and descriptions of causal processes. Unfortunately, popular methods for TSCS data can only produce valid inferences for causal effects under some strong assumptions. In this paper, we propose a new method to address causal questions of interest in these settings and clarify how standard models that do not account for selection on observables can produce biased estimates of these quantities due to measurement conditions. We then describe two extensions: one that allows these past treatment factors (observed and unobserved) to be correlated with the treatment, and then, via simulation, show that our method can standardize a causal effect in small sample settings. We illustrate these methods in a study of how welfare spending affects revenue.

INTRODUCTION

Many inquiries in political science involve the study of repeated measurement of the same countries, people, or groups at several points in time. This type of data, sometimes called time series cross-sectional (TSCS) data, allows researchers to focus on a larger pool of information when estimating causal effects. TSCS data also give researchers the power to ask a wider set of questions than data with a single measurement for each unit (see, e.g., Black and Katz 2011). Using this data, researchers can move past the standard comparisons to question what are the effects of a single event—and instead ask how the history of a process affects the political world. Unfortunately, the most common approaches to modeling TSCS data require strong assumptions to estimate the effects of interest, and these assumptions can be difficult to justify in the context of political science.

This paper makes three contributions to the study of TSCS data. Our first contribution is to define what we mean by causal effects in the context of TSCS data.

Second, we formalize causal effects and discuss the settings that lead to identification of these effects. We then show how to identify these effects in certain settings in the TSCS literature. We describe the assumptions, and show how to derive them from the properties of a common TSCS model, the autoregressive distributed lag (ARDL) model. These standard effects can be automatically identified under a key selection-on-treatment assumption called sequential ignorability. Unfortunately, however, many common TSCS approaches rely on more stringent assumptions, including a lack of causal feedback between the treatment and time-varying covariates. This feedback, for example, might involve a country's level of welfare spending affecting the rate of change of welfare spending. We argue that this type of feedback is common in TSCS data. While we focus on a selection-on-treatment assumption in this paper, we discuss the tradeoffs with this choice compared to standard fixed-effects methods, noting that the latter may also rely on this type of causal feedback.

Our second contribution is to provide an extension to a few methods that have been used to ask about the effect of treatment factors without this and other causal assumptions (that common TSCS models do). We use two methods: (1) structural mean models or SEMMs (Robins 1997), and (2) marginal structural models (MSMs) with inverse probability of treatment weighting (IPTW) (Robins, Hernan, and

Nathaniel Blackwell is an assistant professor, Department of Government and Institute for Government Studies, Harvard University, 77 Massachusetts Avenue, Cambridge, MA 02138. His research interests include political science, statistics, and causal inference. He is currently working on a book about causal inference in political science. Adam N. Glynn is an assistant professor, Department of Political Science, Emory University, 1515 University Drive, Atlanta, GA 30322. His research interests include political science, statistics, and causal inference. He is currently working on a book about causal inference in political science.

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56 / 90

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A Framework for Dynamic Causal Inference in Political Science

Nathaniel Blackwell | [I analyzed it for you](#)

Abstract: "There is a tension in the study of causal dynamics in political science. On the one hand, scholars are interested in understanding the causal mechanisms that drive political outcomes. On the other hand, scholars are interested in understanding the causal effects of political interventions. This tension is reflected in the choice of methods used to study causal dynamics. This paper develops a framework for dynamic causal inference in political science that reconciles these two interests. The framework is based on the idea of a causal process, which is a sequence of events that lead to an outcome. The framework allows for the estimation of causal effects of interventions on outcomes, while also allowing for the estimation of the causal mechanisms that drive the outcomes. The framework is applied to the study of the causal effects of the 2010 earthquake on the political outcomes in Haiti." [Full paper](#)

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Journal of Economic Surveys (2019) 82, 1–30
doi:10.1016/j.econsur.2019.04.001
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How to Make Causal Inferences with Time-Series Cross-Sectional Data under Selection on Observables

MATTHEW BLACKWELL ¹ *Harvard University*
ADAM N. GLYNN ² *Dartmouth University*

Repeated measurement of the same countries, people, or groups over time are often a source of rich data for causal inference. These measurements, however, often reflect time-varying unobserved factors that can confound the relationship between treatment and outcomes. In this paper, we propose a new method for making causal inferences with time-series cross-sectional data under selection on observables. We show that this method can be used to estimate causal effects of interventions on outcomes, while also allowing for the estimation of the causal mechanisms that drive the outcomes. The method is applied to the study of the causal effects of the 2010 earthquake on the political outcomes in Haiti.

INTRODUCTION

Many inquiries in political science involve the study of repeated measurement of the same countries, people, or groups at several points in time. This type of data, sometimes called time-series cross-sectional (TSCS) data, allows researchers to study a large range of phenomena over time, including social, economic, and political phenomena. TSCS data also give researchers the power to study the causal effects of interventions on outcomes. This paper develops a framework for dynamic causal inference in political science that reconciles these two interests. The framework is based on the idea of a causal process, which is a sequence of events that lead to an outcome. The framework allows for the estimation of causal effects of interventions on outcomes, while also allowing for the estimation of the causal mechanisms that drive the outcomes. The framework is applied to the study of the causal effects of the 2010 earthquake on the political outcomes in Haiti.

