Week 12: Repeated Observations and Panel Data

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Princeton

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[1] These slides are heavily influenced by Matt Blackwell, Adam Glynn, Jens Hainmueller, and Erin Hartman.

Where We've Been and Where We're Going...

- Last Week
  - causal inference with unmeasured confounding
- This Week
  - panel data
  - diff-in-diff
  - fixed effects
  - wrap-up
- The Following Week
  - ?
- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causality

Motivation

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Michael Ross University of California, Los Angeles

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  - they are richer or developed earlier
  - provide benefits more efficiently
  - possess some cultural trait correlated with better health outcomes
- If we have data on countries over time, can we make any progress in spite of these problems?

Ross Data

```
##      cty_name year democracy infmort_unicef
## 1 Afghanistan 1965         0            230
## 2 Afghanistan 1966         0             NA
## 3 Afghanistan 1967         0             NA
## 4 Afghanistan 1968         0             NA
## 5 Afghanistan 1969         0             NA
## 6 Afghanistan 1970         0            215
```

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- NB: we won't be using T for treatment today because it is extremely consistently used for time. We will end up using D for treatment which is another common letter for treatment.

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- Panel data: large n, relatively short T
- Time series, cross-sectional (TSCS) data: smaller n, large T
- We are primarily going to focus on similarities today but there are some differences.

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We will start by considering performance of estimators assuming this model is true.

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  1. Heteroskedasticity (see clustering from diagnostics week)
  2. Possible violation of zero conditional mean errors
- Both problems arise out of ignoring the unmeasured heterogeneity inherent in a_i

Pooled OLS with Ross data

```
pooled.mod <- lm(log(kidmort_unicef) ~ democracy + log(GDPcur),
                 data = ross)
summary(pooled.mod)

##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  9.76405    0.34491   28.31   <2e-16 ***
## democracy   -0.95525    0.06978  -13.69   <2e-16 ***
## log(GDPcur) -0.22828    0.01548  -14.75   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7948 on 646 degrees of freedom
##   (5773 observations deleted due to missingness)
## Multiple R-squared:  0.5044, Adjusted R-squared:  0.5029
## F-statistic: 328.7 on 2 and 646 DF,  p-value: < 2.2e-16
```

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- Ignore the heterogeneity $\rightsquigarrow$ correlation between the combined error and the independent variables:

$$E[v_{it}|\mathbf{X}] = E[a_i + u_{it}|\mathbf{X}] \neq 0$$

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- Pooled OLS will be biased and inconsistent because zero conditional mean error fails for the combined error.

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$$\Delta y_i = y_{i2} - y_{i1}$$

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- Look at the change in y over time:

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&= (\mathbf{x}'_{i2} - \mathbf{x}'_{i1})\boldsymbol{\beta} + (a_i - a_i) + (u_{i2} - u_{i1}) \\
&= \Delta\mathbf{x}'_i\boldsymbol{\beta} + \Delta u_i
\end{aligned}
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- Due to 'no perfect collinearity': $\mathbf{x}_{it}$ has to change over time for some units. High variance if its slow moving.
- Differencing will reduce the variation in the independent variables and thus increase standard errors.

First Differences in R (via `plm` package)

```
library(plm)

fd.mod <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur), data = ross,
                      index = c("id", "year"), model = "fd")
summary(fd.mod)

## Oneway (individual) effect First-Difference Model
##
## Call:
## plm(formula = log(kidmort_unicef) ~ democracy + log(GDPcur),
##     data = ross, model = "fd", index = c("id", "year"))
##
## Unbalanced Panel: n=166, T=1-7, N=649
##
## Residuals :
##    Min. 1st Qu.  Median 3rd Qu.    Max.
## -0.9060 -0.0956  0.0468  0.1410  0.3950
##
## Coefficients :
##              Estimate Std. Error  t-value Pr(>|t|)
## (intercept) -0.149469   0.011275 -13.2567  < 2e-16 ***
## democracy   -0.044887   0.024206  -1.8544  0.06429 .
## log(GDPcur) -0.171796   0.013756 -12.4886  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Total Sum of Squares:    23.545
## Residual Sum of Squares: 17.762
## R-Squared      :  0.24561
##       Adj. R-Squared :  0.24408
## F-statistic: 78.1367 on 2 and 480 DF, p-value: < 2.22e-16
```

We Covered

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Next Time: Difference-in-Differences

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Motivation: Studying the Minimum Wage

Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania

By DAVID CARD AND ALAN B. KRUEGER*

On April 1, 1992, New Jersey's minimum wage rose from $4.25 to $5.05 per hour. To evaluate the impact of the law we surveyed 410 fast-food restaurants in New Jersey and eastern Pennsylvania before and after the rise. Comparisons of employment growth at stores in New Jersey and Pennsylvania (where the minimum wage was constant) provide simple estimates of the effect of the higher minimum wage. We also compare employment changes at stores in New Jersey that were initially paying high wages (above $5) to the changes at lower-wage stores. We find no indication that the rise in the minimum wage reduced employment. (*JEL* J30, J23)

https://www.jstor.org/stable/2118030

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- Based on survey data:
  - Wave 1: March 1992, one month before the minimum wage increased
  - Wave 2: December 1992, eight months after increase

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- Idea: compare employment rates in 410 fast-food restaurants in NJ and eastern PA (where there wasn't a wage increase) both before and after the change.
- Based on survey data:
  - Wave 1: March 1992, one month before the minimum wage increased
  - Wave 2: December 1992, eight months after increase
- "What would a skeptic consider convincing evidence?" David Card
- "There was a time when we thought econometric techniques would solve a lot of the data problems. Now I think the feeling is that there are a lot of problems for which it is easier to get better data." Alan Krueger

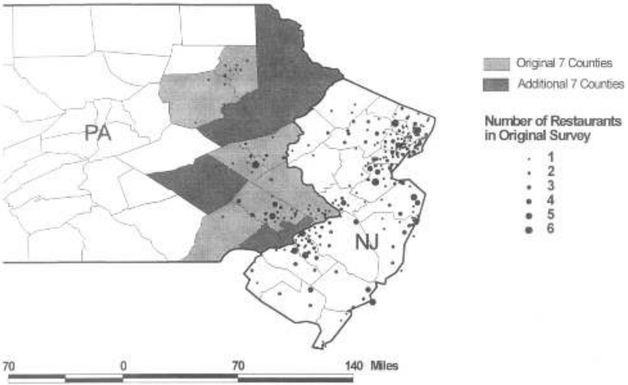

FIGURE 1. AREAS OF NEW JERSEY AND PENNSYLVANIA COVERED BY ORIGINAL SURVEY AND BLS DATA

Source: Card and Krueger 2000

Conversations / David Card and Alan Krueger

Two Economists Catch Clinton's Eye By Bucking the Common Wisdom

They use control groups in their research, and test minimum-wage theories by surveying the managers of fast-food restaurants.

'Let's Not Assume'

The New Jersey Test

Difference-in-Differences

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- $I(t = 2)$ is a dummy variable for the second time period
- β_1 is the quantity of interest: it's the effect of being treated

Diff-in-Diff Mechanics

- Let's take differences:

$$(y_{i2} - y_{i1}) \;\; = \;\; \delta_0(1-0) + \beta_1(x_{i2} - x_{i1}) + (a_i - a_i) + (u_{i2} - u_{i1})$$

- This represents
  - δ_0: the difference in the average outcome from period 1 to period 2 in the untreated group
  - $(x_{i2} - x_{i1}) = 1$ for the treated group and 0 for the control group
  - β_1 represents the additional change in y over time (on top of δ_0) associated with being in the treatment group.