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Journal of the Royal Statistical Society (2019) 182, 1–30
doi:10.1111/rssc.12345
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When Should We Use Unit Fixed Effects Regression Models for Causal Inference with Longitudinal Data?

Kosuke Imai ¹ *Harvard University*
Stewart J. Lieberman ² *Harvard University*

Unit fixed effects regression models are a popular method for making causal inferences with longitudinal data. However, these models are only valid under certain conditions. In this paper, we develop a framework for determining when unit fixed effects regression models are appropriate for making causal inferences with longitudinal data. The framework is based on the idea of a causal process, which is a sequence of events that lead to an outcome. The framework allows for the estimation of causal effects of interventions on outcomes, while also allowing for the estimation of the causal mechanisms that drive the outcomes. The framework is applied to the study of the causal effects of the 2010 earthquake on the political outcomes in Haiti.

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We are going to focus on addressing unobserved time-invariant confounders using the last paper. Next several slides are based on slides graciously provided by In Song Kim and Kosuke Imai.

Directed Acyclic Graph (DAG)

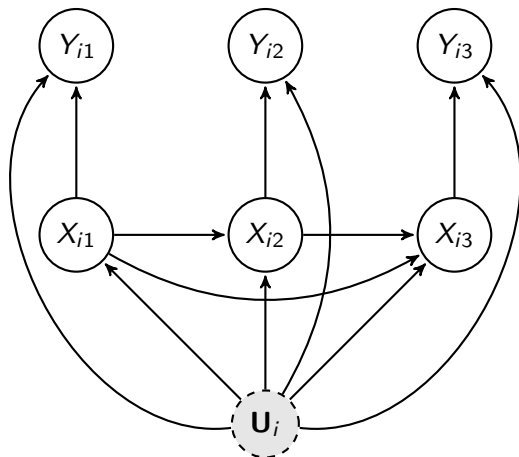
Non-parametric identification assumptions for fixed effects:

$$Y_{it} = g(X_{it}, \mathbf{U}_i, \epsilon_{it}) \quad \text{and} \quad \epsilon_{it} \perp\!\!\!\perp \{\mathbf{X}_i, \mathbf{U}_i\}$$

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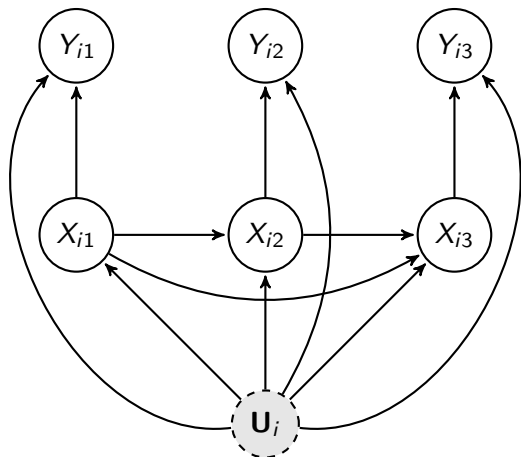


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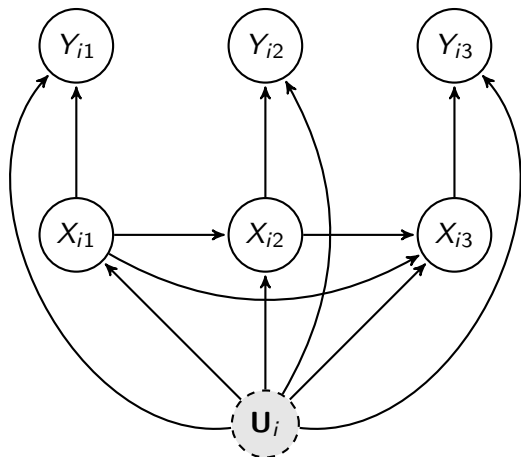
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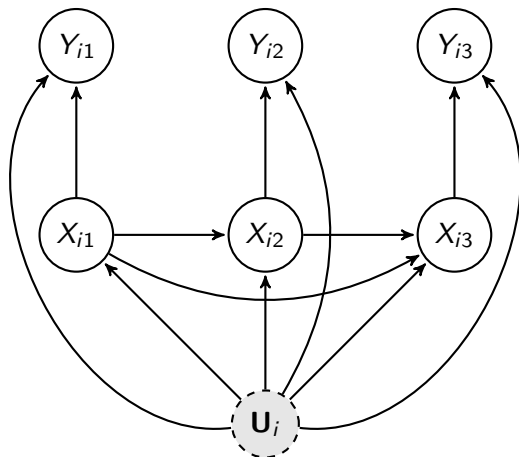
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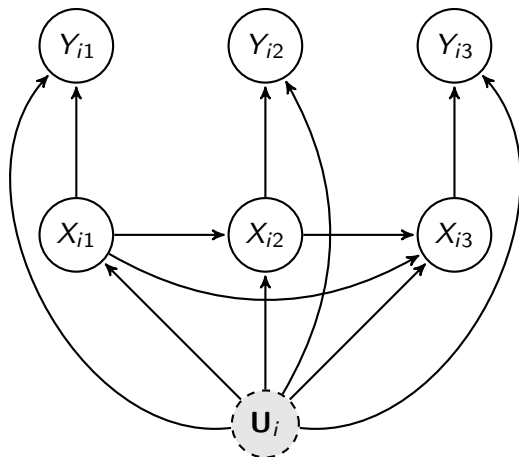
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- 4 Past treatments do not directly affect current outcome