Diff-in-Diff Mechanics

- Let's take differences:

$$
\begin{aligned}
(y_{i2} - y_{i1}) &= \delta_0(1 - 0) + \beta_1(x_{i2} - x_{i1}) + (a_i - a_i) + (u_{i2} - u_{i1}) \\
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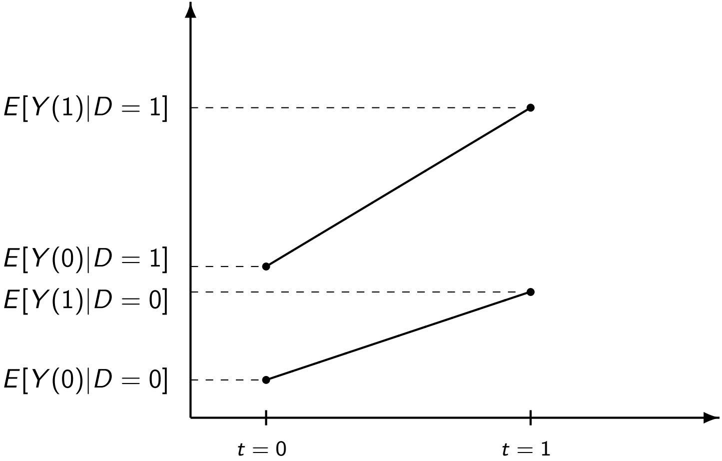
\end{aligned}
$$

- This represents
  - δ_0: the difference in the average outcome from period 1 to period 2 in the untreated group
  - $(x_{i2} - x_{i1}) = 1$ for the treated group and 0 for the control group
  - β_1 represents the additional change in y over time (on top of δ_0) associated with being in the treatment group.

Graphical Representation: Difference-in-Differences

Define $D = 1$ when $x_{i2} - x_{i1} = 1$ and 0 otherwise



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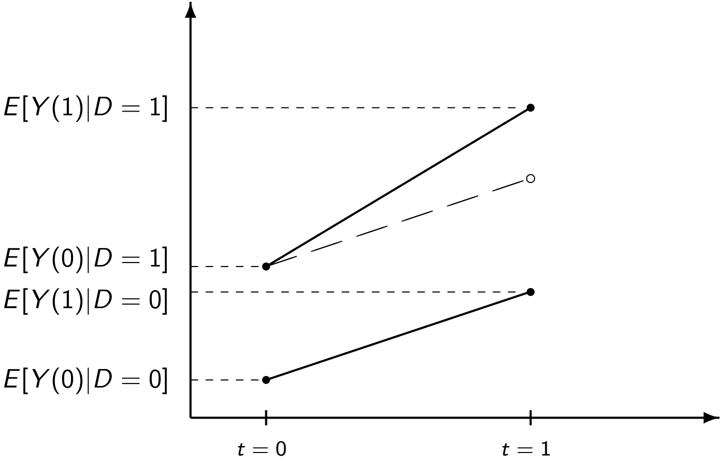

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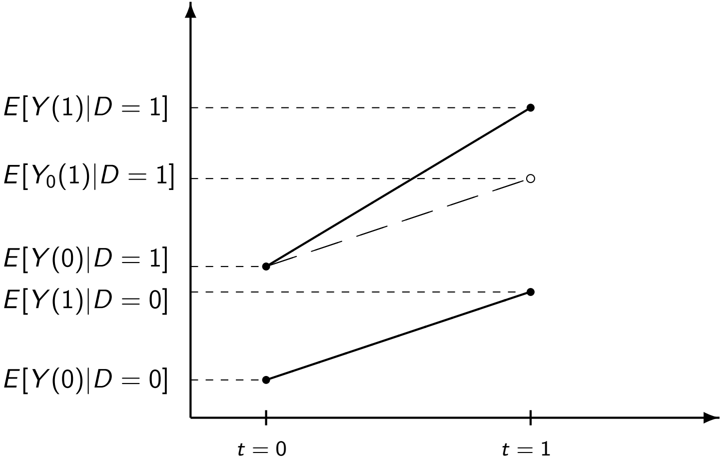

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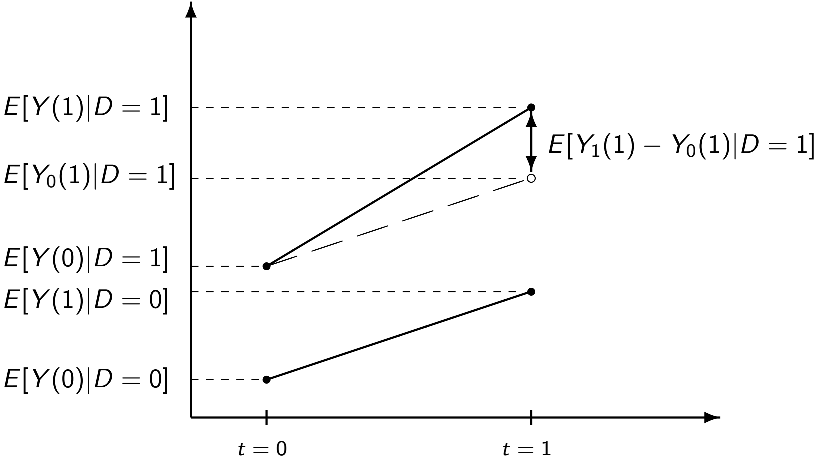

Identification with Difference-in-Differences

Identification Assumption (parallel trends)

$E[Y_0(1) - Y_0(0)|D = 1] = E[Y_0(1) - Y_0(0)|D = 0]$

Identification Result

Given parallel trends the ATT is identified as:

$$\begin{aligned} E[Y_1(1) - Y_0(1)|D = 1] &= \Big\{E[Y(1)|D = 1] - E[Y(1)|D = 0]\Big\} \\ &- \Big\{E[Y(0)|D = 1] - E[Y(0)|D = 0]\Big\} \end{aligned}$$

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Proof.

Note that the identification assumption implies
$E[Y_0(1)|D = 0] = E[Y_0(1)|D = 1] - E[Y_0(0)|D = 1] + E[Y_0(0)|D = 0]$
plugging in we get

$$
\begin{aligned}
& \{E[Y(1)|D = 1] - E[Y(1)|D = 0]\} - \{E[Y(0)|D = 1] - E[Y(0)|D = 0]\} \\
= \; & \{E[Y_1(1)|D = 1] - E[Y_0(1)|D = 0]\} - \{E[Y_0(0)|D = 1] - E[Y_0(0)|D = 0]\} \\
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= \; & E[Y_1(1) - Y_0(1)|D = 1] + \{E[Y_0(0)|D = 1] - E[Y_0(0)|D = 0]\} \\
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- DiD works for additive and time-invariant confounding (i.e. satisfies parallel trends)

Example: Lyall (2009)

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Does Indiscriminate Violence Incite Insurgent Attacks?

Evidence from Chechnya

Jason Lyall
Department of Politics and the Woodrow Wilson School
Princeton University, New Jersey

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- Counterintuitive findings: shelled villages experience a 24% reduction in insurgent attacks relative to controls.