*the result implies that the **counterfactual** outcome for a treated observation in a given time period is estimated using the **observed outcomes of different time periods of the same unit**. Since such a comparison is **valid only when no causal dynamics exist**, this finding underscores the important limitation of linear regression models with unit fixed effects.*

- Imai and Kim (2019)

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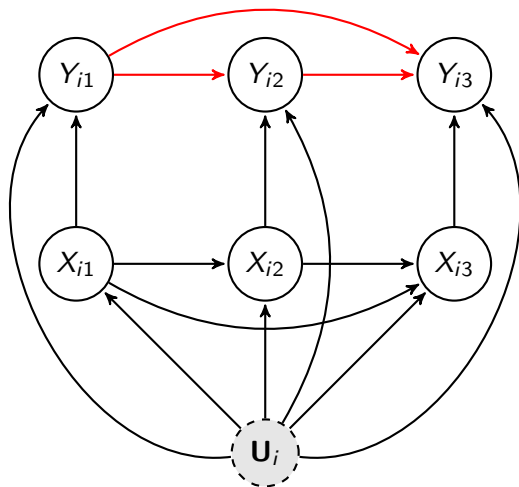
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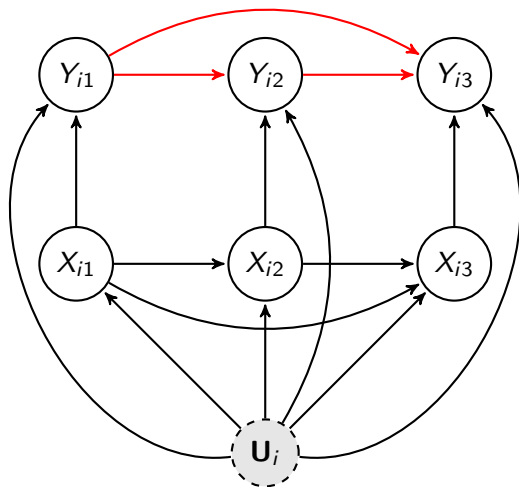
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- Now let's consider each assumption in turn.

Past Outcomes Don't Directly Affect Current Outcome (A2)

- Strict exogeneity still holds.

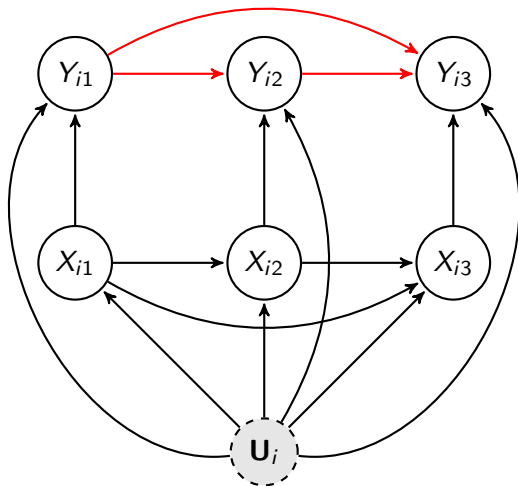


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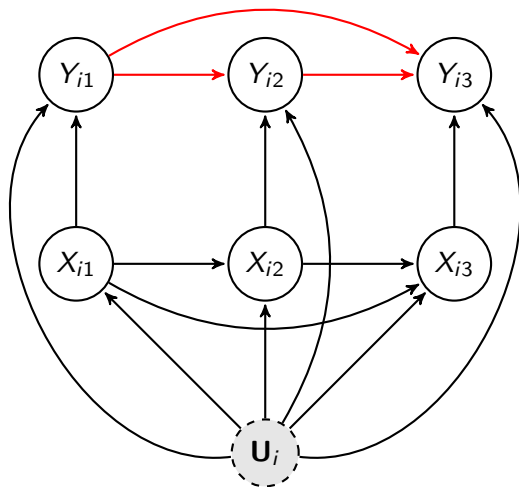
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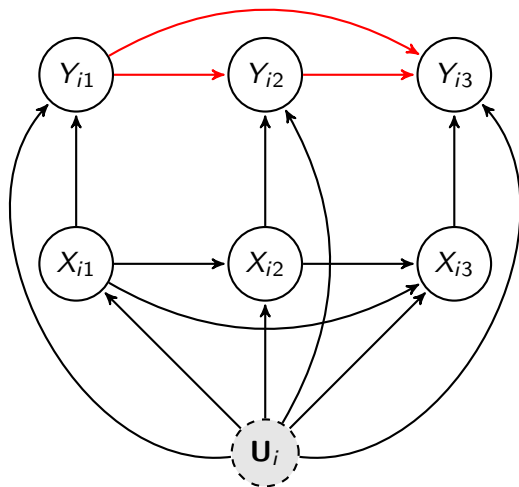
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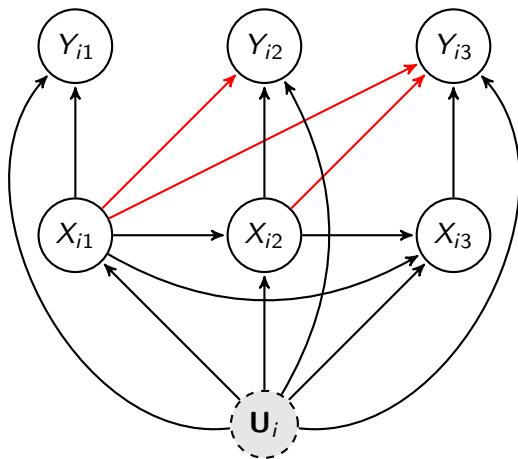
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- No need to adjust for past outcomes.
- Should use cluster robust standard errors for inference.

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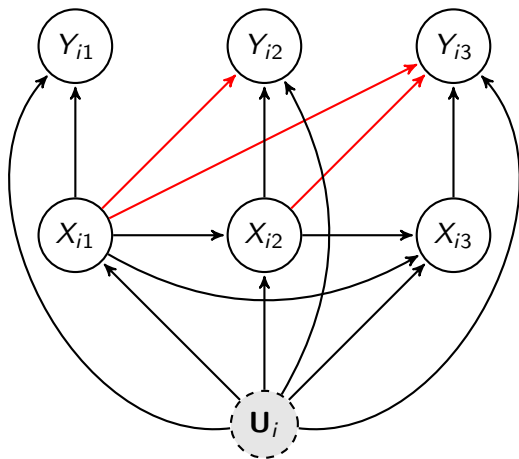
- Strict exogeneity still holds.
- Past outcomes do not confound $X_{it} \rightarrow Y_{it}$ given U_i .
- No need to adjust for past outcomes.
- Should use cluster robust standard errors for inference.
- Conclusion: The assumption can be relaxed

Past Treatments Don't Directly Affect Current Outcome (A4)



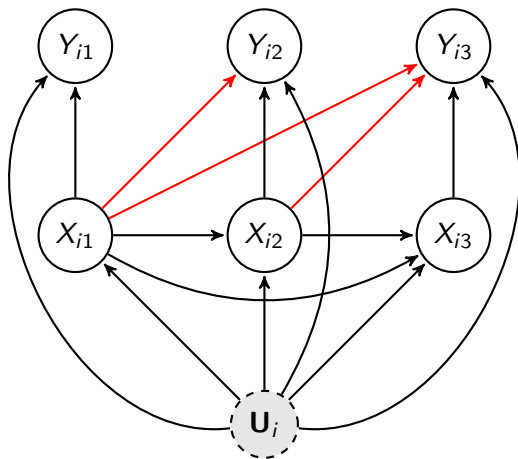
- Need to adjust for past treatments

Past Treatments Don't Directly Affect Current Outcome (A4)



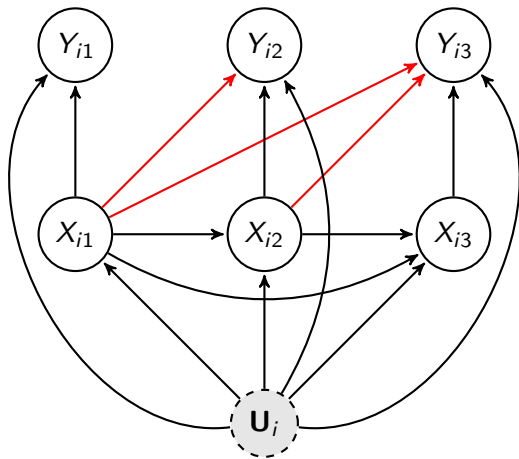
- Need to adjust for past treatments
- Strict exogeneity holds given past treatments and U_i

Past Treatments Don't Directly Affect Current Outcome (A4)