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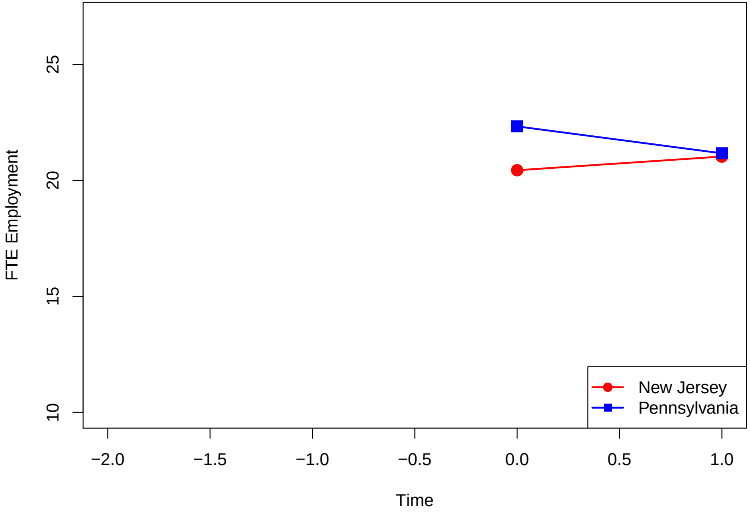
- NJ_i indicates which stores received the treatment of a higher minimum wage at time period $t = 2$

Parallel Trends?



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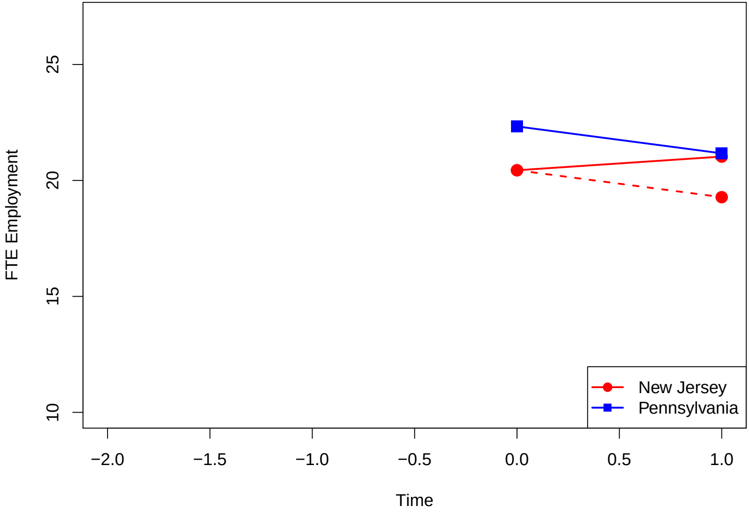

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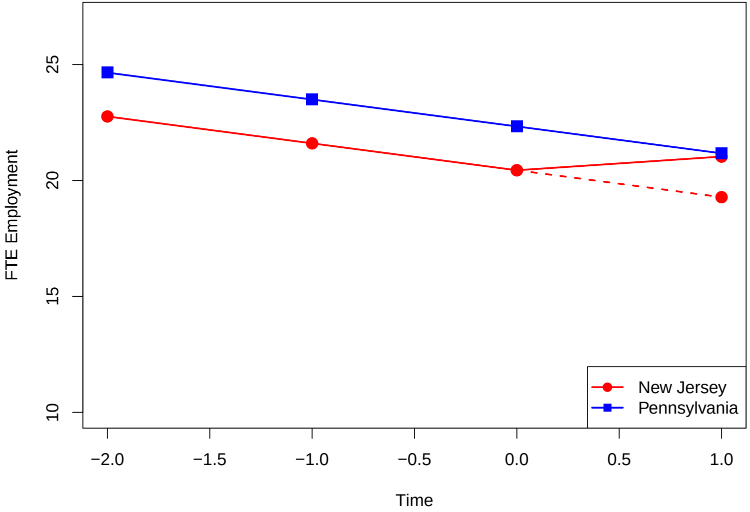

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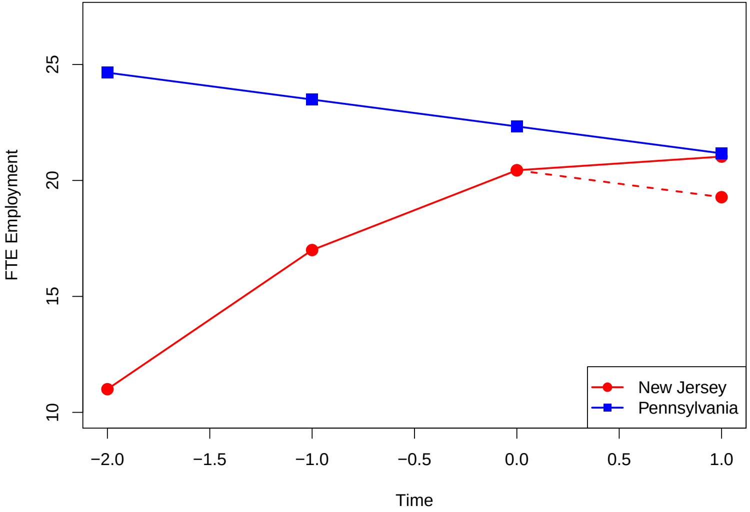
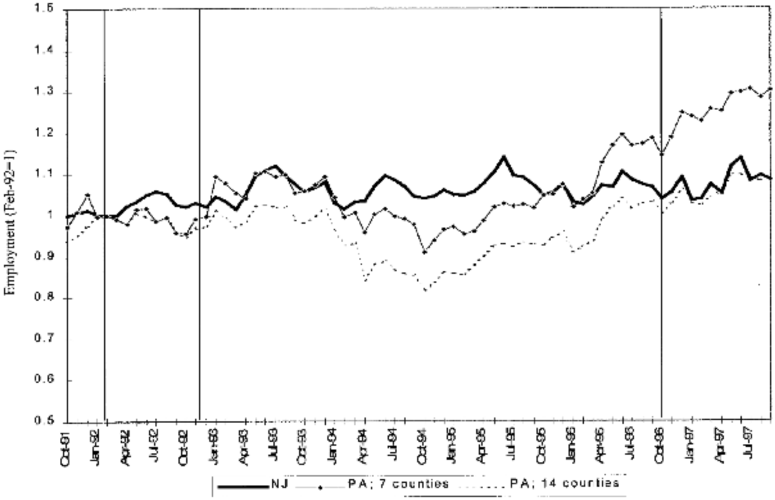


Longer Trends in Employment (Card and Krueger 2000)



First two vertical lines indicate the dates of the Card-Krueger survey. October 1996 line is the federal minimum wage hike which was binding in PA but not NJ

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4) Long-term vs. Short-term Effects
parallel trends are less credible over a long time horizon, but many policies need time to take effect.

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6) Endogenous Control Variables
can add (time-varying) covariates to help with some of above concerns $\rightsquigarrow$ "regression diff-in-diff"

$$y_{i2} - y_{i1} = \delta_0 + \mathbf{z}_i'\tau + \beta(x_{i2} - x_{i1}) + (u_{i2} - u_{i1})$$

but need to be careful that they aren't affected by the treatment.

Concluding Thoughts on Panel Differencing Models

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What to read next?

- Angrist and Pishke Chapter 5 Parallel Worlds: Fixed Effects, Differences-in-Differences and Panel Data
- Morgan and Winship Chapter 11 Repeated Observations and the Estimation of Causal Effects

We Covered

- Difference-in-Differences
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Next Time: Fixed Effects

Where We've Been and Where We're Going...

- Last Week
  - causal inference with unmeasured confounding
- This Week
  - panel data
  - diff-in-diff
  - fixed effects
  - wrap-up
- The Following Week
  - ?
- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causality

Basic Model Review

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- We discussed a differencing approach to this model
- The Fixed effects model is an alternative way to remove time-invariant unmeasured confounding
- We will start by assuming the model and discussing properties and in the next section, we will consider non-parametric identification.