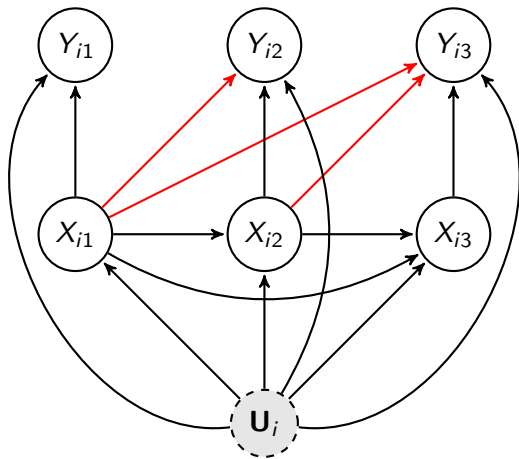
- Need to adjust for past treatments
- Strict exogeneity holds given past treatments and U_i
- Impossible to adjust for an entire treatment history and U_i at the same time

Past Treatments Don't Directly Affect Current Outcome (A4)



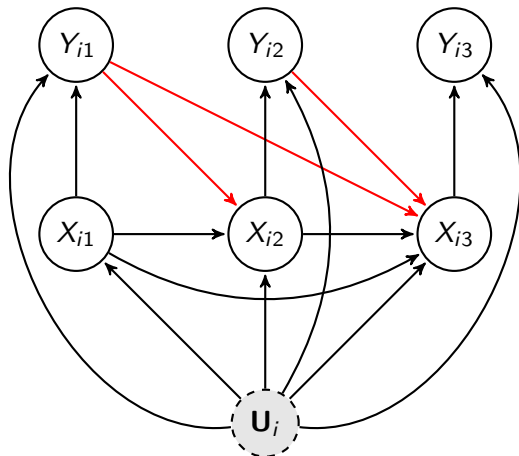
- Need to adjust for past treatments
- Strict exogeneity holds given past treatments and U_i
- Impossible to adjust for an entire treatment history and U_i at the same time
- Adjust for a small number of past treatments $\rightsquigarrow$ often arbitrary

Past Treatments Don't Directly Affect Current Outcome (A4)



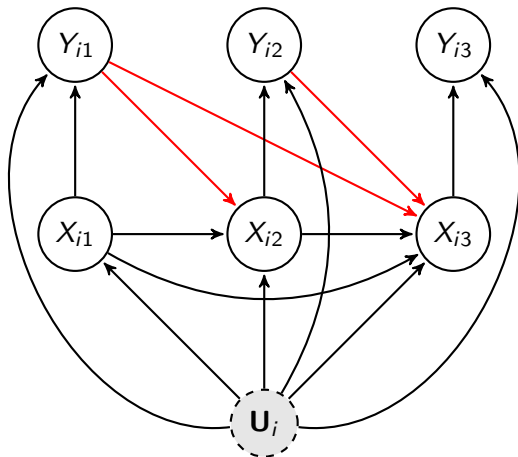
- Need to adjust for past treatments
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- Impossible to adjust for an entire treatment history and U_i at the same time
- Adjust for a small number of past treatments $\rightsquigarrow$ often arbitrary
- Conclusion: The assumption can be partially relaxed

Past Outcomes Don't Directly Affect Current Treatment (A3)

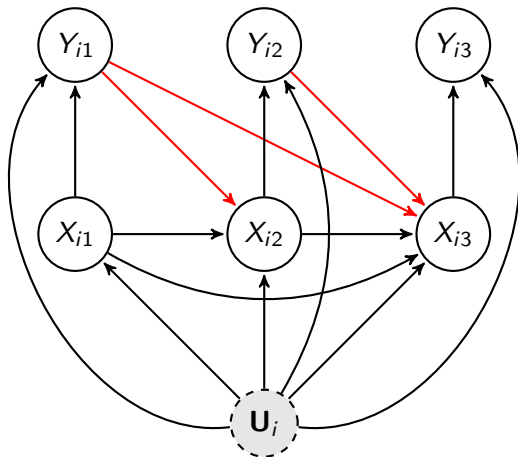


Past Outcomes Don't Directly Affect Current Treatment (A3)

- Correlation between error term and future treatments

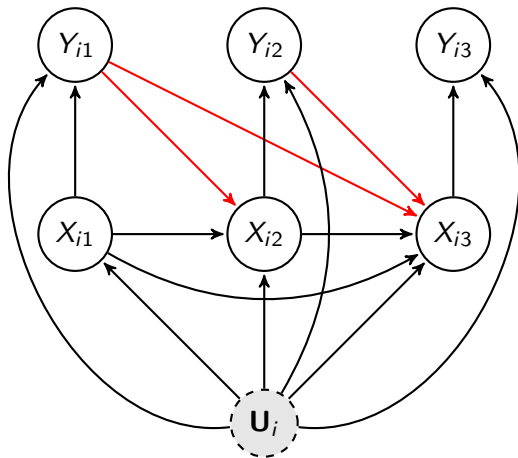


Past Outcomes Don't Directly Affect Current Treatment (A3)