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- Key fact: because it is time-constant the mean of a_i is just a_i

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&= \left(\frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_{it}'\right)\boldsymbol{\beta} + \frac{1}{T}\sum_{t=1}^{T} a_i + \frac{1}{T}\sum_{t=1}^{T} u_{it} \\
&= \overline{\mathbf{x}}_i'\boldsymbol{\beta} + a_i + \overline{u}_i
\end{aligned}$$

- Key fact: because it is time-constant the mean of a_i is just a_i
- This regression is sometimes called the "between regression"

Within Transformation

Within Transformation

- The "fixed effects," "within," or "time-demeaning" transformation is when we subtract off the over-time means from the original data:

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- We are now modeling observations as deviation from their group mean.
- NB: you must demean the X variables not just the Y variables.

Fixed Effects with Ross data

```
fe.mod <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur), data = ross, index = c("id", "year"),
 model = "within")
summary(fe.mod)
```

```
## Oneway (individual) effect Within Model
##
## Call:
## plm(formula = log(kidmort_unicef) ~ democracy + log(GDPcur),
##     data = ross, model = "within", index = c("id", "year"))
##
## Unbalanced Panel: n=166, T=1-7, N=649
##
## Residuals :
##     Min.  1st Qu.   Median  3rd Qu.     Max.
## -0.70500 -0.11700  0.00628  0.12200  0.75700
##
## Coefficients :
##              Estimate Std. Error  t-value  Pr(>|t|)
## democracy   -0.143233   0.033500  -4.2756 2.299e-05 ***
## log(GDPcur) -0.375203   0.011328 -33.1226 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Total Sum of Squares:    81.711
## Residual Sum of Squares: 23.012
## R-Squared      :  0.71838
##       Adj. R-Squared :  0.53242
## F-statistic: 613.481 on 2 and 481 DF, p-value: < 2.22e-16
```

Strict Exogeneity

- FE models are valid if $E[\mathbf{u}|\mathbf{X}] = 0$: all errors are uncorrelated with covariates in every period:

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- This is because the composite errors, $\ddot{u}_{it}$ are function of the errors in every time period through the average, $\overline{u}_i$
- This rules out, for instance, lagged dependent variables, since $y_{i,t-1}$ has to be correlated with $u_{i,t-1}$. Thus it can't be a covariate for y_{it}.

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- Basic message: any time-constant variable gets "absorbed" by the fixed effect. It has nothing to contribute because the comparison is within the units.
- Can include interactions between time-constant and time-varying variables, but lower order term of the time-constant variables get absorbed by fixed effects too

Time-constant variables

- Pooled model with a time-constant variable, proportion Islamic:

```
library(lmtest)
p.mod <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur) + islam,
             data = ross, index = c("id", "year"), model = "pooling")
coeftest(p.mod)

##
## t test of coefficients:
##
##                Estimate  Std. Error  t value  Pr(>|t|)
## (Intercept) 10.30607817  0.35951939  28.6663 < 2.2e-16 ***
## democracy   -0.80233845  0.07766814 -10.3303 < 2.2e-16 ***
## log(GDPcur) -0.25497406  0.01607061 -15.8659 < 2.2e-16 ***
## islam        0.00343325  0.00091045   3.7709 0.0001794 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Time-constant variables

- FE model, where the islam variable drops out, along with the intercept:

```
fe.mod2 <- plm(log(kidmort_unicef) ~ democracy + log(GDPcur) + islam,
               data = ross, index = c("id", "year"), model = "within")
coeftest(fe.mod2)

##
## t test of coefficients:
##
##              Estimate Std. Error  t value  Pr(>|t|)
## democracy   -0.129693   0.035865  -3.6162 0.0003332 ***
## log(GDPcur) -0.379997   0.011849 -32.0707 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Alternate Computation: Least Squares Dummy Variable

- As an alternative to the within transformation, we can also include a series of $n-1$ dummy variables for each unit:

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta} + d_i^{(1)}\alpha_1 + d_i^{(2)}\alpha_2 + \cdots + d_i^{(n)}\alpha_n + u_{it}$$

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- Why are these equivalent? (remember partialing out strategy and Frisch-Waugh-Lovell theorem)

Example with Ross data

```
library(lmtest)
lsdv.mod <- lm(log(kidmort_unicef) ~ democracy + log(GDPcur) +
               as.factor(id), data = ross)
coeftest(lsdv.mod)[1:6,]
coeftest(fe.mod)[1:2,]

##                     Estimate Std. Error    t value      Pr(>|t|)
## (Intercept)       13.7644887 0.26597312  51.751427 1.008329e-198
## democracy         -0.1432331 0.03349977  -4.275644  2.299393e-05
## log(GDPcur)       -0.3752030 0.01132772 -33.122568 3.494887e-126
## as.factor(id)AGO   0.2997206 0.16767730   1.787485  7.448861e-02
## as.factor(id)ALB  -1.9309618 0.19013955 -10.155498  4.392512e-22
## as.factor(id)ARE  -1.8762909 0.17020738 -11.023558  2.386557e-25

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    - ⋆ Twin fixed effects to control for unobserved effects of family background
  - Cluster fixed effects in hierarchical data
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  - if $u_{it} = u_{i,t-1} + e_{it}$ with e_{it} iid (random walk) $\rightsquigarrow$ FD is more efficient.

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- Note that when the second dimension isn't time, fixed effects will be relevant more often.

We Covered

- Fixed Effects!
- Computation for Fixed Effects!

Next Time: Non-parametric Identification and Fixed Effects

Where We've Been and Where We're Going...

- Last Week
  - causal inference with unmeasured confounding
- This Week
  - panel data
  - diff-in-diff
  - fixed effects
  - wrap-up
- The Following Week
  - ?
- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causality

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  - strict exogeneity
- We've seen the trouble with constant effects before, it goes back to Lecture 10 and results on regression with heterogenous treatment effects more generally.

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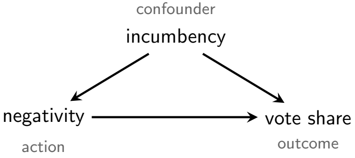

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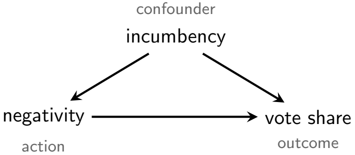

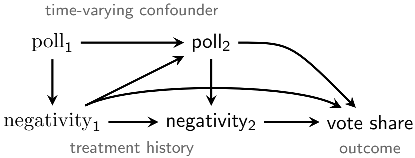

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INTRODUCTION

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INTRODUCTION


AJPS

When Should We Use Unit Fixed Effects Regression Models for Causal Inference with Longitudinal Data?

Core Conundrum

There is a (possibly irresolvable) tension: modeling causal dynamics between treatment and outcomes OR addressing unobserved time-invariant confounders. Three great recent papers:

We are going to focus on addressing unobserved time-invariant confounders using the last paper.

Next several slides are based on slides graciously provided by In Song Kim and Kosuke Imai.

Directed Acyclic Graph (DAG)

Non-parametric identification assumptions for fixed effects:

$$Y_{it} \quad = \quad g(X_{it}, \mathbf{U}_i, \epsilon_{it}) \quad \text{and} \quad \epsilon_{it} \perp\!\!\!\perp \{\mathbf{X}_i, \mathbf{U}_i\}$$

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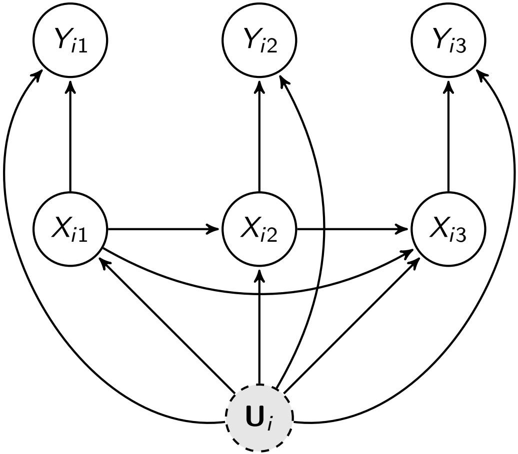

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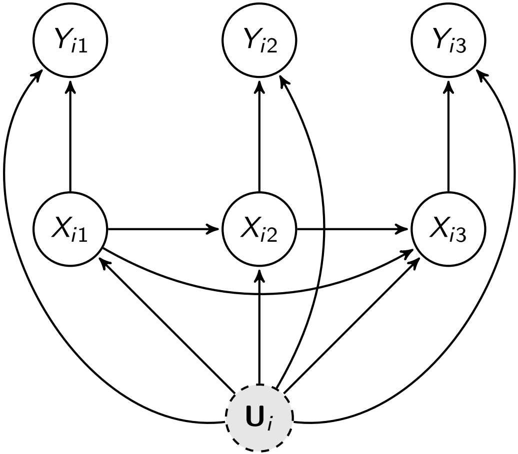

Assumptions:

1. No unobserved time-varying confounders

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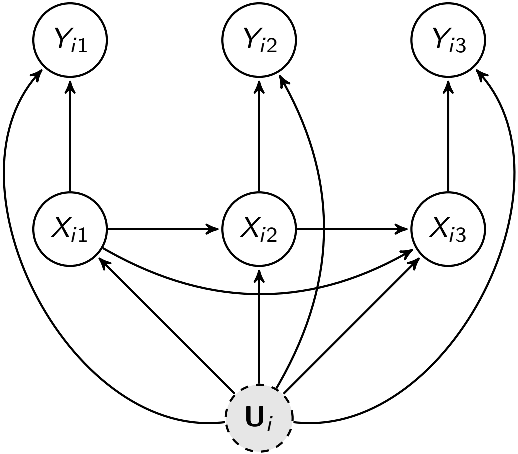