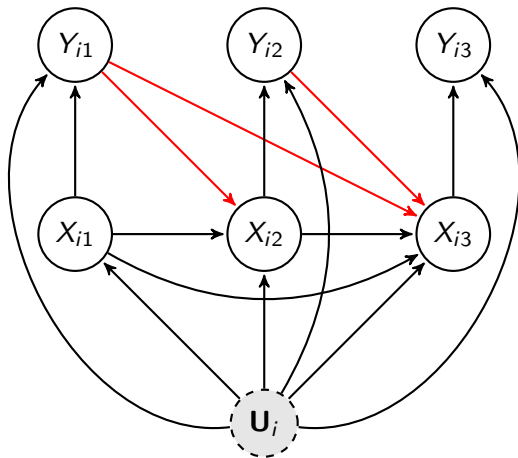
- Correlation between error term and future treatments
- Violation of strict exogeneity

Past Outcomes Don't Directly Affect Current Treatment (A3)



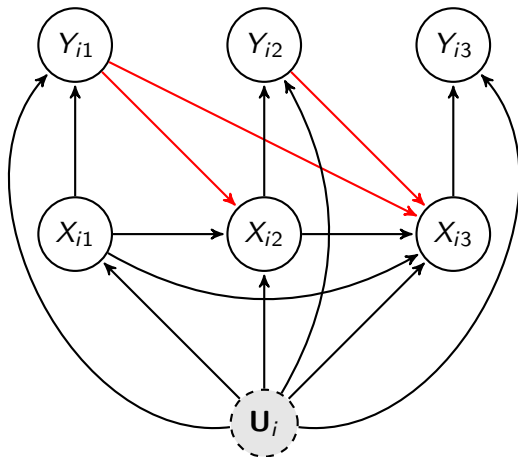
- Correlation between error term and future treatments
- Violation of strict exogeneity
- No adjustment is sufficient

Past Outcomes Don't Directly Affect Current Treatment (A3)



- Correlation between error term and future treatments
- Violation of strict exogeneity
- No adjustment is sufficient
- Implication: No dynamic causal relationships between treatment and outcome variables

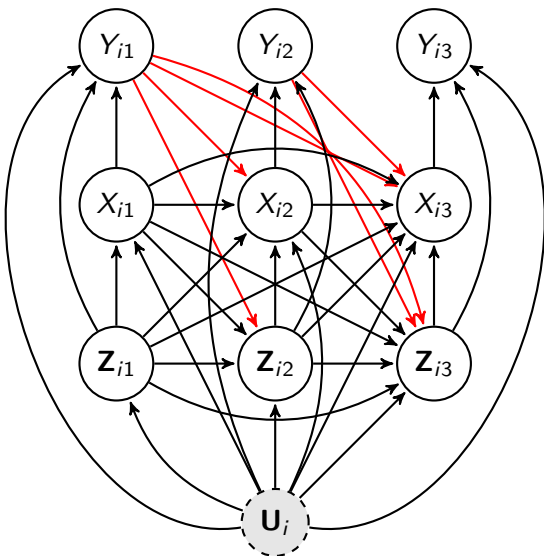
Past Outcomes Don't Directly Affect Current Treatment (A3)



- Correlation between error term and future treatments
- Violation of strict exogeneity
- No adjustment is sufficient
- Implication: No dynamic causal relationships between treatment and outcome variables
- Conclusion: The assumption cannot be relaxed

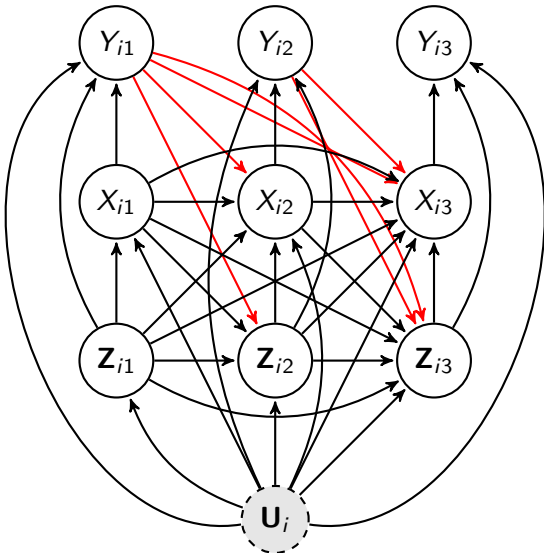
Can't We Just Adjust for Time-Varying Confounders?