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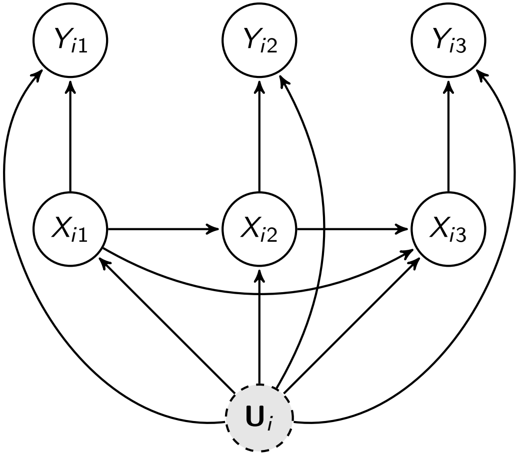

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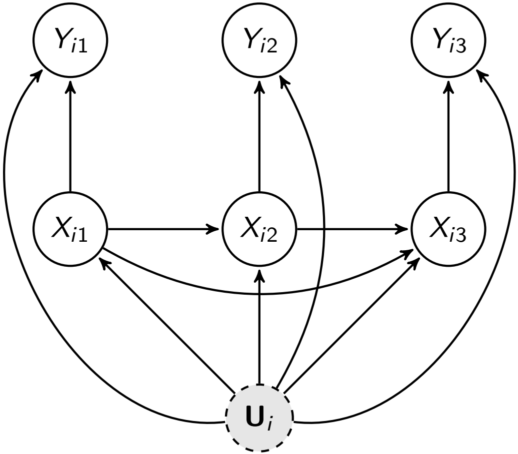

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the result implies that the counterfactual outcome for a treated observation in a given time period is estimated using the observed outcomes of different time periods of the same unit. Since such a comparison is valid only when no causal dynamics exist, this finding underscores the important limitation of linear regression models with unit fixed effects.

- Imai and Kim (2019)

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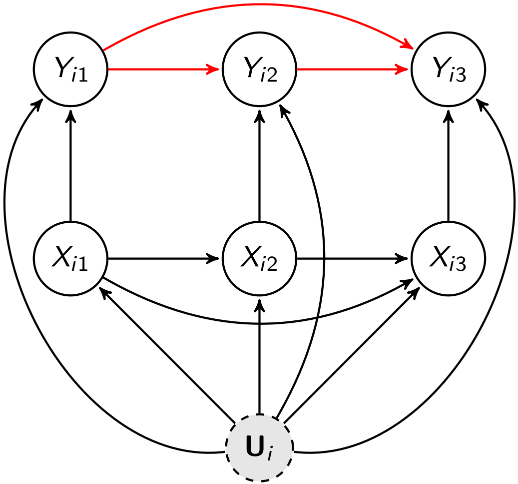
- Now let's consider each assumption in turn.

Past Outcomes Don't Directly Affect Current Outcome (A2)



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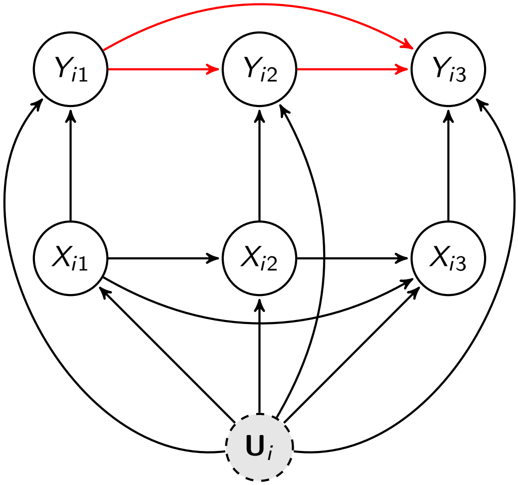

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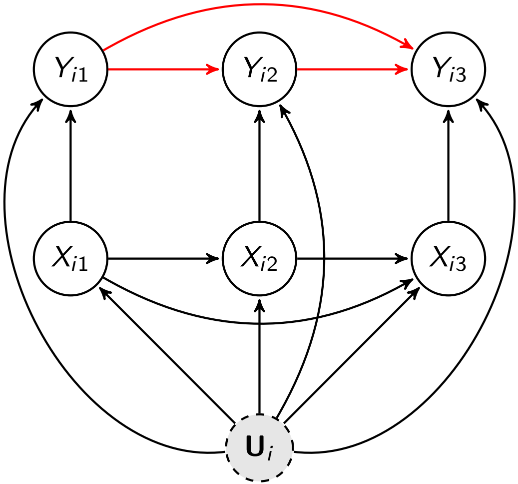

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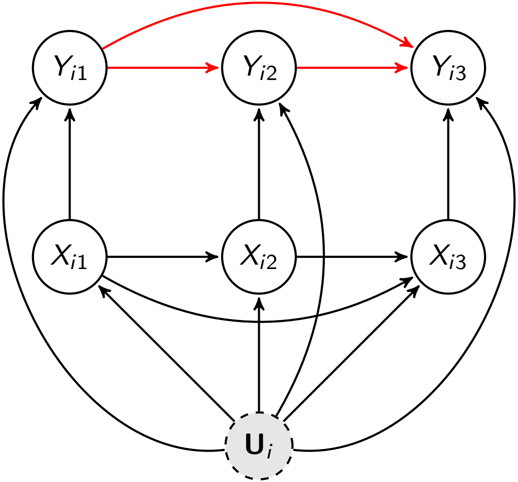

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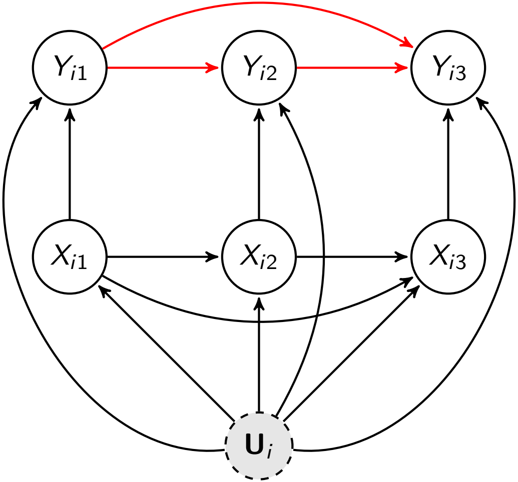

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- Conclusion: The assumption can be relaxed

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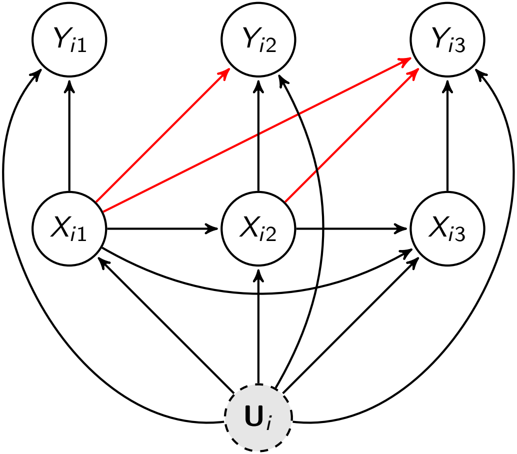

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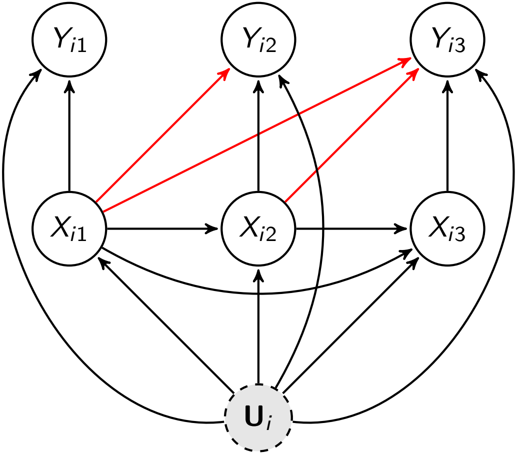

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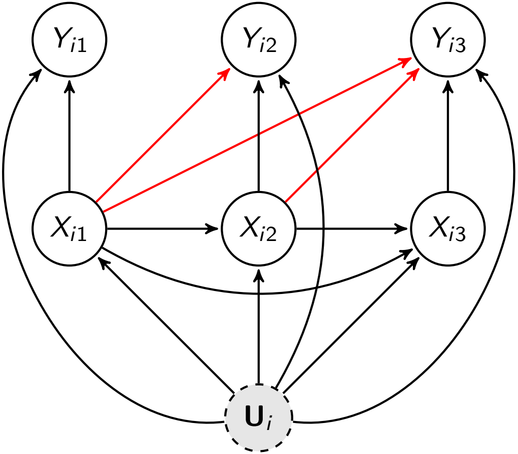

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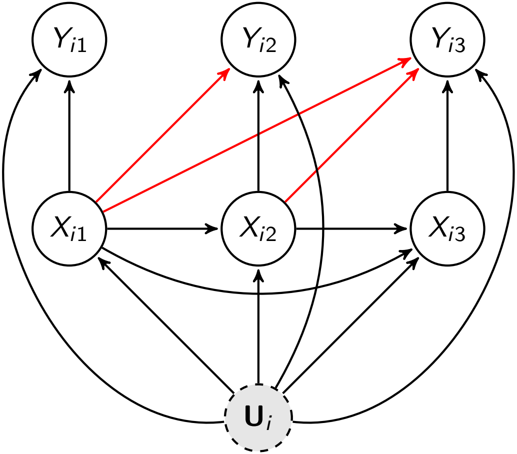

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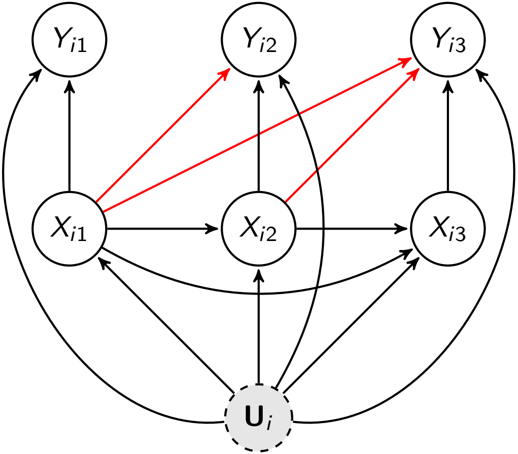

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- Conclusion: The assumption can be partially relaxed

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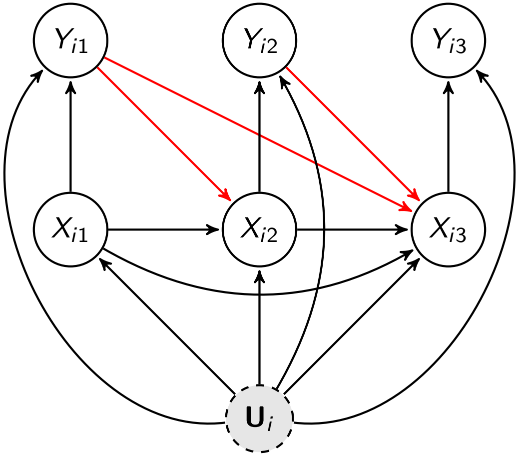

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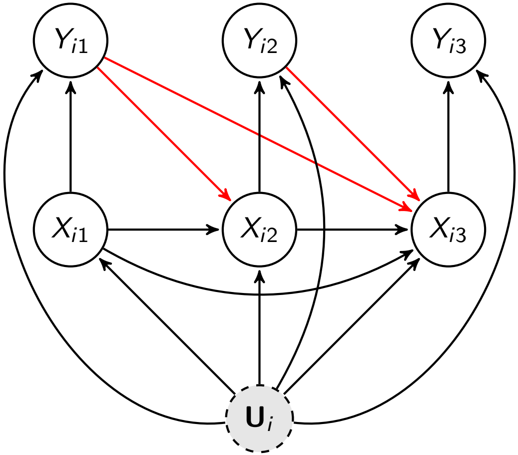

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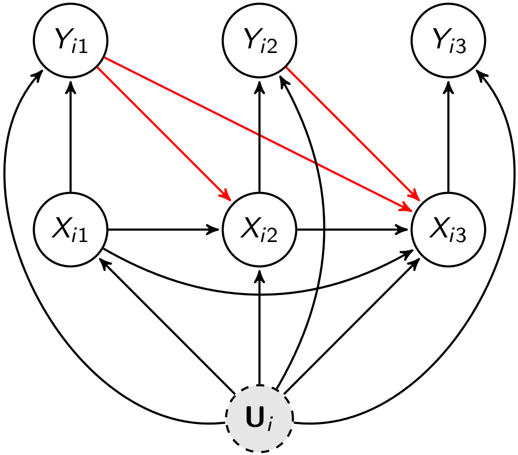

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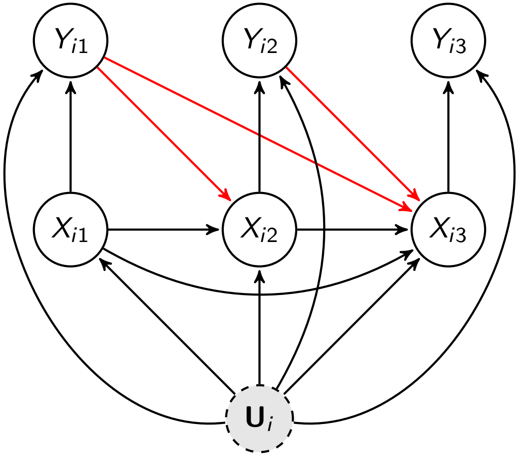

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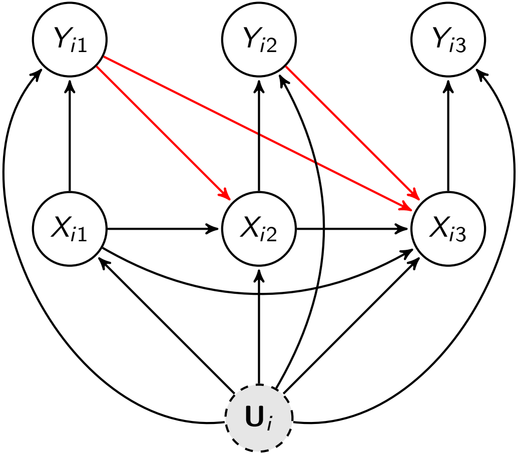

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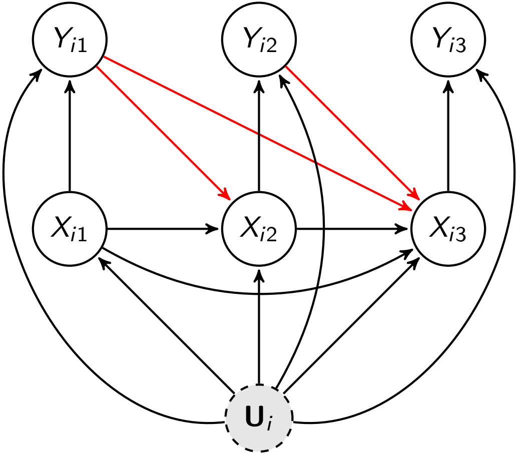

- Correlation between error term and future treatments
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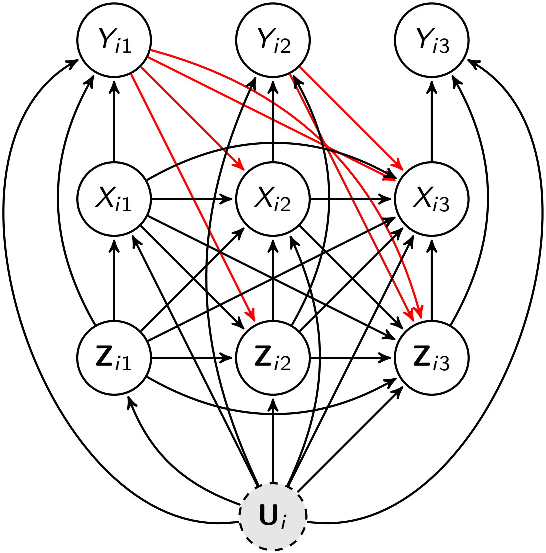

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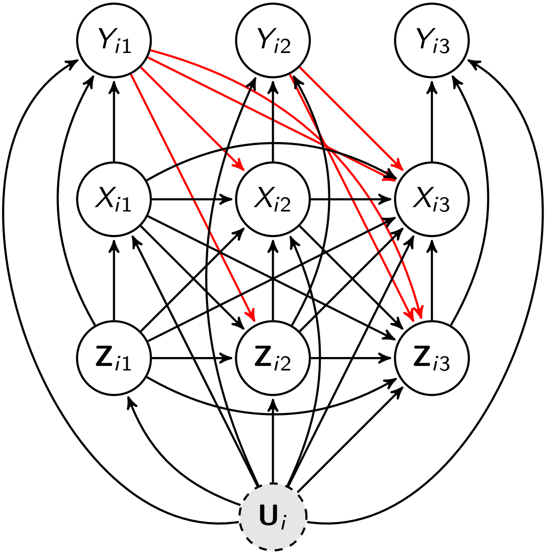

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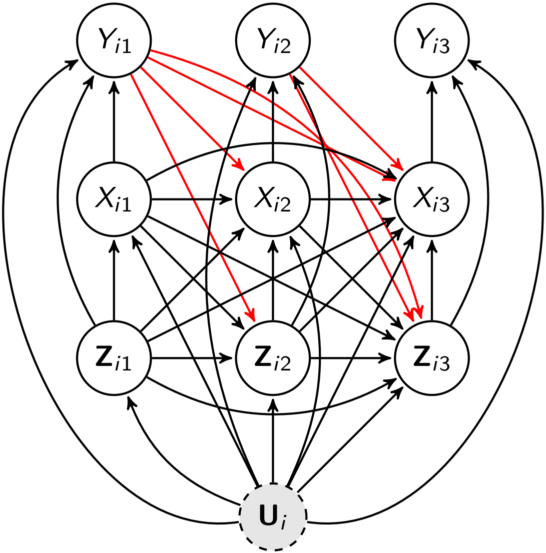

- $Y_{it} = \alpha_i + \beta X_{it} + \gamma^\top \mathbf{Z}_{it} + \epsilon_{it}$
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- past outcomes cannot *indirectly* affect current treatment through $\mathbf{Z}_{it}$

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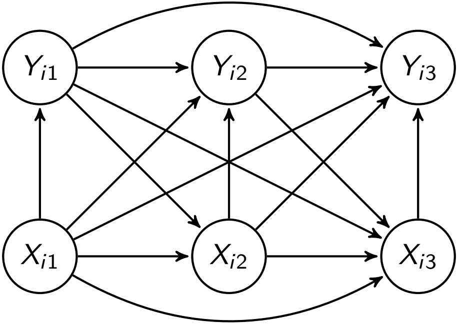

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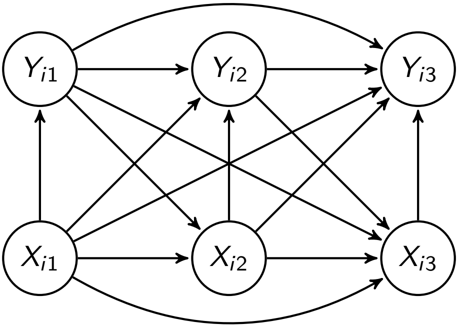

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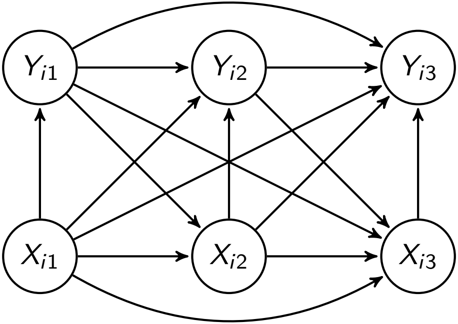

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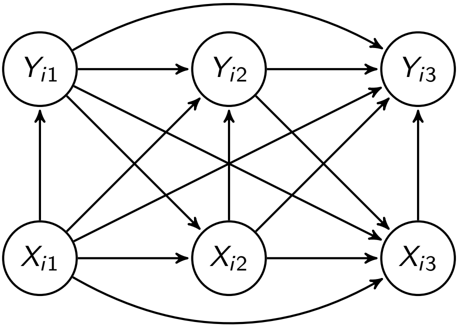

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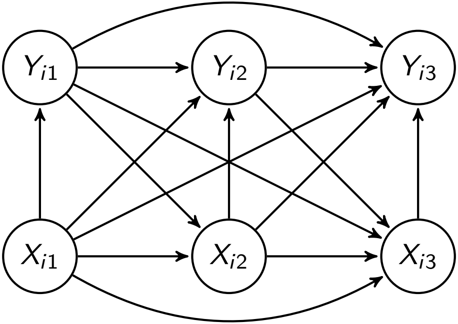

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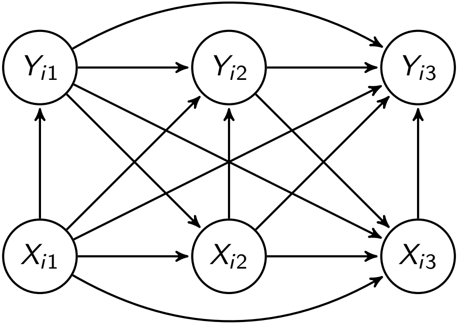

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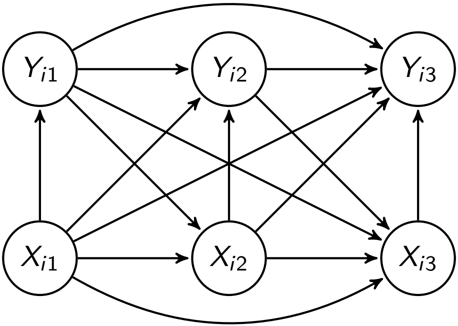

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- Trade-off $\rightsquigarrow$ no free lunch

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    2) causal dynamics between treatment and outcome $\rightsquigarrow$ selection-on-observables

Summary Table (Imai and Kim 2019)

TABLE 1 Identification Assumptions of Various Estimators

	Linearity	Time-Invariant Unobservables	Past Outcomes Affect Current Treatment	Past Treatments Affect Current Outcome
$Y_{it} = \alpha_i + \beta X_{it} + \epsilon_{it}$	Yes	Allowed	Not allowed	Not allowed
$Y_{it} = \alpha_i + \beta X_{it} + \rho Y_{i,t-1} + \epsilon_{it}$	Yes	Allowed	Allowed	Not allowed
$Y_{it} = \alpha_i + \beta_1 X_{it} + \beta_2 X_{i,t-1} + \epsilon_{it}$	Yes	Allowed	Not allowed	Allowed
$Y_{it} = \alpha_i + \beta_1 X_{it} + \beta_2 X_{i,t-1} + \rho Y_{i,t-1} + \epsilon_{it}$	Yes	Allowed	Partially allowed	Partially allowed
Marginal structural models	No	Not allowed	Allowed	Allowed

What to read next?

- Morgan and Winship Chapter 11 Repeated Observations and the Estimation of Causal Effects
- Imai and Kim (2019) “When Should We Use Unit Fixed Effects Regression Models for Causal Inference with Longitudinal Data?” *American Journal of Political Science*, `http://dx.doi.org/10.1111/ajps.12417`

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Next Time: Review

Where We've Been and Where We're Going...

- Last Week
  - causal inference with unmeasured confounding
- This Week
  - panel data
  - diff-in-diff
  - fixed effects
  - wrap-up
- The Following Week
  - ?
- Long Run
  - probability $\rightarrow$ inference $\rightarrow$ regression $\rightarrow$ causality

Q: What conditions do we need to infer causality?

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A: A clear estimand, an identification strategy and an estimation strategy.

Identification Strategies in This Class

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Almost everything assumes: consistency/SUTVA (no interference between units, variation in the treatment is irrelevant) and positivity (there is some chance of all getting treatment)

Some Estimation Strategies

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- Stratification

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- Stratification
- Regression (and relatives)
- Matching (lightly covered)
- Weighting (not covered)

Q: Can you review how instrumental variables deal with issues of confounding?

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A: We use only the units whose treatment status was effectively randomized by the instrument (because they are compliers).

Q: What can make standard errors larger or smaller?

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A: Let's consider the anatomy of a standard error.

Anatomy of the Standard Error