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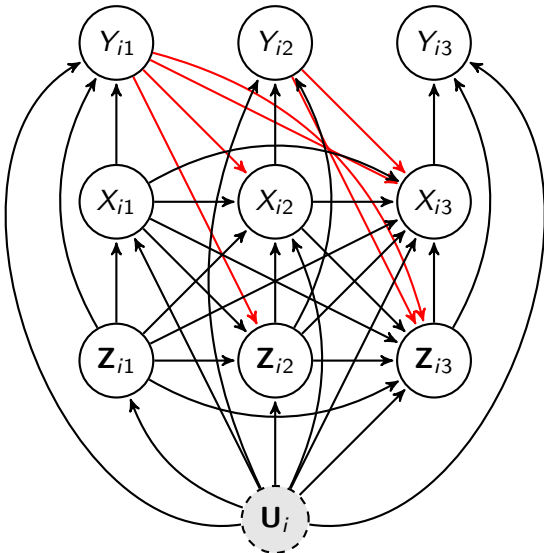
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- past outcomes cannot *indirectly* affect current treatment through $\mathbf{Z}_{it}$

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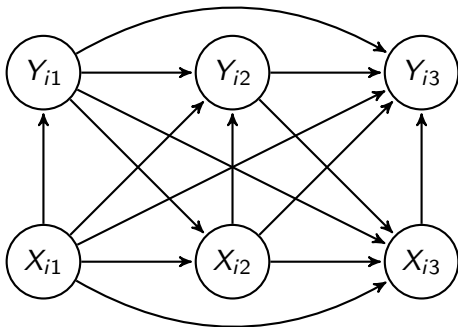
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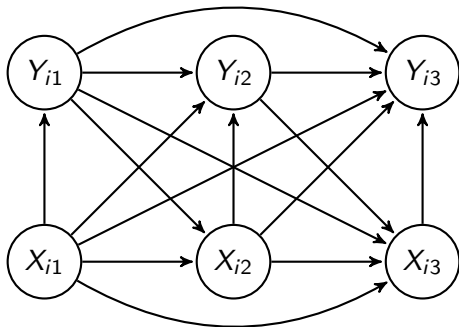
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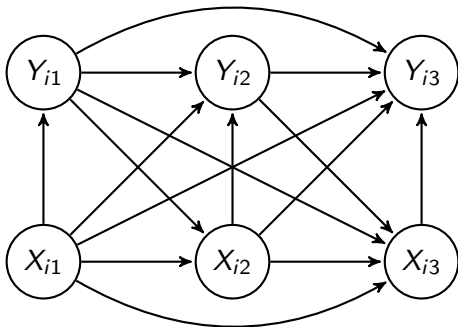
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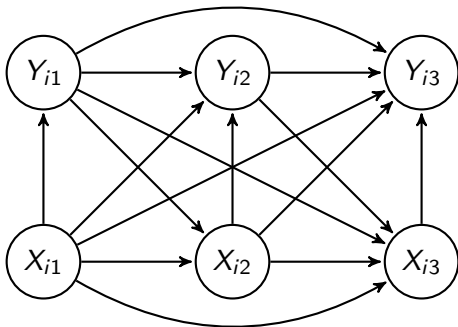
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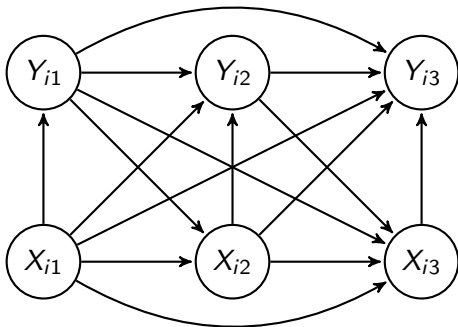
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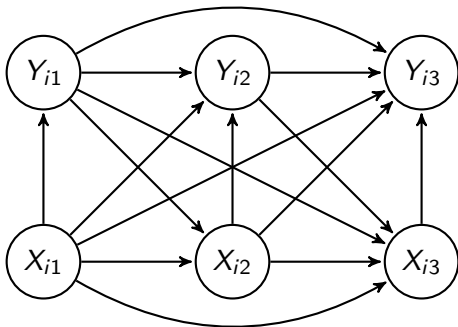


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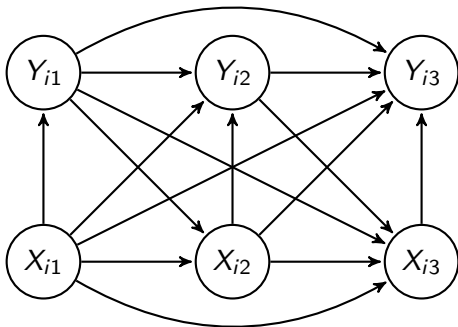


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Conclusions and Nonparametric Estimation

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 - 1) unobserved time-invariant confounders $\rightsquigarrow$ fixed effects
 - 2) causal dynamics between treatment and outcome $\rightsquigarrow$ selection-on-observables

Summary Table (Imai and Kim 2019)

TABLE 1 Identification Assumptions of Various Estimators

	Linearity	Time-Invariant Unobservables	Past Outcomes Affect Current Treatment	Past Treatments Affect Current Outcome
$Y_{it} = \alpha_i + \beta X_{it} + \epsilon_{it}$	Yes	Allowed	Not allowed	Not allowed
$Y_{it} = \alpha_i + \beta X_{it} + \rho Y_{i,t-1} + \epsilon_{it}$	Yes	Allowed	Allowed	Not allowed
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$Y_{it} = \alpha_i + \beta_1 X_{it} + \beta_2 X_{i,t-1} + \rho Y_{i,t-1} + \epsilon_{it}$	Yes	Allowed	Partially allowed	Partially allowed
Marginal structural models	No	Not allowed	Allowed	Allowed

What to read next?