Imagine we have a regression of Y on a variable of interest X and a vector of other variables $\mathbf{Z}$.

$$\widehat{\text{Var}}(\widehat{\beta}_X) = \frac{\frac{1}{(n-k-1)}\sum_{i=1}^{n}\hat{u}_i^2}{(1-R^2_{X\sim\mathbf{Z}})\sum_{i=1}^{n}(X_i-\overline{X})^2}$$

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- the numerator is our estimator for σ_u^2 the unknown error variance. It is formed by the degrees of freedom correction times the sum of the squared residuals.

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Further Reading:

- Lin, W., 2013. 'Agnostic notes on regression adjustments to experimental data: Reexamining Freedman's critique.' *The Annals of Applied Statistics*
- Athey, S. and Imbens, G.W., 2017. 'The Econometrics of Randomized Experiments.' In *Handbook of Economic Field Experiments* (Vol. 1, pp. 73-140).
- Egap Methods Guide: 10 things you need to know about covariate adjustment. `https://egap.org/methods-guides/10-things-know-about-covariate-adjustment`

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4. Try to explain it to someone else

Q: In the real world of research, when you have your data, how do you know which method to use? For example, how do you know that you need to use matching/regression discontinuity?

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A: Let's chat.

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Where are you?

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You've been given a powerful set of tools

Your New Weapons

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  - Probability axioms, random variables, marginal and conditional probability, building a probability model
  - Expected value, variances, independence
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- Univariate Inference
  - Interval estimation (normal and non-normal Population)
  - Confidence intervals, hypothesis tests, p-values
  - Practical versus statistical significance

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  - idea of conditioning
  - kernel and loess regressions
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- Multiple Regression
  - OLS estimator for multiple regression in matrix notation
  - Regression assumptions (classical and agnostic)
  - Properties: Bias, Efficiency, Consistency
  - Standard errors, testing, p-values, and confidence intervals
  - Polynomials, Interactions, Dummy Variables
  - F-tests

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- And you learned how to use `R`: you're not afraid of trying something new!

Using these Tools

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So, Admiral Ackbar, now that you've learned how to run these regressions we can just use them blindly, right?


IT'S A TRAP!

Beyond Linear Regressions

You need more training

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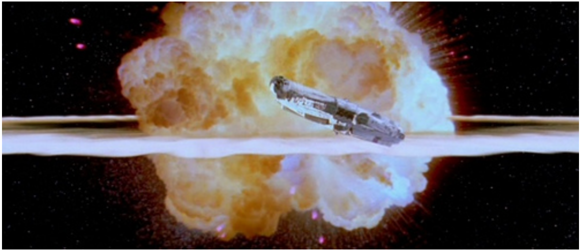
I've tried to offer a perspective on technique in addition to just a list of methods. See Lundberg, Johnson and Stewart "What's Your Estimand? Defining the Target Quantity Connects Statistical Evidence to Theory"

Thanks!

Thanks so much for an amazing semester.



I will see you in the final synchronous session!