- Morgan and Winship Chapter 11 Repeated Observations and the Estimation of Causal Effects
- Imai and Kim (2019) “When Should We Use Unit Fixed Effects Regression Models for Causal Inference with Longitudinal Data?” *American Journal of Political Science*,
<http://dx.doi.org/10.1111/ajps.12417>

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Next Time: Review

Where We've Been and Where We're Going...

- Last Week
 - ▶ causal inference with unmeasured confounding
- This Week
 - ▶ panel data
 - ▶ diff-in-diff
 - ▶ fixed effects
 - ▶ wrap-up
- The Following Week
 - ▶ ?
- Long Run
 - ▶ probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causality

- 1 Differencing Models
- 2 Difference-in-Differences
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Q: What conditions do we need to infer causality?

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A: A clear estimand, an identification strategy and an estimation strategy.

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Almost everything assumes: consistency/SUTVA (no interference between units, variation in the treatment is irrelevant) and positivity (there is some chance of all getting treatment)

Some Estimation Strategies

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- Stratification

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Some Estimation Strategies

- Stratification
- Regression (and relatives)
- Matching (lightly covered)
- Weighting (not covered)

Q: Can you review how instrumental variables deal with issues of confounding?

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A: We use only the units whose treatment status was effectively randomized by the instrument (because they are compliers).

Q: What can make standard errors larger or smaller?

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A: Let's consider the anatomy of a standard error.

Anatomy of the Standard Error

Imagine we have a regression of Y on a variable of interest X and a vector of other variables $\mathbf{Z}$.

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Further Reading:

- Lin, W., 2013. 'Agnostic notes on regression adjustments to experimental data: Reexamining Freedman's critique.' *The Annals of Applied Statistics*
- Athey, S. and Imbens, G.W., 2017. 'The Econometrics of Randomized Experiments.' In *Handbook of Economic Field Experiments* (Vol. 1, pp. 73-140).
- Egap Methods Guide: 10 things you need to know about covariate adjustment.
<https://egap.org/methods-guides/10-things-know-about-covariate-adjustment>

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- 3 Derive the equations/math
- 4 Try to explain it to someone else

Q: In the real world of research, when you have your data, how do you know which method to use? For example, how do you know that you need to use matching/regression discontinuity?

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A: Let's chat.

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Where are you?

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You've been given a powerful set of tools



Your New Weapons

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- Basic probability theory
 - ▶ Probability axioms, random variables, marginal and conditional probability, building a probability model
 - ▶ Expected value, variances, independence
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- Univariate Inference

- ▶ Interval estimation (normal and non-normal Population)
- ▶ Confidence intervals, hypothesis tests, p-values
- ▶ Practical versus statistical significance

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- Simple Regression

- ▶ regression to approximate the conditional expectation function
- ▶ idea of conditioning
- ▶ kernel and loess regressions
- ▶ OLS estimator for bivariate regression
- ▶ Variance decomposition, goodness of fit, interpretation of estimates, transformations

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• Multiple Regression

- ▶ OLS estimator for multiple regression in matrix notation
- ▶ Regression assumptions (classical and agnostic)
- ▶ Properties: Bias, Efficiency, Consistency
- ▶ Standard errors, testing, p-values, and confidence intervals
- ▶ Polynomials, Interactions, Dummy Variables
- ▶ F-tests

Your New Weapons

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- Diagnosing and Fixing Regression Problems
 - ▶ Non-normality
 - ▶ Outliers, leverage, and influence points, Robust Regression
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- And you learned how to use R: you're not afraid of trying something new!

Using these Tools

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So, Admiral Ackbar, now that you've learned how to run these regressions we can just use them blindly, right?



IT'S A TRAP!



Beyond Linear Regressions

You need more training



Beyond Linear Regressions

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Beyond Linear Regressions

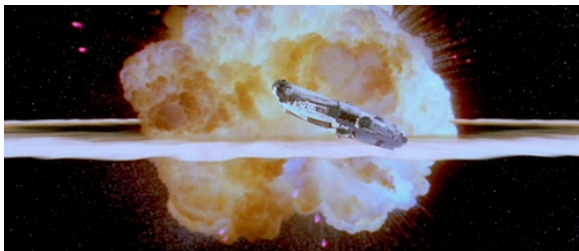
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I've tried to offer a **perspective on technique** in addition to just a list of methods. See Lundberg, Johnson and Stewart “[What's Your Estimand? Defining the Target Quantity Connects Statistical Evidence to Theory](#)”

Thanks!

Thanks so much for an amazing semester.



I will see you in the final synchronous session!